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The Transition of Land to Urban Use

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This paper investigates the economics of the transition of land from rural to urban use. A simple model is employed to examine the developer's problem: When and at what density should vacant land be developed to maximize the present value of the land? A series of rules emerges from the analysis relating to the timing and density of new development. In the latter half of the paper, the rules are tested against recent Canadian experience and perform well.

Despite a large and growing literature in urban economics, remarkably little work has been done on housing development at the urban periphery. In fact, Shoup's paper (1970) on the timing of urban development represents the only attempt to deal with this issue rigorously. In the first part of this paper we build on Shoup's work, developing a partial equilibrium model which yields simple testable rules for the timing and structural density of new housing development.[1] In the second part, we test these rules, employing data on Canadian cities during the period 1961–75. We also use the model to estimate the elasticity of substitution between land and capital in the production of housing.

The Model

We assume that a landowner will develop at the time and density which maximize the present value of his land, subject to the constraint that

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[1] Throughout the rest of the paper we use "density" to refer to structural, not population, density. We take "optimal" to mean profit maximizing.

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building will freeze the land forever in that particular use. It is also assumed initially that (1) rents prior to development (agricultural rents) are zero, (2) the price of capital is constant, (3) property taxes are zero, (4) the building does not depreciate, and (5) rental rates are expected to increase at a constant rate. The objective of the landowner is to maximize per unit of land the difference between the present value of rents and the present value of construction costs with respect to the development time, T, and the capital applied to the land, K:

$$\max_{T,K} L(T, K) = \int_{T}^{\infty} r(t)Q(K)e^{-it}\, dt - pKe^{-iT}, \tag{1}$$

where $L(T, K)$ = present value of a unit of land if it is developed at time T with capital stock K; $r(t)$ = rental rate of a unit of housing at time t; $Q(K)$ = output of housing on a unit of land with capital, K [$Q'(K) > 0$, $Q''(K) < 0$];[2] i = interest rate; and p = price of a unit of capital. From assumption 5, it follows that

$$r(t) = r(0)e^{\eta t}, \tag{2}$$

where η = expected rate of change of rental rates ($\eta > 0$). The first-order condition with respect to T is $-r(T)Q(K)e^{-iT} + ipKe^{-iT} = 0$,[3] or

$$\frac{pK}{r(T)Q(K)} = \frac{1}{i}. \tag{3}$$

The value of a property (land and building) is the present value of the expected future rents:[4]

$$\begin{aligned} P(s) &= \int_{s}^{\infty} r(t)Q(K)e^{-i(t-s)}\, dt, && \text{for } s \geq T; \\ P(s) &= [P(T) - pK]e^{-i(T-s)}, && \text{for } s < T, \end{aligned} \tag{4}$$

where $P(s)$ = value of the property at time s, given construction at time T and at density K. From equation (2) it follows that the value of the property at development time is

$$P(T) = \frac{r(T)Q(K)}{i - \eta}. \tag{5}$$

[2] Since we are treating the rental rate on housing as separable from the ratio of capital to land, the negative second derivative can be justified both by technology and tastes. It costs more to produce additional units of housing both because construction costs increase with density and because people prefer to live at lower density.

[3] The second-order condition (for a maximum) requires $L_{TT} < 0$. This implies $r'(T) > 0$, or that at development time rental rates are increasing.

[4] Throughout the analysis we ignore the "Hahn problem" that asset prices may not equal the discounted present value of the net earnings stream.

Substituting equation (5) into equation (3) gives

$$\frac{pK}{P(T)} = \frac{i - \eta}{i}. \tag{6}$$

Equation (6) requires that at the optimal development time (independent of structural density) the ratio of the cost of the capital to the value of the property must equal the ratio of the difference between the interest rate and the expected rate of increase of rental rates to the interest rate. The intuition behind this rule is straightforward: A developer will wait until the interest saved by postponing development one period, ipK, equals the rent foregone, $(i - \eta)P(T)$.

