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Logic's Little Words

As Newton's name is associated with the law of gravitation, so Aristotle's is associated with the law of contradiction. As Einstein's name is to the theory of relativity, so Aristotle's is to the theory of the syllogism. Two words lie at the heart of the law of contradiction: "is" and "is not." Two pairs of words are central to the theory of the syllogism—Aristotle's account of correct and incorrect reasoning. They are "if" and "then," "since" and "therefore."

As a rule of thought, the law of contradiction tells us primarily what *not* to do. It is a law *against* contradiction, a law that commands us to *avoid* contradicting ourselves, either in our speech or in our thought. It tells us that we should not answer a question by saying both yes and no. Stated in another way, it tells us that we should not affirm and deny the same proposition. If I say or think that Plato *was* Aristotle's teacher, I should avoid saying or thinking that Plato *was not* Aristotle's teacher. To say or think that would be to deny something that I have affirmed.

You may ask why this rule of thought is so basic and so sound. Aristotle's answer is that the law of contradiction is not only a rule of thought but also a statement about the world itself—about the realities we try to think about.

The law of contradiction, as a statement about reality, says what is immediately obvious to common sense. A thing—whatever it may be—cannot both exist and not exist at the same time. It either exists or it does not exist, but not both at once. A thing cannot have a certain attribute and not have that attribute at the same time. The apple in my hand that I am looking at cannot, at this instant, be both red in color and not red in color.

This is so very obvious that Aristotle calls the law of contradiction self-evident. Its self-evidence, for him, means its undeniability. It is impossible to think that the apple is both red and not red at the same time, just as it is impossible to think that a part is greater than the whole to which it belongs. It is impossible to think that a tennis ball that you hit over the fence is to be found in the grass that lies beyond and, at the same time, to think that it cannot be found there because it no longer exists.

The law of contradiction as a statement about reality itself underlies the law of contradiction as a rule of thought. The law of contradiction as a statement about reality *describes* the way things are. The law of contradiction as a rule of thought *prescribes* the way we should think about things if we wish our thinking about them to conform to the way things are.

When a pair of statements are contradictory, both cannot be true, nor can both be false. One must be true, the other false.

Plato either was or was not Aristotle's teacher. All swans are white or some are not. However, if instead of saying that some swans are not white, which contradicts the statement that all swans are white, I had said no swans are white, a contradiction would not have resulted. People who are not acquainted with Aristotle's distinction between contradictory and contrary statements may be surprised by this.

It is possible for both of these statements— "All swans are white" and "No swans are white"—to be false, though both cannot be true. Some swans may be white and some black, in which case it is false to say that all swans are white or that none is. Aristotle calls a pair of statements contrary, not contradictory, when both cannot be true, but both can be false.

Is there a pair of statements, both of which can be true, but both of which cannot be false? Yes, according to Aristotle, the statement that some swans are white and the statement that some swans are not white can both be true, but both cannot be false. Swans must be either white or not white, and so if only some are white, some must be not white. Aristotle calls this pair of statements subcontrary.

Suppose, however, that instead of saying that some swans are white and some swans are not white, I had said "Some swans are white" and "Some swans are black." Would that pair of statements have been subcontrary—impossible for both to be false? No, because some swans might be gray, or green, yellow, or blue. *White* and *black* are not exclusive alternatives. It is not true that any visible object must be either white or black.

This being the case, it will not do to pose as the contrary of "All swans are white" the statement "All swans are black," for neither may be true and both can be false. To state the contrary of "All swans are white," one must say "No swans are white," not "All swans are black."

Unlike "black" and "white," some pairs of terms, which are contrary terms, do exhaust the alternatives. For example, all integers or whole numbers are either odd or even. There is no third possibility. When one uses terms that are exclusive alternatives, it is possible to state a contradiction without using "is" and "is not." The statement that any given whole number is an odd number is contradicted by the statement that that number is an even number, because if it is odd, it is not even, and if it is even, it is not odd, and it must be one or the other.

I cannot exaggerate the importance of Aristotle's rules concerning statements that are incompatible with one another in one of these three ways—through being contradictory of one another, through being contrary to one another, or subcontrary to one another. The importance is that observing these rules not only helps us to avoid making inconsistent statements but also helps us to detect inconsistencies in the statements made by others and to challenge what they say.

When a person we are conversing with contradicts himself or herself or makes contrary statements, we have every right to stop him and say, "You cannot make both of those statements. Both cannot be true. Which of the two do you really mean? Which do you want to claim as true?"

It is particularly important to observe that general statements—statements containing the word "all"—can be contradicted by a single negative instance. To contradict the generalization that all swans are white, one needs only to point to a single swan that is not white. That single negative instance falsifies the generalization.

Scientific generalizations are put to the test in this way. The claim that they are true can be upheld only so long as no negative instances are found to falsify them. Since the search for negative instances is an unending one, a scientific generalization can never be regarded as finally or completely verified.

