American Economic Review: Papers & Proceedings 2014, 104(5): 37–43
http://dx.doi.org/10.1257/aer.104.5.37

The Natural Rate of Interest and Its Usefulness for Monetary Policy†

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Knut Wicksell characterizes the natural rate as the real interest rate that yields price stability and would equate real saving and investment in an (otherwise equivalent) nonmonetary Walrasian economy.

> *There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods.*
>
> —Knut Wicksell
> *Interest and Prices*, 1898 p. 102

In identifying the price-stabilizing rate with the real general equilibrium rate, Wicksell gets close to describing what has been labeled the divine coincidence of the benchmark New Keynesian Model (Woodford 2003; Blanchard and Galí 2007). In this context, the notion of a natural rate of interest is unambiguous and most clearly meaningful. Therefore, for motivation and as a point of comparison with the richer model used later in the quantitative analysis, we begin by describing the natural rate in the canonical New Keynesian (NK, hereafter) model without wage stickiness and capital accumulation as the real interest rate prevailing in an economy with flexible prices.

Following the derivations of the canonical NK model in Galí (2008), we characterize the expressions for the marginal cost, the Euler equation for consumption, and the Phillips Curve in terms of the gaps between output and the real interest rate from their natural counterparts. Substituting the consumer's static labor supply condition $w_t - p_t = (s^{-1}y_t + \varphi n_t)$ into the usual Cobb-Douglas specification of log real marginal cost mc_t, and rearranging, mc_t can be expressed in terms of (log) output y_t and technology a_t as $\left(s^{-1} + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha)$.[1] Setting this expression equal to the inverse desired markup $-\mu$ (presumed constant over time) and solving for the associated flexible price level of output y_t^n, we characterize natural output as $y_t^n = \psi_{ya}^n a_t + \vartheta_y^n$, where $\psi_{ya}^n \equiv \frac{1 + \varphi}{s^{-1}(1 - \alpha) + \varphi + \alpha}$ and ϑ_y^n is an inessential constant.

The consumer Euler equation with $c_t = y_t$, including a time-varying second-order precautionary saving term associated with the conditional variance, $\text{var}_t[y_{t+1}]$, is $y_t = E_t[y_{t+1}] - s(i_t - E_t[\pi_{t+1}] - \rho_t) - \frac{1}{2}s^{-1}\,\text{var}_t[y_{t+1}]$. The *natural rate of interest* r_t^n must satisfy $E_t[\Delta y_{t+1}^n] = s(r_t^n - \rho_t) + \frac{1}{2}s^{-1}\,\text{var}_t[y_{t+1}^n]$; where ρ_t is the (possibly time-varying) subjective rate of time preference. Simple algebraic manipulations lead to $r_t^n = \rho_t + s^{-1}E_t(\Delta y_{t+1}^n) - \frac{1}{2}s^{-2}\,\text{var}_t[y_{t+1}^n]$. Substituting in for the growth of natural output

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† Go to http://dx.doi.org/10.1257/aer.104.5.37 to visit the article page for additional materials and author disclosure statement(s).

[1] We follow Galí's (2008) notation, with n and φ denoting labor and the inverse Frisch elasticity, respectively, but depart from him by using s for the intertemporal elasticity of substitution. The log-linear aggregate production function reads $y_t = a_t + (1 - \alpha)n_t$.

one can write the following definition for the natural rate r_t^n:

$$(1) \qquad r_t^n = \rho_t + s^{-1}\psi_{ya}^n E_t[\Delta a_{t+1}] - \frac{1}{2}s^{-2}(\psi_{ya}^n)^2 \operatorname{var}_t[a_{t+1}].$$

If we write the Euler equation in terms of the output gap, $\tilde{y}_t \equiv y_t - y_t^n$, i.e., the difference between actual and natural output, we can also see that[2]

$$(2) \qquad \tilde{y}_t = -s \sum_{k=0}^{\infty} E_t(r_{t+k} - r_{t+k}^n).$$

The last expression makes clear that the output gap is the sum of all future real interest rate gaps, defined as the deviations of the ex ante real rate, $i_t - E_t\pi_{t+1}$, from the natural rate, r_t^n. Finally, from the NK Phillips curve, $\pi_t = \beta E_t[\pi_{t+1}] + k\tilde{y}_t$, closing the output gap $\tilde{y}_t$ also stabilizes inflation.