A corresponding condition relating to the size of the structure is derived by differentiating equation (1) with respect to K and applying equation (2):

$$\frac{r(T)Q'(K)}{i - \eta} e^{-iT} - pe^{-iT} = 0.\text{[5]} \tag{7}$$

Substituting equation (4) into equation (7) and rearranging terms gives

$$\frac{pK}{P(T)} = \frac{Q'(K)K}{Q(K)} = \alpha(K), \tag{8}$$

where $\alpha(K)$ = output elasticity of capital in producing housing.

Equation (8) requires that, independent of development time, a developer will construct that building for which the output elasticity of capital equals the ratio of the cost of capital to the value of the property. The intuition for this rule is also immediate. The increase in cost from increasing capital one unit, p, must equal the discounted value at construction time of the increase in rents, $P(T)(Q'/Q)$.

Combining equations (6) and (8) gives a relationship between the output elasticity of capital, the interest rate, and the expected rate of growth of rental rates,

$$\alpha(K) = \frac{i - \eta}{i}, \tag{9}$$

which holds when development occurs at both the optimal density and the optimal time. A necessary condition for a local maximum is that $d\alpha/dK$ be

[5] The second-order condition, $r(T)Q''(K)e^{-iT}/(i - \eta) < 0$, implies that the (discounted) marginal revenue product of capital curve must be downward sloping.

TABLE 1
COMPARATIVE STATICS RESULTS

	p	i	$r(0)$	η
T	+	?	−	?
K	0	−	0	+

NOTE.—The comparative statics results are based on the assumption that the elasticity of substitution between land and capital is less than one.

nonpositive,[6] or equivalently that the elasticity of substitution between land and capital in the production of housing services be less than or equal to one (assuming a constant returns-to-scale production function for housing). Also, if land is developed optimally, the ratio of the value of land to the value of the property at development time equals the ratio of the expected rate of growth of rental rates to the interest rate:

$$\frac{V(T)}{P(T)} = \frac{\eta}{i},^{7} \tag{10}$$

where $V(T)$ = value of land at time T if it is developed optimally.

The comparative statics results for optimal timing and density of development are given in table 1. An increase in the price of capital postpones development time; an increase in current rental rates hastens it;[8] the effects of the interest rate and the expected rate of growth of rental rates on timing are indeterminate. The density of development is not affected

[6] $L_{TT} < 0$, $L_{KK} < 0$, and

$$\begin{vmatrix} L_{TT} & L_{TK} \\ L_{KT} & L_{KK} \end{vmatrix} > 0$$

is a set of sufficient conditions for a local maximum. These conditions (and $Q'' < 0$) imply $d\alpha dK < 0$. The intuitive interpretation of $d\alpha/dK < 0$ is now given (see fig. 1). Suppose $d\alpha/dK > 0$. Consider $K \varepsilon (0, K^*)$; $p > [Q'/Q]\, P[T]$ (p is the marginal cost of an extra unit of capital, $[Q'/Q]\, P[T]$ is the present value of the marginal revenue product of capital), which implies that a reduction in K increases the value of land. Similarly, for $K \varepsilon (K^*, \infty)$, an increase in K increases the value of land. Thus K^* is not a maximum (it is in fact a saddlepoint).

[7] The model also yields a form of the Wicksell solution relating to maturing wine. Land should be developed when the value of land, if developed in its optimal use, is growing at the interest rate. The development value of land can be represented by

$$D(s) = \int_s^{\infty} r(t)Q(K)e^{-i(t-s)}\,dt - pK,$$

where s = development time. Differentiating totally with respect to s, we have

$$\frac{dD(s)}{ds} = -r(s)Q(K) + i\int_s^{\infty} r(t)Q(K)e^{-i(t-s)}\,dt + \left[\int_s^{\infty} r(t)Q'(K)e^{-i(t-s)}dt - p\right]\frac{dK}{ds}.$$

If land is developed in the optimal use and at the optimal time, it follows from eqq. (4), (5), and (7) that $dD(T)/ds = -(i-\eta)P(T) + iP(T) + 0$. Substituting eq. (10) gives: $dD(T)/ds \cdot 1/D(T) = i$.