Human beings are prone to generalize, especially in their thinking about other human beings who differ from themselves in sex, race, or religion. If they are men, they will permit themselves to say—unthinkingly, one hopes—that all women are such and such. If they are white persons, they will permit themselves to say that all blacks are so and so. If they are Protestants, they will permit themselves to say that all Catholics are this or that. In every one of these cases, one negative instance suffices to invalidate the generalization; and the more negative instances one can point to, the easier it is to show how wild the generalization was in the first place.

The use of contrary terms, such as "black" and "white," or "odd" and "even," brings into play another set of words that control our thinking according to certain rules—"either-or" and "not both." For example, when we toss a coin to decide something, we know that when it lands, it must be either heads or tails, not both. That is a strong disjunction. There are, however, weak disjunctions, in which something may be either this or that, and perhaps both, though not in the same respect or at the same time. To say of tomatoes that they are either red or green permits us to say that one and the same tomato can be both red and green, but at different times.

Disjunctions, especially strong disjunctions, enable us to make simple, direct inferences. If we know that a whole number is not odd, we can infer immediately that it must be even. Similarly, if we know that a whole number is not a prime number, we can infer immediately that it must be divisible by numbers other than itself and one. When we see that the tossed coin has landed heads up, we know at once that we, who bet on tails, have lost the toss. We do not have to turn the coin over to be sure of that.

Inferences of this sort Aristotle calls immediate inferences because one goes immediately from the truth or falsity of one statement to the truth or falsity of another. No steps of reasoning are involved. If one knows that it is true that all swans are white, one also knows immediately that some swans are white; and in addition one knows that at least some white objects are swans.

One can make mistakes in this simple process of inference, and mistakes are frequently made. For example, from the fact that all swans are white, it is correct to infer that some white objects are swans, but quite incorrect to infer that all white objects are swans.

That incorrect inference Aristotle calls an illicit conversion. The class of white objects is larger than the class of swans. Swans are only some of the white objects in the world. To make the mistake of thinking that because all swans are white, we can also say that all white objects are swans is to treat the two classes as coextensive, which they are not.

Two pairs of words are operative in immediate inference as well as in the more complex process of reasoning. They are "if" and "then," and "since" and "therefore." In order to express the logical correctness of an immediate inference (the inference that some swans are white from the fact that all swans are white), we say, "*If* all swans are white, *then* it *must follow* that some swans are white." To express the incorrectness of an illicit conversion, we say, "*If* all swans are white, *then* it *does not follow* that all white objects are swans."

"If-then" statements of these two kinds are statements of logically correct and logically incorrect inferences. The important point to note here is that the truth of these "if-then" statements about logically correct and logically incorrect inferences does not in any way depend upon the truth of the statements connected by "if" and "then."

The statement that all swans are white may in fact be false, and it would still be logically correct to infer that some swans are white, *if*—but only *if*—all are. Even if the statement that all white objects are swans were in fact true instead of false, it would still be logically incorrect to infer that all white objects are swans from the fact that all swans are white.

So much for the use of "if" and "then"—the latter accompanied by the words "it *must* follow" or "it *does not* follow"—to express our recognition of correct and incorrect inferences. What about "since" and "therefore"? When we substitute "since" and "therefore" for "if" and "then," we are actually making the inference that we did not make when we said only "if" and "then."

To stay with the same example that we have been using, I have made no actual inferences about swans or white objects in all the "if-then" statements I have made about them. I do not make an actual inference until I say, "*Since* all swans are white, it *therefore* follows that some swans are white." My assertion that all swans are white enables me to assert that some swans are white.

Only when I make assertions of this kind, connected by "since" and "therefore," does the truth or falsity of my first statement affect the truth or falsity of my second. My inference may be logically correct, but the conclusion of my actual inference may be actually false because my initial statement, introduced by the word "since," is false in fact. The truth may be that no swans are white, and so it was false to conclude that some are, even though it was logically correct to do so.

When I say, "If all swans are white . . . ," I am only saying *if all are*, not *that all are*. But when I say "Since all swans are white . . . ," I am saying *that all are*. Should I be right in making that assertion, I would also be right in asserting that some swans are white.

What has just been said about Aristotle's rules of immediate inference helps me to summarize briefly the rules of reasoning that constitute his theory of the syllogism. Here is a model syllogism:

Major premise: All animals are mortal.
Minor premise: All men are animals.
Conclusion: All men are mortal.

Let us consider two more examples of reasoning syllogistically—from a major and a minor premise to a conclusion. First, this one in which the reasoning is logically valid, but the conclusion is false because the minor premise is false.

Major premise: Angels are neither male nor female.
Minor premise: Some men are angels.
Conclusion: Some men are neither male nor female.

And this one in which a true conclusion follows logically from two true premises.

Major premise: Mammals do not lay eggs.
Minor premise: Human beings are mammals.
Conclusion: Human beings do not lay eggs.