There are two takeaways from this simple model. First, the natural rate is increasing in the subjective rate of time preference, ρ_t, and the expected growth rate of technology, $E_t[\Delta a_{t+1}]$, and decreasing in the conditional variance of future technology, $\operatorname{var}_t[a_{t+1}]$. Increases in patience, i.e., declines in ρ_t (often labelled discount factor or "beta" shocks), lower r^n, as does a reduction in expected productivity growth and higher uncertainty about future productivity (due to an increase in precautionary savings). Second, an interest rate path in which the actual real rate is always equal to the natural rate achieves both an output gap of zero (in the sense that output is at natural, i.e., flexible price equilibrium level) and zero inflation.

Equation (1) shows that an uncertainty shock is isomorphic to the discount shock and indeed may provide one attractive structural interpretation of that rather reduced-form construct. Although movements in r_t^n due to realistic *aggregate* technological uncertainty is rather small for reasonable calibrations, it can be far larger if heterogeneous agents face *idiosyncratic* shocks (Aiyagari and Gertler 1991 and Huggett 1993). Thus in a richer economy uncertainty shocks can have a significant depressing effect on the natural rate.[3]

I. A State-of-the-Art DSGE Model

Though the canonical New Keynesian model of the last section provides motivation and intuition for the natural rate and its determinants, it is far too stylized to be taken directly to the data. For that purpose, we build on the well-known framework by Smets and Wouters (2007). Compared with the stylized model of the last section, the Smets and Wouters' model includes price and wage stickiness, backward-looking components in wage and price setting, habit formation, nonseparable utility in consumption and leisure as well as investment subject to adjustment costs. In addition to stationary variations in the level of technology, it is buffeted by shocks to the marginal efficiency of investment and stochastic cost-push shocks in wage and prices.

There is an additional disturbance that Smets and Wouters call a "risk shock." As described below, in our model risk shocks play a prominent role in explaining business cycle fluctuations and a major role in triggering the Great Recession. Although this shock is of course not identical to the uncertainty shock in the stylized model (indeed, we log-linearize, thereby removing the role of risk) it is analogous on the consumer side, in that it lowers the required return to saving and reduces consumption. The "risk shock," however, is not simply isomorphic to a "beta shock" in that reductions in consumption by the latter are associated with expansions in investment.

We introduce some important departures in specification from Smets and Wouters (2007), which are now described, together with the empirical rationale for their inclusion and the data brought to inform them. The reader is referred to the online Appendix for additional details. First, policymakers are assumed

[2] We impose that the gap-creating consequences of sticky prices do not last forever; that is, $\lim_{T\to\infty} E_t[\tilde{y}_{t+T}] = 0$.

[3] At least two other disturbances that could provide an underlying foundation for discount shocks come to mind: Eggertsson and Krugman (2012) sketch a two-agent economy in which a loss of net worth forces deleveraging and hence reduced consumption on the part of the borrower group, requiring a fall in the real interest rate to induce a compensating increase in the consumption of the lenders. But even when agents purchase durables with their own funds, efforts to reduce the durables stock in response to downward revisions of permanent income also lower the natural rate much in the fashion of a beta shock (Hall 2011).

to respond to four quarter averages of current, expected, and two lags of inflation, as well as the deviation of GDP from the model's linear trend. Second, based on the evidence presented in Gurkaynak, Sack, and Swanson (2005), we incorporate Forward Guidance (henceforth, FG) regarding the future conduct of monetary policy. Following the methodology in Campbell et al. (2012) and Campbell, Fisher, and Justiniano (2012), agents receive news regarding the future paths of the federal funds rate, governed by two latent variables referred to as the *Target* and *Path* factors.[4] FG is informed with market-based expectations of the fed funds rate obtained from Fed Funds, Eurodollar, and OIS futures contracts. Hence, the model accounts for agents' evolving expectations regarding the duration of the zero lower bound (ZLB) since the Great Recession.