[8] Since η is held constant, rental rates are increased proportionally from time zero.

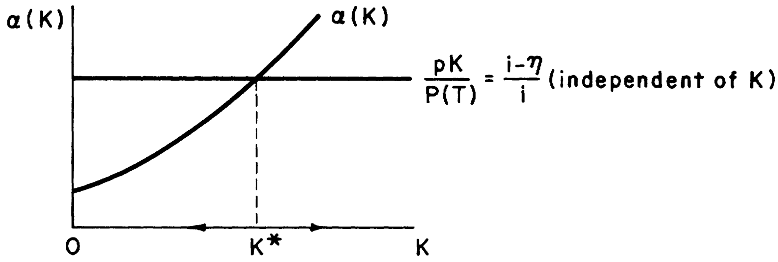

Fig. 1

by either the price of capital or current rental rates;[9] however, an increase in the interest rate will reduce density, while an increase in the expected rate of growth of rental rates will increase density.

The model can be easily extended. To make it more realistic we now allow for (1) differential property taxes before and after development and (2) a price of capital which is expected to change at a constant rate. The objective of the developer now becomes

$$\max_{T,K} L(T, K) = \int_T^\infty r(t)Q(K)e^{-it}\,dt - \tau_b \int_0^T P(t)e^{-it}\,dt - \tau_a \int_T^\infty P(t)e^{-it}\,dt - p(T)Ke^{-iT},^{10} \tag{11}$$

where τ_b, τ_a = property tax rate before, after development; $p(t) = p(0)e^{\delta t}$; $p(t)$ = price of capital at time t; and δ = expected rate of change in the price of capital. The derivation is almost identical with that of the simplest case, and so we present only the results, which are also quite similar. Equations (8), (9), and (10) become, respectively:

$$\alpha(K) = \frac{p(T)K}{P(T)}, \tag{12}$$

$$\alpha(K) = \frac{i + \tau_b - \eta}{i + \tau_b - \delta}, \tag{13}$$

$$\frac{V(T)}{P(T)} = \frac{\eta - \delta}{i + \tau_b - \delta}. \tag{14}$$

[9] Although somewhat surprising, this result follows immediately from eqq. (6) and (8). From eq. (8), we have that the output elasticity of capital must equal the ratio of the cost of capital to the value of the property at development time; eq. (6) implies that this same ratio is independent of density if the land is developed at the optimal time. This means that if, for example, the price of capital rises, a developer will construct the same building but develop at a later time, when rental rates are higher.

[10] There is a discontinuity in $P(s)$ at $s = T$. The value of the property immediately after development is denoted by $P(T)$. In deriving our results we assume that, immediately prior to development, land is valued for tax purposes at $P(T) - p(T)K$. Note that this will be less than the market value if land is not developed optimally.

TABLE 2
Some Comparative Statics Results for the Extended Model

	τ_a	δ	τ_b
T	+	?	−
K	0	−	−

Note.—As in the simple case, we have assumed that $\alpha'(K) < 0$, which is one of the conditions for a maximum. Another condition for a maximum is $(\eta\text{-}\delta)(i + \tau_b - \eta) > 0$. Results are based on the realistic case, $i + \tau_b > \eta > \delta$.

The comparative statics results of table 1 are not affected by these modifications; however, we present here the results for the additional variables which were introduced (see table 2). Increases in the predevelopment property tax rate and the expected rate of change in the price of capital reduce density, while the postdevelopment property tax rate has no effect on density. An increase in the predevelopment property tax rate speeds up development; an increase in the postdevelopment property tax rate postpones it. The effect on timing of the expected rate of change in the price of capital is indeterminate.

Empirical Applications

Large differences were observed in the ratio of the value of land to the value of property among Canadian metropolitan areas during the period 1972–75 (see table 3). Typically, the ratio was high in the rapidly growing regions—the West and Ontario—and much lower in areas which experienced moderate growth—the Prairies, Quebec, and the Maritimes. Although this accords qualitatively with the model, we also present a more rigorous test by comparing the actual ratios of land to property values with those predicted by the model.