Considering these three different pieces of reasoning, we can observe at once that syllogistic reasoning is more complicated than immediate inference. In immediate inference, we go at once from a single statement to another single statement, and both statements will have the same terms. In syllogistic reasoning, we go from two statements, in which there are three different terms, to a conclusion in which two of these three terms occur.

In the first example above, the three terms in the major and minor premise were "animals," "men," and "mortal." And the two terms in the conclusion were "men" (a term in the minor premise) and "mortal" (a term in the major premise). That is always the case in syllogistic reasoning, and it is always the case that the third term, which occurs in both premises ("animals"), has been dropped out of the conclusion.

Aristotle calls the term that is common to the major and the minor premise the middle term. It is dropped out of the conclusion because it has served its function in the reasoning process. That function is to connect the other two terms with each other. The middle term mediates between them. That is why Aristotle calls syllogistic reasoning mediated as contrasted with immediate inference. In immediate inference, there is no middle term because there is no need of mediation.

I will not bother to spell out how this works in the three examples of syllogistic reasoning just given. You can do that for yourself. The only additional rules that you must note are these. First, that if the major or the minor premise is negative (if it contains some form of "is not" instead of "is," or "no" instead of "all"), then the conclusion must also be negative. You cannot draw an affirmative conclusion if one of the premises is negative.

The second rule is that the middle term must function connectively. Here is an example in which the middle term fails to do so.

Major premise:	No men are by nature beasts of burden.
Minor premise:	No mules are by nature men.
Conclusion:	No mules are by nature beasts of burden.

Not only is the conclusion false in fact, but it is also a logically incorrect conclusion. An affirmative conclusion must be drawn from two affirmative premises, but no conclusion at all can be validly drawn from two negative premises. The reason is that the negative in the major premise excludes all men from the class of things that are by nature beasts of burden; and the negative in the minor premise excludes all mules from the class of men. Hence we cannot correctly infer anything at all about the relation between the class of mules and the class of things that are by nature beasts of burden.

It is interesting to observe in the example just given that the major and minor premises are both true, while the conclusion that does not logically follow from them is false. It is quite possible for both premises to be false in fact and for a false conclusion to follow logically from them. For example:

Major premise:	No fathers have daughters.
Minor premise:	All married men are fathers.
Conclusion:	No married men have daughters.

What all these examples (and many others that we might consider) show us is something that has already been pointed out and is, perhaps, worth repeating. Reasoning may be logically correct regardless of whether the premises and the conclusion are true or false in fact. Only if both premises are in fact true is the conclusion that follows logically from them also in fact true.

If either premise is false, then the conclusion that follows logically from them may be either true or false. We cannot tell which it is. On the other hand, if the conclusion that follows logically from certain premises is in fact false, then we can infer that one *or* both of the premises from which it is drawn must also be false.

This leads us to one more important rule of reasoning that

Aristotle pointed out. In syllogistic reasoning, as in immediate inference, the validity of the inference is expressed by an "if" and a "then." In the case of syllogistic reasoning, we are saying that *if* the two premises are true, *then* the conclusion that logically follows from them is also true. We have not yet asserted the truth of the premises. We have asserted only the validity of the inference from the premises to the conclusion. Only when we assert the truth of the premises by substituting "since" for "if," can we also substitute "therefore" for "then" and assert the truth of the conclusion.

The rule with which we are here concerned has two parts. On the one hand, it says that we have a right to assert the truth of the conclusion if we assert the truth of the premises. On the other hand, it says that we have a right to question the truth of the premises if we deny the truth of the conclusion. I say "question the truth of the premises" rather than "deny the truth of the premises" because when we deny the truth of the conclusion, we know only that either one of the premises is false or that both may be, but we do not know which is the case.

The double-edged rule just stated is particularly applicable to a kind of reasoning that Aristotle called hypothetical. It usually involves four terms, not three.

Alexander Hamilton, in one of the *Federalist* papers, said: "If men were angels, no government would be necessary." If, having said that, Hamilton went on to deny that men were angels, no conclusion would follow. Denying the *if* statement (which is called the antecedent in hypothetical reasoning) does not entitle you to deny the *then* statement (which is called the consequent).

However, Hamilton obviously thought that government is unquestionably necessary for a society of human beings. He would, therefore, have had no hesitation in denying that men are angels. He would have been right in doing so because denying the consequent (or the *then* statement) in hypothetical reasoning does entitle you to deny the antecedent (or the *if* statement).

The truth that Hamilton is getting at can also be expressed in a single complex statement that conceals rather than reveals the reasoning behind it. That complex statement is as follows: "Because men are not angels, government is necessary for human society." The reasoning that goes unexpressed involves a series of statements about the difference between men and angels as well as statements about the special characteristics of men that make government necessary for human society. The kind of compressed argument that omits or conceals indispensable premises Aristotle called an enthymeme.