Market and Shadow Land Rents with Congestion

By RICHARD J. ARNOTT AND JAMES G. MACKINNON*

It is well known that when there are imperfections in the economy, shadow prices should be used instead of market prices in cost-benefit analysis. A major imperfection in urban economies is that residents do not pay the social cost of the congestion they create. As a result, market and shadow land rents may be different. This paper investigates the relationship between the two. The conventional view, expressed by Robert Solow, is that the shadow rent on land in residential use always exceeds the market rent. One major result of our paper is that this conclusion is shown to be incorrect. We also examine the shadow rent on land in transportation, and show that it may be negative.

This subject is of considerable interest for policymakers. Many urban public expenditures, such as the construction of roads and government buildings and the creation of public recreational land, involve the acquisition of land. If efficient decisions are to be made on such expenditures, it is clearly necessary for governments to know the shadow rent on land. Also, when there are divergences between shadow and market rents, governments may want to intervene in the market to alter private decisions, which are socially inefficient since they are based on market rents. Governments may also want to change the pricing of transportation so as to reduce or eliminate the divergence between the private and social costs of congestion, which is responsible for the divergence between market and shadow land rents. William Vickrey (1963) has proposed various schemes which would do this. In Section III of this paper the income-equivalent benefit of charging (almost) optimal congestion tolls is computed using a numerical simulation model. To the extent that this model is realistic, we can provide some indication of the expenditure that would be justified to implement a Vickrey-type scheme.

Previous work on the relationship between market and shadow land rents is not entirely satisfactory. Solow and Vickrey investigated analytically the effects of employing an incorrect planning rule to determine road width in a model with a very restrictive technology, where all land is in commercial use. Yoshitsugu Kanemoto (1975, 1976) undertook similar analyses for models in which land use is industrial. Richard Muth (1975) used a simulation model to investigate whether too much or too little land is allocated to streets. Solow constructed a simulation model which permitted him to investigate whether, on average, shadow land rents exceed market land rents on land in residential use, but not how shadow and market rents are related as a function of location. Kanemoto (1977) investigated the relationship between shadow and market rents on residential land for the case where road widths are optimal given that congestion is not priced. Since it is doubtful that roads are approximately of second best optimal width, his results are not generally applicable. All these papers are, at least implicitly, concerned with the relationship between market and shadow land rents, but this is, to our knowledge, the first paper to deal explicitly with that relationship as a function of location, in a residential location theory model which is reasonably realistic.

In Section I we investigate the analytical relationship between the market and shadow rent on residential land in the presence of unpriced congestion. We use a model similar to Solow's. The conventional argument concerning this relationship is shown to be

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faulty. In Section II a numerically solvable residential location model and the technique used to solve it are described. In Section III this simulation model is used to investigate the relationship between the market and shadow rents on residential land and the shadow rent on land used for transportation, and to estimate the benefits from charging congestion tolls. In Section IV other factors affecting the relationship between market and shadow rents are discussed briefly.

I. The Relationship Between Shadow and Market Rents

Shadow rents can be computed in a number of different ways. In this paper, the following procedure is adopted. To ascertain the shadow rent on residential land at a particular location, add a small amount of residential land at that location, and solve for a new equilibrium using lump sum transfers to ensure that all city residents achieve their previous level of utility. The shadow rent is then the money saved by the government as a result of the land being added, divided by the amount of land that was added. Alternatively, a small amount of land could be subtracted, and the shadow rent would be the additional money spent by the government in order to keep residents at their previous utility level, divided by the amount of land that was subtracted. In the limit, of course, these two procedures give identical results.

A major conclusion is that when congestion is unpriced, the value to society from adding land to roads is, in general, *not* equal to the direct transport savings associated with the land addition. A (compensated) transport improvement or the (compensated) addition of residential land results in residents expanding lot size on average, and in an increase in traffic flow at every location. *This increased flow causes an increase in the excess burden associated with congestion not being priced.* The conventional argument ignores this, and by doing so incorrectly computes shadow rents. It comes to the erroneous conclusion that the shadow rent on residential land always exceeds the corresponding market rent (except at the boundary where they are equal). These points are now elaborated.

Consider a very simple residential location theory model similar to those dealt with by Solow and Kanemoto (1977). The city has a fixed population of households with identical tastes and incomes, who derive satisfaction from land and from other goods. All markets are competitive, so that residents have equal utility in equilibrium. Every day, all residents must commute to and from the central business district (*CBD*). That is the only travel which occurs. Also, land is homogeneous. Thus, locations are differentiated solely on the basis of accessibility to the *CBD*, which is measured by x. A resident may consume land at only one location. The lot size of the resident at x is denoted by $T(x)$, market and shadow land rents by $r(x)$ and $s(x)$, respectively, nonland consumption of the resident at x by $C(x)$, and the population between x and $x + dx$ by $n(x)dx$. The boundary of the city is at x^b, and the land rent at the boundary, $r(x^b)$, is equal to $\bar{r}$, the exogenous rent on land in agricultural use.