Our predictions are based on equation (14):

$$\frac{V(T)_j}{P(T)_j} = \frac{\eta_j^e - \delta_j^e}{i^e - \delta_j^e},\text{[11]} \qquad (15)$$

where η_j^e, δ_j^e = expected rate of change of rental rates and construction costs in city j; and i^e = expected (real) interest rate (assumed invariant across cities). We assume for each city that the expected values of η and δ, for 1972 through 1975, were equal to their actual values over the period 1961–71 with an adjustment to allow for convergence with national average growth rates:

$$\eta_j^e = \eta_j + a(\bar{\eta} - \eta_j), \qquad (16)$$

$$\delta_j^e = \delta_j + b(\bar{\delta} - \delta_j), \qquad (17)$$

[11] This is eq. (14) with τ_b set equal to zero. In Canada agricultural land is typically given special treatment when valued for tax purposes. As a result, the effective property tax rate on undeveloped land is normally very low.

TABLE 3

An Application of the Model to Canadian Metropolitan Areas: 1961–75

City	Average Value of $V(T)/P(T)$* 1972–75	Annual Change in Rental Rates (%)† 1961–71	Annual Change in Construction Costs (%)‡ 1961–71	Predicted Value of $V(T)/P(T)$§
Victoria	.356	3.435	1.694	.355
Vancouver	.398	2.172	1.000	.275
Edmonton	.260	1.466	.772	.219
Calgary	.266	1.360	.727	.213
Saskatoon	.178	.491	−.268	.232
Regina	.168	.216	−.343	.212
Winnipeg	.268	.854	.368	.199
Sudbury	.258	3.459	1.599	.369
Windsor	.272	4.232	3.206	.255
London	.286	1.716	.726	.254
Kitchener	.324	2.414	.465	.361
St. Catherines	.287‖	2.948	1.591	.302
Hamilton	.414	2.672	.624	.374
Toronto	.366	2.458	1.197	.287
Ottawa	.289	1.025	.249	.231
Montreal	.099	.356	.447	.134
Quebec	.127	.979	.672	.176
Chicoutimi#	.093	1.178	1.135	.136
Halifax	.186	2.532	1.556	.251
St. John	.188	1.791	.990	.231
St. John's	.226	1.421	.549	.241

Sources.—Central Mortgage and Housing Corporation 1962, table 97; 1969, table 82; 1971, table 87; 1973, table 90; 1975, table 88.

Note.—Data apply to new single-detached dwellings financed under the National Housing Act.

* $P(T)$ is based on cost estimates of owner and builder applications at the time of their loan approval. It includes land, construction and other costs, but excludes the mortgage insurance fee (which was less than 1% of the total cost). These estimates appeared to correspond quite closely to actual selling prices. Land costs, $V(T)$, reflect the prices paid for lots regardless of the extent of servicing or method of financing services. Although degree of servicing may have varied across cities, this should not bias our results since differences in servicing would be reflected in land values. Note that the extent of servicing will not affect the ratio of the value of land to the value of the property.

† This was assumed to equal the rate of increase in $P(T)$ per unit of finished floor area (deflated by the CPI). Finished floor area in 1961 was computed from estimated construction costs per square foot using the assumption that the ratio of "other costs" to total costs was equal to its 1968 value (1969 value for Ottawa). These other costs were generally between 1 percent and 2 percent of the total cost.

‡ The rate of increase in construction cost per square foot deflated by the CPI.

§ Derived from the estimated equation (see eqq. [15], [16], and [17] and table 4).

‖ Includes Niagara Falls.

Includes Jonquière.

where η_j, δ_j = rate of change of rental rates and construction costs between 1961 and 1971 in city j; and $\bar{\eta}$, $\bar{\delta}$ = average rate of change of rental rates and construction costs between 1961 and 1971.