Third, we introduce a slow moving inflation drift in the policy rule. This primarily accounts for the stability of long-run expected inflation since 1997, but will also capture the effects, if any, of unmodeled unconventional policy actions on agents' inflation expectations. The drift is informed by matching model-based average expected inflation over the next 40 quarters with median 10 year expected inflation from the Survey of Professional Forecasters.

Fourth and finally, instead of matching the model's concepts of price and wage inflation with a single series for each, we rely on multiple indicators. This approach diminishes the importance of cost-push shocks for cyclical fluctuations, as shown by Boivin and Giannoni (2006) for the case of goods prices, and by Justiniano, Primiceri, and Tambalotti (2013) for wages. For this reason, our estimate of the natural rate is fairly robust to the interpretation of these disturbances as efficient or inefficient (Section IV).

The estimation sample is 1990:I–2013:II, allowing for a break in all parameters in 2008:III, and centering the prior for the short second subsample at the first subsample estimates.[5] To the seven observables used in Smets and Wouters (2007) we add two price and one wage series, long-run expected inflation, and 4 and 10 quarters of market-based federal funds rate expectations in the first and second subsamples, respectively.[6]

A. *Defining the Natural Rate in the Richer Economy*

Unlike the canonical model in the first part of the paper, a richer economy, which is subject to *inefficient cost-push shocks* (hereafter, markup shocks) and to nominal rigidities in both price and wage setting, does not have a unique, unambiguous definition of the natural interest rate (or output). One might define the natural rate as the rate that would prevail if both wages and prices were perfectly flexible. However, with markup shocks the associated flexible wage and price equilibrium would not be "welfare relevant" (Woodford 2003). We choose to define the natural real interest rate and level of output as those that would have prevailed in an economy with neither nominal rigidities nor price and wage markup shocks. In the absence of a distorted steady state, this definition also corresponds to the efficient economy. However, since we do not undo the effect of steady-state markups, real disturbances affect the equilibrium of the natural and efficient economies differently (Woodford 2010, Section 3.4.3).

II. The Natural Rate Is Volatile and Procyclical

Figure 1 presents the filtered (one-sided) and smoothed (two-sided) estimates of the natural rate (on an annualized basis), which follows a highly procyclical pattern characterized by fairly pronounced swings. Perhaps surprisingly, we do not observe a substantially larger drop during the Great Recession than in the previous two downturns. However, in stark contrast with earlier recessions, it has remained negative since 2008. This last finding is mainly explained by the highly persistent negative risk shock that according to our model triggered the Great Recession and is responsible for the ensuing slow recovery.

[4] The target factor is the only common component correlated with changes in the current federal funds rate.

[5] More precisely, we first estimate through 2008:III using a prior almost identical to Smets and Wouters (2007). This mode becomes the center of the prior for the second subsample, with the initialization of the filter based on the mean and variance of the state in 2008:III.

[6] Priors and posterior moments of the model parameters are available upon request.

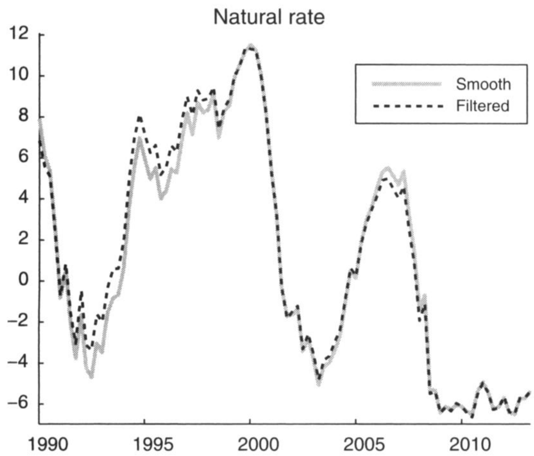

FIGURE 1. ONE-SIDED (*filtered*) AND TWO-SIDED (*smoothed*) ESTIMATES OF THE NATURAL RATE