There is only one transport artery (or, equivalently, many identical ones), so that the number of travellers on it at rush hour, $Q(x)$, equals the number of residents who live between x and the boundary of the city; i.e.,

$$Q(x) = \int_x^{x^b} n(x')dx' \tag{1}$$

Transport costs between x and $x + dx$ are $g[w(x), Q(x)]dx$, where $w(x)$ is the width of the road. The dependence of transport costs on w and Q reflects flow congestion in transportation. As the number of travellers on the road increases, so do transport costs ($g_Q > 0$), and as the width of the road increases, transport costs decline ($g_w < 0$).

Now consider what happens when an amount of residential land $T(x^*)$ is added to the residential land available at location x^*. This is exactly the amount of land occupied by each resident at x^*. *If it is assumed that lot sizes remain fixed*, the net effect of the addition must be the movement of one resident from the boundary of the city to x^*. The shadow rent of the added lot, $s(x^*)T(x^*)$, is the income that the government can extract and still leave everyone with the same allocations they had before (or, in the case of the mover, with the same allocation as other residents at x^*). This is the sum of three components: the reduction in aggregate transport costs, the mover's reduction in expenditure on other goods (which is typically negative), and the income derived from the vacated lot at the edge of the city (which can now be used for agriculture).

The mover's transport costs are reduced by

$$(2) \qquad \int_{x^*}^{x^b} g\, dx$$

Since he no longer travels between x^b and x^*, the number of travellers on the road between x^b and x^* decreases by one, which reduces travel costs of residents between x^* and x^b. The reduction in transport costs between x and $x + dx$, where x lies between x^* and x^b, is

$$(3) \qquad Q(x)\, g_Q(x) dx$$

The transport costs of those living between 0 and x^* remain the same. Thus the total reduction in transport costs is

$$(4) \qquad \int_{x^*}^{x^b} g\, dx + \int_{x^*}^{x^b} Q g_Q dx$$

The income derived from the vacated lot is $r(x^b)\, T(x^b)$. The reduction in the mover's expenditure on other goods is

$$(5) \quad C(x^b) - C(x^*) = r(x^*)T(x^*) - r(x^b)T(x^b) - \int_{x^*}^{x^b} g\, dx$$

from the mover's budget constraints at x^* and x^b. Adding up these three terms yields

$$(6) \quad s(x^*)\, T(x^*) = \int_{x^*}^{x^b} Q g_Q dx + r(x^*)\, T(x^*)$$

Rearrangement of (6) then yields

$$(7) \quad (s(x^*) - r(x^*))\, T(x^*) = \int_{x^*}^{x^b} Q g_Q dx$$

Expression (7) indicates that the shadow rent on the added lot exceeds the market rent by an amount equal to the value of transport savings to the nonmovers resulting from the decrease in congestion between x^* and x^b. Since these savings must be positive for all x^* less than x^b, $s(x)$ must exceed $r(x)$ at every location, except at the boundary where they are equal. The slope of the market rent gradient reflects a resident's reduction in transport costs resulting from his moving from $x + dx$ to x. The slope of the shadow rent gradient, however, reflects the sum of the resident's reduction in transport costs and the reduction in nonmovers' transport costs resulting from the move. Since, with the specified assumptions, congestion is always reduced by such a move, the shadow rent gradient is always steeper than the market rent gradient at a particular location. Thus, the shadow rent exceeds the market rent by an increasingly large amount as distance to the city center is reduced.

The foregoing argument is correct, given the assumption that lot sizes remain fixed. However, the addition of land increases the supply of land in residential use. As a result, rents fall on average, causing residents to substitute land for other goods in consumption. Average lot size increases, and more people travel on the road at all locations, relative to the situation after the addition of land but before lot sizes adjust. Consequently, the amount of unpriced congestion at all locations increases, and also the excess burden associated with the unpriced congestion. By assuming that lot size is fixed, the conventional method of calculating the shadow rent on residential land yields a measure of thc shadow rent that is consistently biased upwards by an amount equaling the increase in excess burden.

