Taxation and the Conservation of Resources

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TAXATION AND THE CONSERVATION OF RESOURCES[1]

SUMMARY

I. Objectives of the Study

Analysis of the relations between existing social institutions and conservation of resources is essential for intelligent public action in the field of conservation. Such relations, if unrecognized, may lead to socially undesirable resource utilization, while knowledge of them may make it possible either to utilize social institutions effectively as tools of conservation policy or to modify them to prevent interference with conservation objectives. The tax system is only one of many social institutions that are important in this connection. The great increase in tax burden that will result from the present war makes analysis urgent at the present time. The way in which war taxation is handled may either jeopardize past and future public action in the field of conservation, or it may bring about overdue tax reforms through utilizing the present tax consciousness and political willingness to accept changes in social institutions.

A further reason making analysis of the relation between taxation and conservation especially attractive is the existence of an "empty box" in taxation theory. If conservation is understood as defined below (Section III), we are dealing with the effect of

1. Giannini Foundation Paper No. 110.

different forms of taxation on the distribution of rates of production over time. The impact, the shifting, the incidence, and the burden of taxation[2] appear in a new light, if the "instantaneous" approach to these problems is supplemented with the tools of "time economics."

In taxation theory from Ricardo, Thünen, and Senior to Wicksell and Seligman, the tools of long-run equilibrium analysis predominate.[3] Marshall deals with the short-run effects of a tax implicitly.[4] Only a few studies[5] recognize the importance of the short run, and still fewer deal explicitly with the effects of taxation upon changes of rates of production over time. Among the latter, Hotelling, in his famous pioneering study,[6] deals with two types of taxes in the field of "exhaustible" resources under simplifying assumptions as to the interrelation of rates of use in different instants of time. Fairchild is concerned with the effects of the general property tax on deferred-yield utilization plans in forestry.[7] His study gives an excellent illustration of tax shifting over time in a special case, but its significance for resource utilization in general is not elaborated. Fagan's recent study[8] is confined to the effects of taxation on the disposition over time and on prices of goods already produced. Only two "market periods" are assumed, and the effects of taxation on the time distribution of rates of pro-

2. We follow the terminology of Seligman's standard work, The Shifting and Incidence of Taxation. Fifth edition, pp. 1–15. New York, 1932.

3. Ricardo, D., Principles of Political Economy and Taxation. London, 1929. J. H. von Thunen, Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalokonomie. Third edition. Jena, 1930. Knut Wicksell, Finanztheoretische Untersuchungen nebst Darstellung und Kritik des Steuerwesens Schwedens. Jena, 1896. E. R. A. Seligman, op. cit.

4. Marshall, A., Principles of Economics. Eighth edition, p. 415. London, 1930.

5. Edgeworth, S. Y., "Theory of Taxation." Economic Journal, March, 1897, and June, 1897.

Nicholson, J. S. Rates and Taxes as Affecting Agriculture. London, 1905.

Great Britain. Treasury. Report of Committee on National Debt and Taxation (Colwyn Report). London, 1927.

Fagan, Elmer D. and Roy W. Jastram, "Tax Shifting in the Short Run." This Journal, August, 1939.

6. Hotelling, Harold, "The Economics of Exhaustible Resources," Journal of Political Economy, April, 1931. Particularly sections 13 and 14.

7. Fairchild, Fred Rogers, and associates, Forest Taxation in the United States. United States Department of Agriculture, Miscellaneous Publication 218. October, 1935.

8. Fagan, Elmer D., "Tax Shifting in the Market Period." The American Economic Review, March, 1942.

duction are not considered. The present study is an attempt to make a contribution to taxation theory in the latter direction. The impact, the shifting, the incidence, and the burden of taxation are of interest here only as they throw light upon this problem.

II. Classification and Interrelation of Resources

The effects of taxation upon the time distribution of production are highly important, not only in the utilization of "natural" resources, but also in the use — in contrast to sale — of durable producer and consumer goods. Changes in the time distribution of use of durable goods brought about by taxation are only vaguely touched upon in Seligman's discussion of "transformation" of taxes and of tax evasion through "deterioration" of taxed goods.[9] Space does not permit a detailed classification of resources, but a sketch appears necessary to prevent some common misunderstandings, and to serve as a basis for the definition of "conservation."