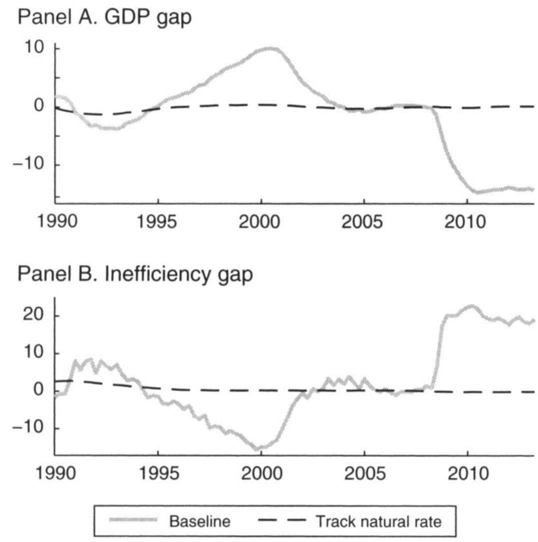

FIGURE 2. OUTPUT AND INEFFICIENT GAPS UNDER ESTIMATED INTEREST RULE AND WHEN TRACKING THE NATURAL RATE

III. The Natural Rate and Its Relevance

As discussed earlier, tracking the natural rate would accomplish full macroeconomic stabilization in models where the divine coincidence holds. Unfortunately, even in the absence of markup shocks, this policy does not necessarily deliver such a desirable outcome in the presence of both price and wage rigidities (Woodford 2003, p. 443; Galí 2008, Chapter 6) and in the presence of a distorted steady state (Woodford 2010, Section 3.4.3). Additional trade-offs arise from the presence of markup shocks, which do not affect the natural economy and hence the natural rate of interest.

Even if setting the nominal interest rate to target the natural rate is not guaranteed to achieve full stabilization of inflation and the output gap, according to our model pursuing this policy, had it been feasible, would have considerably diminished the volatility of these variables in the last 25 years—including the Great Recession.

To see this, we compare macroeconomic outcomes depending on the assumptions regarding the conduct of monetary policy. The solid line in panel A of Figure 2 shows the inferred output gap—defined as the difference between actual and the natural level of output—under the estimated Taylor rule. The dashed line captures instead the counterfactual output gap that would have arisen had the Federal Reserve tracked the natural rate period by period.[7] This policy would have accomplished a considerable reduction in the output gap, even during the Great Recession, as well as its variability. Panel B of Figure 2 shows the *inefficiency gap* (also known as the labor wedge), which following Galí, Gertler, and Lopez-Salido (2007), is defined as the wedge between the marginal rate of substitution between consumption and leisure and the marginal productivity of labor. This turns out to be essentially the reciprocal of the output gap and once again it is drastically reduced under the counterfactual policy.

Furthermore, the ratios of actual (nominal) wage inflation variance to the counterfactual variance had policy tracked the natural rate are 3.7 and 23.3 in the first and second subsamples respectively. The fall in the corresponding ratios for price inflation is more moderate at 1.9 and 1.7. Thus these findings suggest that a considerable degree of wage and price inflation stabilization could also have been achieved if the Federal Reserve had effectively tracked the natural rate.

[7] We feed the smoothed sequence of all shocks (including markups) and replace the estimated interest rule with $i_t = r_t^n + 1.0001\, E_t\, \pi_{t+1}$, where i_t is the nominal short-term interest rate, r_t^n is the natural interest rate, and $E_t\, \pi_{t+1}$ is the next period's expected inflation.

Therefore, despite the departure from the divine coincidence, the natural rate of interest seems to be a useful benchmark for policymakers. Abstracting for the time being from important considerations about the implementability of such a policy (Section IV), our findings suggest that tracking the natural rate would have stabilized the output and inefficient gaps as well as inflation in prices and wages. Overall, these results echo the findings in Justiniano, Primiceri, and Tambalotti (2013), and as in their case, rest on the predominance of demand fluctuations, such as the risk shock, which move price and quantities in the same direction, and on the smaller contribution of price and wage markup shocks to fluctuations in economic activity. However, our natural rate need not correspond with the optimal real rate arising from the solution of the Ramsey problem faced by the policymaker, as characterized by Benigno and Woodford (2005). Yet, Justiniano, Primiceri, and Tambalotti (2013) show in a similar model that closing the output gap—as implied by our natural rate—aligns with optimal policy. Verifying that this approximation holds in our model is an interesting topic for future research.