A similar argument can be made for the shadow rent on land used in transportation. If one assumes that lot sizes do not adjust in response to the addition of land to the road at some location x^*, the shadow rent on land in road use there is computed as

$$(8) \qquad -Q(x^*)\, g_w(x^*)$$

which is the number of road users at that location, $Q(x^*)$, times the travel savings to each when the road is widened by one unit for a distance of one unit, $-g_w(x^*)$, or simply the direct transport savings associated with the addition of the land. But since it ignores lot size adjustment, which increases the excess burden associated with unpriced transportation congestion, this measure also consistently overstates the true shadow rent.[1]

II. A Simulation Model

The conventional argument suggests that the shadow rent on land in residential use always exceeds the market rent. In the preceding section it was argued that the shadow rent, correctly measured, is less than the shadow rent measured according to the conventional argument. The obvious question that arises is: can the shadow rent on residential land be less than the market rent? To answer this question, a simulation model which requires numerical solution is constructed. The model is described in this section, and the results of several simulations are presented in Section III.

This model is similar to Solow's, which was also solved numerically, but is substantially more complicated and realistic. It incorporates three of his four suggested extensions: "... (1) the explicit inclusion of housing in addition to land as a residential cost; (2) the allowance for both time costs and out-of-pocket costs of commuting; (3) the use of a congestion-cost function that rises more than proportionally with traffic density ..." (p. 617). Solow's fourth suggestion, that two or more income classes be dealt with, was incorporated in one simulation run, but since this paper focuses on issues of efficiency rather than of distribution, only the one-group model is described here.

The city has a population of 1 million households, each of which has an annual after-tax income of $13,000, and an indirect utility function

$$U = YP_h^{-.3}\tau \tag{9}$$

where Y is income net of money transport costs (inclusive of toll charges, where applicable), P_h is the rental price of housing, and τ is defined by

$$\tau = 1 - .125\,Time \tag{10}$$

where *Time* is the amount of time, in hours, it takes to get to the *CBD*. Thus if Y and P_h did not depend on the household's location, which of course they do, utility would decline linearly with the time it takes to get to the *CBD*, reaching zero at a travel time of eight hours. Equation (9) implies that the household spends 30 percent of its income net of transport costs on housing, and 70 percent on other goods. The functional form (10) is justified in the authors' article (1977b). What matters for the purposes of this paper is that utility declines, *ceteris paribus,* as the time spent commuting increases.

Housing is treated as a nondurable good, which is produced in a perfectly competitive market at each location. The rent on housing is determined by a constant elasticity of substitution (*CES*) cost function,

$$P_h = (.1r^{.3} + .9\,P_S^{.3})^{1/.3} \tag{11}$$

where r is the rent on land, which varies across locations, and P_S is the rent on a unit of structure, which is assumed to be $1,300. The above cost function implies that the elasticity of substitution in the production of housing is .7, a figure in line with empirical work by Muth (1971) and Roger Koenker.

For purposes of numerical solution, the city is divided into a number of concentric rings each half a mile wide, around a *CBD* with a radius of one mile. One member of

[1]Cost-benefit analysts are concerned with values rather than rents. Because of the durability of housing, it may take a long time for the city to adjust to a lot addition or the widening of a road. In a stationary economy, value may usefully be viewed as a weighted average of the capitalized value of long-run rents (we compute long-run rents) and the capitalized value of short-run rents. (By implicitly assuming fixed lot sizes, Solow computes short-run rents.) The weight depends on the speed of adjustment of the economy. The various qualitative propositions we develop concerning rent relationships apply also to value relationships.

every household which lives in ring i is assumed to commute from the midpoint of the ring, making 250 round trips or 500 one-way trips per year. Since money transport costs are assumed to be 15¢ per mile, regardless of congestion, the net income of a household which lives in ring i is

$$(12) \quad Y = 13{,}000 - (500)(.15) \cdot (1.0 - .25 + .5i) = 12{,}943.75 - 37.5i$$

Dividing the city into discrete rings inevitably introduces some inaccuracy, but since the maximum difference between where a household lives and where it is assumed to live is only one quarter of a mile, this is quite small.

The treatment of congestion in existing urban models is not very satisfactory. Only flow congestion is considered. Congestion associated with intersections and with entry to and exit from the traffic flow, and queuing phenomena are ignored. The latter may be important, and may have striking implications for the relationship between shadow and market land rents. Dan Usher has developed a location theory model in which the only form of congestion is queuing congestion, and obtained the result that the market rent on land *always* exceeds the shadow rent. Consider a very simple version of his model, in which the city is a long narrow parking lot. At one end of the parking lot is a gate, through which traffic flow is limited. Every morning residents get in their cars and wait their turn to go through the gate into the *CBD*. Waiting is the only cost of travel. Clearly, residents will be willing to pay more for locations nearer the gate, because the length of their wait depends on their position in the queue. Thus market rents decline monotonically with distance from the gate. Now suppose that a new parking place is created. Its shadow rent is clearly just the opportunity rent on land in other uses, because moving someone from another space into the new one does not change aggregate congestion at all; it simply frees up a parking space elsewhere. The market rent on the last space in the queue is also the opportunity rent on land in other uses, which is equal to the shadow rent on every space in the queue. But the market rent on all spaces except the last exceeds the market rent on the last space. Thus in Usher's model, market rents always exceed shadow rents, except at the boundary where they are equal.