The values of a and b are estimated by substituting equations (16) and (17) into equation (15). The values are presented in table 4. Both coefficients have the expected sign and magnitude (i.e., positive and less than one) and are significant at the .5 percent level. The discount rate is estimated to be 4.1 percent also significant at the .5 percent level. The estimated equation explains 60 percent of the variance in the ratio of the value of land to property across Canadian metropolitan areas.

Another application of the model relates to the production function of housing. If the production function is assumed to be CES, we can derive from the model an estimate of the elasticity of substitution between land

TABLE 4
REGRESSION RESULTS

Coefficient	Estimated Value	t-Statistic
a	.608	7.308
b	.468	3.524
i^e	4.060	18.694
$R^2 = .598$		

NOTE.—The nonlinear estimation was performed using the TSP routine ITERAT. The values of $\bar{\eta}$ and $\bar{\delta}$ were 1.412% and .608%, respectively (Central Mortgage and Housing Corporation 1975, table 106).

and capital. Let $Q(K) = [\gamma + (1 - \gamma)K^{\rho}]^{1/\rho}$, where γ = land's distribution coefficient; and $\rho = \sigma - 1/\sigma$, σ = elasticity of substitution between land and capital. Thus, using equation (12): $p(T)K/P(T) = Q'(K)K/Q(K) = (1 - \gamma)K^{\rho}/[\gamma + (1 - \gamma)K^{\rho}]$. Taking the reciprocal and subtracting 1 from both sides gives $V(T)/p(T)K = \gamma/(1 - \gamma)K^{\rho}$, or

$$\ln \frac{V(T)}{p(T)K} = \ln\left(\frac{\gamma}{1 - \gamma}\right) - \rho \ln K. \tag{18}$$

Equation (18) is estimated with 1975 and 1976 data on new single-detached dwellings in 23 Canadian metropolitan areas,[12] where K is assumed to equal the ratio of average finished floor area to average lot size. The results are given by equations (19a) and (19b):

$$\ln \frac{V(T)}{p(T)K} = \underset{}{1.849} + \underset{(3.165)}{1.687} \ln K \quad R^2 = .323, \quad 1975 \tag{19a}$$

$$\ln \frac{V(T)}{p(T)K} = \underset{}{2.397} + \underset{(3.320)}{1.927} \ln K \quad R^2 = .344, \quad 1976 \tag{19b}$$

where numbers in parentheses are the t-statistics. Both coefficients on ln K are significant at the .5 percent level. For 1975 the implied value of the elasticity of substitution is .372; for 1976 it is .342.[13] These low values are encouraging for they suggest that the model does indeed generate a local maximum.

Conclusions

We have presented a simple but powerful model to explain aspects of new urban development. The simplest version of the model predicts that development will take place when the ratio of the value of the land to the value of the postdevelopment property is equal to the ratio of the expected growth

[12] Central Mortgage and Housing Corporation 1976, table 91. Data on average lot size were provided to us by the Central Mortgage and Housing Corporation. These data will be made available by the authors on request.

[13] These elasticities of substitution are significantly less than one at the .5 percent level. The implied distribution coefficients for land are .864 for 1975 and .917 for 1976.

rate of rents to the interest rate. When adjusted to allow for changing construction costs, the model generates a result which corresponds quite closely to the experience of Canadian cities during the early 1970s. We also obtained some rather surprising comparative statics results relating to development density. For example, density does not depend on current construction costs, current rental rates, or postdevelopment property tax rates, but rather on interest rates and expected rates of change of construction costs and rental rates. Finally, we applied our model in estimating the elasticity of substitution between land and capital in the production of housing. Our estimates are quite low. They average about .36.

There are possible extensions to the model which we did not consider. We assume that building freezes land in a particular use forever. This assumption could be modified to allow for eventual replacement of old housing with new structures. A related assumption is that buildings do not depreciate. We also avoid the problem of uncertainty by assuming that building decisions are based on expected values of the relevant variables. This treatment is appropriate only if developers have rational expectations and are risk neutral. In spite of these limitations the model predicts the Canadian experience quite well and should provide a useful step in explaining development in urban areas.

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