IV. Discussion

In Section III, we show that if the Federal Reserve had tracked the natural rate, it would have accomplished a substantial degree of macroeconomic stability. However, Figure 1 shows that the natural rate of interest falls to −4 percent or lower in each of the three recessions similar to the findings in Curdia et al. (forthcoming). This implies that it would not have been possible to stabilize the output gap and to maintain a steady inflation rate, say of 2 percent per year, as this would have required a negative nominal rate during each trough. This suggests that methods to deal with the ZLB should be part of the standard policy framework. In characterizing the optimal monetary policy in the presence of a binding ZLB constraint, Eggertsson and Woodford (2003) propose a commitment by the central bank to raising the nominal interest rate slowly at the time the natural rate will become positive in order to affect expectations already when the ZLB is binding.[8] In this light, the natural rate remains a useful statistic for two reasons: (i) to understand the path of interest rate required to implement the desired paths for the output gap and inflation and (ii) to forecast how long the ZLB will continue to constraint policy. In this second respect, the highly persistent drop of the natural rate of interest after the Great Recession (Figure 1) has critical implications for the policy proposed by Eggertsson and Woodford. A natural way to implement this proposal is for the central bank to make explicit statements, often called *forward guidance*, about the future path of short-term interest rates. Indeed, our model suggests that forward guidance contributed considerably to real activity in 2003–2004 as well as since the more explicit language adopted by the FOMC in August 2011.[9]

Other concerns for using the natural rate of interest as a useful benchmark for monetary policy are the availability of real time estimates of this latent variable as well as the seemingly implausible requirement of discerning efficient from inefficient cost-push shocks.[10] On the first issue, note that one-sided and two-sided estimates of the natural rate are reasonably close (Figure 1). On the second, it is interesting that the main results in this paper are quite robust to assuming all cost-push shocks are not efficient (baseline) or the polar opposite case in which they are treated as efficient fluctuations, and therefore included in the definition of the natural rate of interest and output. As mentioned earlier, this robustness property stems from the diminished importance of these shocks—compared to models without multiple indicators and idiosyncractic disturbances—in explaining real fluctuations.

Finally, in this short article we do not directly address other important issues such as the

[8] Krugman (1998) and Werning (2012) have also advocated the use of this type of policy to provide more accommodation when the policy rate is stuck at its zero bound. Bianchi and Melosi (2013) show that committing to systematically inflating away the portion of public debt accumulated during severe economic downturns would also be a powerful device to raise short-term inflation expectations.

[9] According to our estimates, forward guidance (given by the *path factor*) contributed almost a full percentage point to quarterly GDP growth (on a quarterly basis) in 2003 and since 2011. Conversely, it dragged GDP by roughly 1 percent at the trough of the Great Recession.

[10] As noted by Woodford (2003, p. 454), it is often difficult quantitatively to tell whether a particular real disturbance distorts the economy towards inefficiency or simply affects the efficient level of output and interest rates.

appropriate characterization of the uncertainty surrounding our estimates of the natural rate, including its sensitivity to alternative specifications of the model or its implications for equilibrium determinacy.

V. Concluding Remarks

A fairly rich DSGE model indicates that since 1990 the natural real rate of interest, defined as the real rate of an economy with neither nominal stickiness nor cost-push distortions has been quite variable and highly procyclical. The natural rate turned negative in the last three recessions and has remained persistently depressed since 2008. We find that the natural rate could be a useful benchmark for the Federal Reserve in so far as tracking this rate would have significantly stabilized the output and inefficient gaps while also decreasing the variability of price and wage inflation. Nevertheless, the recurrently binding zero lower bound and the difficulty of computing the natural rate in real time pose nontrivial challenges for the practical use of this rate to guide monetary policy.

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