The conventional modelling of congestion in urban models not only ignores forms of congestion other than flow congestion; its treatment of flow congestion is also unsatisfactory. The assumed specifications of flow congestion imply that there is no maximum feasible flow; flow can always be increased at some cost in time. But traffic engineering studies (see Institute of Traffic Engineers, pp. 271–76) suggest that there is a maximum flow, and that if traffic tries to exceed it, flow is actually reduced and queuing must occur. To model this phenomenon realistically would be very difficult, since the length of the queues must vary with the time of day. Moreover, when a person enters the queue depends not only on where he lives but also on when he begins the journey to or from work, which should be determined endogenously. Because of these difficulties, we have chosen with some reluctance to follow the conventional modelling of congestion. It should be emphasized that the results obtained in this paper are contingent on the treatment of congestion.

The specification of the flow-congestion function is now considered. Most authors do not distinguish between the time and money costs of commuting. Observation suggests, however, that congestion has a large effect on time costs, but only a small effect on money costs. It is assumed here that money costs are unaffected by congestion, but that time costs are related to flow per unit width of road. Specifically,

$$(13) \quad t(x) = t[Q(x)/w(x)]$$

where $t(x)$ is the time required to travel a mile at x, $Q(x)$ is the number of households living beyond x, and $w(x)$ is the width of the road in feet times forty. Width is measured in this way so that $Q(x)/w(x)$ equals unity with "normal" traffic flow. The factor of 40 enters since the rush hour is assumed to be one hour long, and since normal traffic flow is defined to be forty cars per hour per foot of road width.

A number of additional stylized facts were employed in choosing the specific functional form of (13). Before introducing them, it is necessary to develop some terminology. Define the private congestion cost at x, $PC(x)$, to be the increase in the time required for an individual to cover a mile at x due to the presence of others on the road; i.e., $PC(x) = t[Q(x)/w(x)] - t[0/w(x)]$. Total travel time on a mile of road at x is $t(x)Q(x)$. When one more traveller is added, the increase in total travel time is

$$t(x) + t_Q(x)\,Q(x) \tag{14}$$

The first term in (14) is the private cost to the traveller; the second term, the congestion externality (in time units) imposed by the marginal traveller on other persons on the road. This latter term is defined to be the marginal congestion externality at x, $MCE(x)$.

The additional stylized facts that we considered were that:

(i) free flow speed be reasonable;

(ii) the elasticity of private congestion with respect to flow exceed unity $((\partial PC/\partial Q)(Q/PC) > 1)$; and

(iii) the ratio of the marginal congestion externality to private congestion should be an increasing function of Q, where[2]

$$\partial\left(\frac{MCE}{PC}\right)\Big/\partial Q > 1$$

The flow congestion function employed in this paper is

$$t(x) = t_0 \exp\,[\alpha(Q(x)/w(x))^{\beta}] \qquad \beta \geqq 1, \alpha > 0, t_0 > 0 \tag{15}$$

It is easy to show (see the authors, 1976) that this functional form satisfies the above stylized facts; however, the flow congestion function used most commonly in the literature,[3]

$$t(x) = t_0 + k(Q(x)/w(x))^{a}, \qquad a > 0, k > 0, t_0 > 0 \tag{16}$$

does not. Thus (15) is the better functional form.

In the simulation runs, t_0 was set at 1/35, which implies a free flow velocity of 35 miles per hour; the other two parameters of (15) were chosen to result in a reasonable speed gradient for rush hour traffic, and were varied over the simulation runs. The amount of land devoted to streets was also varied over the simulation runs, while the amount of land devoted to housing was set equal to half the potentially available land in each ring beyond the *CBD*. This reflects the fact that much land in real cities is used for purposes other than housing and roads.

The model was solved using a variant of the technique employed by the authors (1977a,b), which makes use of a simplicial search algorithm called the Vector Sandwich Method (see MacKinnon) which is similar to the algorithms developed by Herbert Scarf. At each iteration, the algorithm "tries out" a rental price of housing at the center of the city. From this can be calculated the utility level achievable by a household living at the city center, which must be the utility level achieved everywhere else as well. Flow through the first ring is always 1 million people, so that time cost for households in the first ring can be calculated, and that, along with the utility level they must achieve, implies the rents on housing and land they must face, which in turn determines how many people can fit in the first ring. Flow through the second

[2]Solow argued for the use of (but did not himself employ) "a congestion-cost function that rises more than proportionally with traffic density ..." (p. 617). Since Solow took $Q(x)$ as a proxy for density, rather than flow, we took his statement to mean that $(\partial PC/\partial Q)(Q/PC) > 1$. Thus (ii) is Solow's stylized fact. Vickrey (1969) implies that the ratio of MCE to PC should be an increasing function of Q. Thus (iii) is Vickrey's stylized fact.

