PART V

DIFFICULT PHILOSOPHICAL QUESTIONS

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Infinity

Difficult philosophical questions are questions that it is impossible to answer in the light of common experience and by the use of common sense. To answer them requires sustained reflection and reasoning.

How do such questions arise? For Aristotle they arose in part from the refinements of common sense that his own philosophical thought developed. In part, they were questions he asked in response to the views of others that were current in his day.

Among the students of nature who preceded him were two Greek physicists, Leucippus and Democritus, who first proposed the theory of atoms. According to their theory, everything in the world of nature is composed of tiny, invisible particles of matter, separated by a void—space totally devoid of matter. They called these particles atoms to indicate that these units of matter were not merely very small, but absolutely small. Nothing smaller, in their view, can exist, for each atom is an indivisible unit of matter. It cannot be cut up into smaller units.

Atoms, according to Democritus, differ from one another only in size, shape, and weight. They are constantly in motion. And they are infinite in number.

Confronted with this theory, Aristotle raised two objections to it. In the first place, he challenged the central notion in the theory of atomism. If an atom is a solid unit of matter with no void or empty space inside it, then, he argued, it cannot be uncuttable or indivisible. Either an atom has some empty space inside it, in which case it is not a unit of matter; or, lacking empty space, the matter is continuous, in which case it is divisible.

The reasoning here can be illustrated by taking something larger than an atom. I am holding in my hand one matchstick. I break it into two smaller pieces of wood. Each of these pieces of wood is now a separate unit of matter. No longer being one piece of wood, they can no longer be broken into two. But each of the two pieces of wood can be further divided, and so on without end.

Whatever is continuous, Aristotle held, is infinitely divisible. Anything that is one—a single unit of matter—must be continuous. If it were not, it would not be one unit of matter, but two or more. By this reasoning, Aristotle thought he showed that there could be no atoms. There may be very small units of matter, but however small these particles may be, they can be divided into smaller particles, if each is a unit of matter—one and continuous.

In the second place, Aristotle objected to the view that there are an infinite number of atoms in the world. The number may be very large, so large that it cannot be counted in any time that a counter might use to do so. But it cannot be an infinite number because, Aristotle maintained, an infinite number of things cannot actually coexist at any moment of time.

These two objections that Aristotle raised against the atomists of his day may at first appear to be inconsistent. On the one hand, Aristotle appears to be saying that any continuous unit of matter must be infinitely divisible. On the other hand, he appears to be saying that there cannot be an infinite number of units in existence at any one time. Is he not both affirming the existence of an infinity and also denying it?

The apparent contradiction is resolved by a distinction that is characteristic of Aristotle's thought. We have come upon this distinction in an earlier chapter of this book (see chapter 7). It is the distinction between the potential and the actual—between what can be (but is not) and what is.

Aristotle thinks that there can be two infinities—both potential, neither actual. One is the potential infinite of addition. The other is the potential infinite of division.

The potential infinite of addition is exemplified in the infinity of whole numbers. There is no whole number that is the last number in the series of whole numbers from one, two, three, four, and so on. Given any number in that series, however large it may be, there is a next one that is larger. It is possible to go on adding number after number without end. But it is only *possible*, you cannot *actually* carry out this process of addition, for to do so would take an infinite time—time without end.

Aristotle, as we shall see in the next chapter, did not deny the infinity of time. On the contrary, he affirmed the eternity of the world—that it has no beginning or end. But an infinite time does not exist at any one moment. Like the infinite series of whole numbers, it is only a potential, not an actual, infinite.

So, too, the infinity of division is a potential, not an actual, infinite. Just as you can go on adding number after number without end, so you can go on dividing anything that is continuous without end. The number of fractions between the whole numbers two and three is infinite, just as the number of whole numbers is infinite. Both infinities, however, are potential, not actual. They do not actually exist at any moment of time.

At this or any other moment, Aristotle maintained, there cannot be an actual infinity of coexisting things, as there would be if the atomists were correct in their view. They held, it must be remembered, that at this very moment an actually infinite number of atoms coexist. It is that and that alone which Aristotle denied.

His reasoning on this score ran as follows. Either the number of actually coexisting things is definite or indefinite. If it is infinite, it is indefinite. But nothing can be both actual and indefinite. Therefore, there cannot be an actual infinity of any sort— an actually infinite number of coexisting atoms, an actually infinite world, an actually infinite space that is filled with actually existing units of matter.

The only infinities that there can be, according to Aristotle, are the potential infinities that are involved in the endless processes of addition or division. Since one moment of time succeeds another or precedes another, and since two moments of time do not actually coexist, time can be infinite.