[3]Actually, the flow congestion function in the literature is of this form, but treats money expenditure rather than time expenditure.

ring can now be determined, hence its prices and population, and so on through the rings. The algorithm then searches for a rental price of housing at the center such that, when all residents are housed, the rent on urban land at the edge of the city is equal to the predetermined rent on agricultural land of $750 an acre.

In order to compute the shadow rent on residential land as a function of location, the model is first solved as above, and the level of utility achieved in this base solution stored. The model is then solved a number of times, with 100 acres of residential land added to each even-numbered ring in turn (odd-numbered rings were not included to save computer time).[4] In these simulations, utility is prespecified to equal the level achieved initially. The algorithm varies lump sum transfer payments from the government so that this is achieved, as well as varying the rent on housing at the center so that, when all residents are housed, the land rent at the edge of the city equals the agricultural land rent. The shadow rent on residential land in ring i is then calculated as

$$(17)\quad (-TP + \Delta DLR + 75{,}000)/100$$

where TP is the total transfer payments by the government to the households, ΔDLR is the change in differential land rents between the base solution and the compensated solution with added land, and 75,000 is the value of the added land in agricultural use (100 acres at $750 per acre). Note that the government is treated as owning all the land in the city. This is equivalent to assuming absentee landlords, who are also compensated in the new equilibrium (so that the land rents they receive are unchanged).

The shadow rent on land in transportation is calculated the same way as the shadow rent on land in residential use, except that only 50 acres are added to the land used for roads in every even-numbered ring in turn. Since there were generally around 22 occupied rings, calculating the shadow rents on land in transportation and in residential use for all even-numbered rings, required that the simulation model be solved around twenty-three times. This required about four minutes of processing time on a Burroughs B6700.

III. Simulation Results

In the first subsection, results are presented for the base case city; in the second, the effects of some alternative specifications of the road width and congestion functions are considered; and in the third, the efficiency gain from imposing a congestion toll is evaluated.

A. *The Base Case City*

In the base case city, the parameters of the congestion function (15) are $\alpha = .6$ and $\beta = 1.0$ (in all runs $t_0 = 1/35$). The characteristics of this function were discussed in the last section. At the normal flow where $Q/w = 1$, private congestion is 1.41 minutes time loss per mile, and the marginal congestion externality is 1.87 minutes time loss per mile. Thus MCE/PC is 1.33 for $Q/w = 1$; for $Q/w = 0$ it is 1.00, and for $Q/w = 2$ it is 1.71.

The road width function, which gives forty times the width of the road in feet, as a function of location, is

$$(18)\quad w(x) = 211{,}200(.1x + .5)\pi$$

Thus the width of the road increases with x. Since the city is circular, the proportion of land used for the road is $(.05 + .25/x)$, which decreases with x. This function seems roughly realistic: 30 percent of the land is used for roads at the edge of the CBD, compared with 10 percent five miles from the center of the city.

Some of the characteristics of the simulated city are presented in Table 1. The

[4] The choice to add 100 acres rather than say, to subtract 50, is purely arbitrary. One hundred acres is small relative to the land available in any ring (about 6 percent of the land available in ring 2, and about .8 percent of the land available in ring 24), but not so small that round-off errors become important in the evaluation of (17). We experimented with other values, and found the calculated shadow rents quite insensitive to the choice, and certainly more than accurate enough for the uses to which they will be put.

TABLE 1—CHARACTERISTICS OF THE BASE CASE CITY[a]

	First Ring	Middle Ring	Boundary Ring
Distance from the city center in miles	1.25	6.25	11.75
	(1.25)	(6.25)	(11.25)
Land rent per acre	19540	4456	768
	(34048)	(4257)	(771)
Structural density	1.377	.489	.143
	(2.032)	(.474)	(.143)
Housing price, 1,000 square feet equivalent	1928	1504	1238
	(2171)	(1495)	(1238)
Housing quantity, 1,000 square feet equivalent	2.008	2.500	2.937
	(1.848)	(2.506)	(2.923)
Land share in housing	.2003	.1385	.0867
	(.2284)	(.1369)	(.0867)
Speed (mph)	8.24	23.21	34.46
	(8.24)	(24.47)	(34.46)
Cumulative travel time in hours per trip	.0304	.3809	.5689
	(.0304)	(.3663)	(.5335)
Proportion of land used for roads	.2500	.0900	.0713
	(.2500)	(.0900)	(.0713)
Cumulative toll ($/year)	0	0	0
	(74.35)	(590.53)	(636.65)

[a] Numbers in parentheses refer to the same city with a congestion toll imposed.

numbers in parentheses refer to the same city with a congestion toll imposed (see Section IIIc). The base case city is in most respects quite similar to actual cities with 1 million households (for example, Toronto). The radius of the city is 11.75 miles. Structural density (square feet of floor area per square foot of residential land) varies from 1.377 at the center to 0.143 at the boundary of the city, a factor of about 10. Land rent varies by a factor of about 25, while the rent on housing varies by a factor of about 1.5. All these gradients are convex, as expected. Traffic speed increases from 8.24 miles per hour in the innermost ring to 34.46 miles per hour in the boundary ring, while cumulative travel time reaches about 35 minutes per trip at the boundary.

Figure 1 shows the relationship between the shadow rent on land in transportation computed correctly (curve I), the shadow rent on residential land computed correctly (curve II), and the market rent on residential land (curve III). Curves I and II differ because the road width is not second best optimal; comparison of those two curves suggests that road width is less than optimal up to about 6 miles from the edge of the *CBD*, beyond which it is greater than optimal.[5] Certainly the road is too wide at the edge of the city, since traffic flow goes to zero there, as does the shadow rent on land in transportation.

More interesting is the comparison between curves II and III. Curve II lies above curve III from the center of the city to ring 8, but below curve III from ring 9 to the boundary of the city; that is, *the market rent on residential land is less than the corresponding shadow rent in the inner section of the residential area, but exceeds the shadow rent in the outer section.* An intuitive explanation of this is the following. Compare the effects of adding a lot at x_1, which is near the center of the city where there is considerable transportation congestion, equal in size to existing lots at x_1, to the effects of adding

[5] This statement is only true in the sense that widening the road at a location where the shadow rent on land in road use exceeds the shadow rent on land in residential use will result in a Pareto improvement when no other changes are made. If the road width is altered at other locations, the shadow rents on land in both road and residential use will change at this location, so that, when road width at other locations has been adjusted optimally, it may actually be desirable to narrow the road at this location.

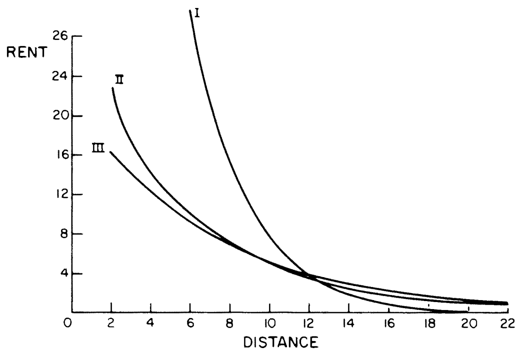

FIGURE 1

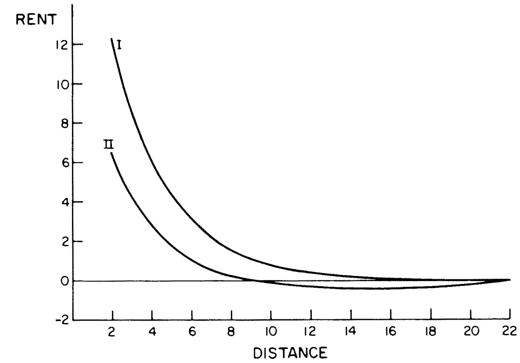

FIGURE 2

a lot at x_2, which is near the boundary of the city where congestion is negligible, equal in size to existing lots at x_2. Note that this implies that the amount of land added at x_2 must be considerably greater than the amount added at x_1. We have argued that the difference between the shadow rent and market rent on a residential lot can be separated into two components: first, the reduction in transport costs, induced by the lot addition, beyond the location where the lot was added, lot sizes fixed, which by itself would cause the shadow rent to exceed the market rent; and second, the effects resulting from general equilibrium lot size adjustment, which by themselves would cause the shadow rent to be less than the market rent. From (7), the first effect is larger the nearer is the lot to the city center because more people's travel costs are reduced. The magnitude of the second effect depends on how much people will expand their lots as a result of the addition of the lot; this can be expected to be more or less the same whether the lot is added near the center of the city or near the boundary. On balance then, one would expect the shadow rent on a lot minus its market rent to fall the further is the lot away from the city center. Furthermore, since the first effect becomes negligible near the boundary of the city, while the second does not, one would expect the market rent on a lot near the boundary to exceed the corresponding shadow rent.

Figure 2 shows the difference between the shadow and market rent on residential land as a function of location. Curve I shows this relationship when shadow rents are computed assuming lot size fixed; curve II, when shadow rents are computed correctly. The figure demonstrates the conventional result that the shadow rent on residential land computed assuming lot size fixed always exceeds the market rent. Also, in this simulated city, the shadow rent computed on the assumption of fixed lot size everywhere overstates the true shadow rent on residential land. The same is true of the shadow rent on land in transportation (not shown).

B. *Other Cities*

A number of other simulations were performed using different parameter values. In most cases, they yielded results qualitatively similar to those of the base case. One somewhat bizarre city did yield a new result, however.

This city differed from the base case in two ways. First, the parameters α and β of the congestion function were 0.2 and 3.0, respectively. With these parameters, MCE/PC is 3.00 for $Q/w = 0$, 3.33 for $Q/w = 1$, and 6.02 for $Q/w = 2$. The effect of these parameter changes is to make speed less sensitive to increases in flow for flows less than normal flow, and more sensitive to such increases for flows greater than normal flow. Second, the road width function was

$$(19)\quad w(x) = \begin{cases} 211{,}200\,(.603 - .03768x)\pi & \text{for } x \leqq 5.75 \\ 211{,}200\,(.7537)\pi & \text{for } x > 5.75 \end{cases}$$

This road narrows to a distance 5.75 miles from the city center, and then suddenly widens and remains of constant width.

With these congestion and road width functions, this city is extremely congested in the inner portion of the residential area. Speed in the first residential ring is only 0.64 miles per hour. Congestion gradually decreases as one moves away from the center, until 5.75 miles away speed is 16.71 miles per hour. Then traffic speed jumps up as a result of the sudden widening of the road, so that at 6.25 miles from the center it is 34.70 miles per hour, nearly free flow velocity. Travel time to the boundary is 108 minutes. In the innermost ring, MCE/PC is 12.2: for every minute a traveller loses due to congestion, he causes others to lose 12.2 minutes.

With such a bizarre city, one should expect unusual and extreme results, and this is indeed the case. Figure 3 is comparable to Figure 1. The most remarkable feature of this city is the gradient of the shadow rent on land in transportation. It is roughly \$3 million per acre 0.75 miles from the inner residential boundary, and falls sharply until it is −\$24,389 per acre just before the road widens. Thus, *the shadow rent on land in transportation may be negative.* This can come about because the increased congestion elsewhere caused by widening the road at one location may exceed the direct savings.

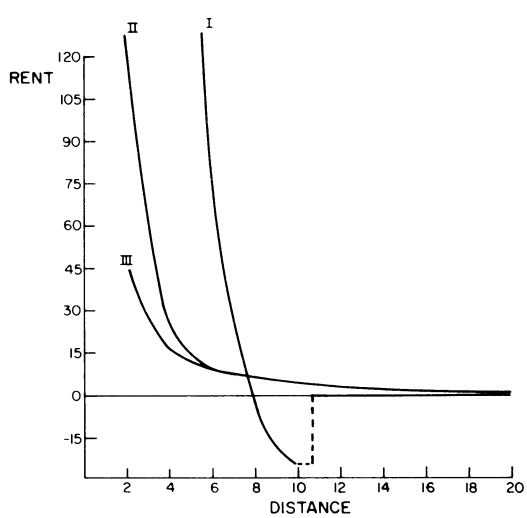

FIGURE 3

At this point, it is useful to review what the simulation runs discussed in this subsection and the preceding one have shown. They have shown that:

1) the shadow rent on residential land correctly computed may be less than the shadow rent on residential land computed assuming lot size fixed (see Figure 2);

2) the market rent on residential land may exceed the shadow rent in quite realistic circumstances (see Figures 1 and 2);

3) the shadow rent on land in transportation correctly computed may be less than the shadow rent on land in transportation computed assuming lot size fixed (no figure, but implied by Figure 3); and

4) the shadow rent on land in transportation may be negative (see Figure 3).[6]

[6]Subsequent to the writing of this paper, Arnott analytically solved what was characterized in this paper as the Solow model (which is simpler than the simulation model in this paper). His results support the verbal arguments in the text of this paper, and suggest that most of the qualitative characteristics of the simulation runs presented here hold generally (contingent on the modelling of congestion). His results are summarized as follows. i) The measures of shadow rents computed ignoring lot size adjustment nearly always overstate the corresponding shadow rent computed correctly (nearly indicating a couple of cases where they are equal. ii) There is a critical location, x^*. Between x^* and the center of the city the shadow rent on residential land exceeds the corresponding market rent, while between x^* and the boundary the shadow rent on residential land is less than the corresponding market rent. iii) There is always a region near the boundary of the city where the shadow rent on land in road use is negative. iv) Between x^* and the center of the city, the difference between the shadow and market rent on residential land monotonically increases. It should be noted that while these results probably extend to the more detailed model treated in this paper, this has not been proved.

C. *Imposition of a Congestion Toll*

It would be difficult to impose an *optimal* congestion toll in the model of this paper. The problem is that congestion affects travel time, while any congestion toll must be in terms of money, and the shadow value of time varies with the location of the household. It is however possible to impose a congestion toll which is reasonably close to being optimal, certainly closer than tolls which realistically could be charged, by charging travellers the *MCE* they create times the shadow value of time of someone living at the center of the city (an arbitrary choice). If the congestion toll were really optimal, the shadow and market rents on residential land would be identical. With the toll that was imposed, the divergence between the two was always less than 10 percent, so that the toll is reasonably close to being optimal.

The toll was imposed on the base case city, and residents were compensated so as to keep them at their pretoll utility levels. Thus the efficiency gain is simply the change in government revenue. The characteristics of the base case city with the congestion toll are shown in parentheses in Table 1. Imposition of the toll increases the curvature of the rent and structural density gradients, and makes the city smaller and denser. Cumulative travel time to any point in the city is reduced as a result of the toll (but not by very much). The congestion toll per mile falls off very sharply away from the center of the city. A trip to the middle of the first ring costs 15 cents (or 60 cents per mile), while a trip to the boundary ring costs $1.27 (or 12.4 cents per mile).

The magnitude of the efficiency gain from the imposition of the toll is $8,801,761, or approximately $8.80 per household per year. Since the toll is not quite optimal, this is a lower bound. However, since excess burden tends to increase more than proportionately with the size of a distortion, it seems doubtful that the efficiency gain with an optimal congestion toll would be significantly larger than this.

For a model in which the average household is paying over $500 in tolls each year (see Table 1), $8.80 is a remarkably small figure for the efficiency gain from a congestion toll. The reason for this is that this model allows households very limited opportunities for escaping the toll. Every household must send someone to work in the *CBD* during the rush hour, so that congestion in the first ring is unchanged by the toll. People cannot avoid the toll by abandoning unnecessary trips, taking less congested roads, shifting trips to nonpeak hours, or taking jobs outside the *CBD*. All they can do is move their residences so that, on average, trips to work are shorter and cause less congestion. In view of this, we would caution strongly against attaching any practical significance to the figure of $8.80. Its smallness probably reflects the deficiencies of the treatment of congestion in residential location theory more than the deficiencies of congestion tolls. An accurate measure of the efficiency gains from congestion tolls could be computed by a technique similar to ours, but the model would have to be a great deal more complicated.

IV. Realistic Complications[7]

In a simple residential location theory model, Solow argued that in the long run the shadow rent on residential land exceeds the market rent. This paper has shown that this argument is incorrect, and has provided counterexamples. The actual relationship between market and shadow rents is considerably more complex than that implied by the conventional analysis. This conclusion would be reinforced if one were to introduce realistic complications.

One such complication is the presence of other distortions. In the models discussed above, unpriced transportation congestion was the only distortion in the urban economy. But other distortions, such as inefficient zoning and the property tax, are also important. The addition of land at some location may either increase or de-

[7]Ronald Grieson brought several of these points to our attention.

crease the excess burden associated with these distortions, so that shadow rents may be either lower or higher than in the case analyzed in this paper.

There are many other features of the urban economy which one would want to capture in a model that was used for evaluating policy. These include the dynamic features caused by the durability of structures, migration into and out of the city, the complications of multiple workplaces determined endogenously, the very complex nature of congestion (which has been discussed in this paper), and the possible benefits associated with the open space afforded by urban roads.

V. Conclusions

This paper has investigated the relationship between shadow and market land rents. It was argued that the conventional wisdom, which asserts that shadow rents on residential land always exceed market rents, and that shadow rents on land in transportation are always positive, is incorrect because it does not measure shadow rents correctly. On the contrary, the simulation results demonstrated that the shadow rent on residential land may be less than the market rent, and that the shadow rent on land in transportation may actually be negative. The simulation model which was employed is more realistic than models which have been used by other authors, incorporating as it does the major extensions suggested by Solow. In particular, the form of the congestion function accords better with the available stylized facts than previous functional forms. However, as the small efficiency gain from imposing a congestion toll suggested, there are still many aspects of the model which are unrealistic. It is particularly necessary that further work be done on modelling congestion.

The results of the paper have a number of interesting policy implications. The correct calculation of the shadow rent on land in transportation and in residential use is evidently very difficult. However, this does not argue for the use, in cost-benefit analysis, of shadow rents computed assuming that lot size is unaffected by the addition of land, since these may be grossly incorrect; rather, the implication is that expenditure is justified in the development of urban simulation models to permit calculation of the true shadow rent with reasonable accuracy. An argument based on the conventional and incorrect calculation of shadow rents is that if planners allocate land to roads up to the point where the shadow rent on land in road use equals the market rent on residential land, then since the shadow rent on residential land exceeds the market rent, too much land is allocated to roads. However since the shadow rent on residential land correctly computed does not necessarily exceed the market rent, this argument is false.

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