The concept "resource" presupposes that there is a subject with a certain end in view appraising the usefulness of his environment for obtaining this end on the basis of his knowledge of suitable ways and means. A resource, therefore, is a highly relative concept changing with the "ends-means scheme," that is, with the appraising subject, with the end in view, and with the knowledge to obtain the end. The subject, which will be called the "planning agent," may be a real person (e.g. an individual entrepreneur), an abstract entity (e.g. the directorate of a corporation), or a social group, as a whole or represented by its government. The end may be not only material, but also spiritual human wants. The ways and means are called in economics the "state of the arts."

Following common usage, three broad classes of resources — natural, cultural, and human — may be differentiated. Natural resources represent an appraisal of the natural environment, cultural resources an appraisal of the cultural environment, and human resources an appraisal of all aspects of human usefulness in a given ends-means scheme. Durable producer and consumer goods may be termed tangible cultural resources. Social institutions, rights, and similar relations may be regarded as intangible cultural resources. A significant portion of the problems of natural and tangible cultural resources can be discussed quantitatively in

9. Seligman, E. R. A., op. cit. pp. 1–15, and Part II, Chapters 1–3.

terms of monetary value. This is not true for the problems of human and intangible cultural resources, and hence the latter are best dealt with in social sciences other than economics. In this study, therefore, the term "resources" will be used to refer to natural resources and tangible cultural resources.

Resources are usually classified with respect to their exhaustibility.[1] Exhaustibility and inexhaustibility are concepts which are of interest only if they are employed in the economic sense. A resource may be exhausted — in the sense that further utilization is indefinitely discontinued — long before the resource is physically used up or even appreciably diminished. This is because the costs of producing any possible quantity of the resource in an additional instant[2] of time may be larger than the revenues which could be obtained from these quantities. We are dealing here with expected changes of revenues and costs over time, not with movements along "instantaneous" curves. The interrelation of revenues and costs of differently dated rates of use, the causes of which will be discussed presently, makes it advisable to speak about revenue and cost functions rather than curves or schedules.

All factors which cause changes of revenue and costs over time are in principle of equal interest for the problem of exhaustibility. Among these factors are, obviously, changes of wants and changes in the state of the arts. In resource utilization, however, two factors are of special economic interest which operate even if changes of wants and changes in the state of the arts are excluded. There is, first, the use or, more precisely, the cumulative use[3] of resources itself; second, variations in the physical quantity and quality of resources over time, regardless of use. Cumulative use may cause

1. Fernow, B. E., Economics of Forestry, p. 10 New York, 1902. Gray, L. C., "Economic Possibilities of Conservation." This JOURNAL, May, 1913. R. H. Hess., Conservation and Economic Evolution," p. 117 (in R. T. Ely, R. H. Hess, C. K Leith, and T. N Carver, The Foundations of National Prosperity. New York, 1917). E. W. Zimmerman, World Resources and Industries, p. 796. New York, 1933.

2. An "instant" may be defined as a period of time during which changes in revenues and costs can be neglected. The actual extent of an instant in terms of "clock" time may be any period — a day or a part thereof, a week, a month, a year, or a number of years (a crop rotation, for instance), depending on the nature of the problem to be solved.

3. Cumulative use is the integral of rates of use over instants of time. Conversely, rate of use is the first derivative of cumulative use with respect to time. If $X(t)$ stands for cumulative use and $x(t)$ for rate of use, then $x = \frac{dX}{dt}$.

changes in revenues because the accumulation of durable products and of scrap affects demand; accumulation of cut diamonds and of metal scrap influence the demand for uncut diamonds and for ores.[4] Cumulative use may cause changes in costs because it becomes necessary to drive mine shafts deeper, to follow thinner veins, to utilize ores of lower metal content, to drill oil and water wells deeper, or to pump from greater depth. Changes in quantity and quality of a resource over time, regardless of use, may affect revenues and costs in many ways. For example, the growth of timber affects both revenues and costs; the "blowing off" of natural gas affects costs of both gas and oil; changes over time in the total quantity of fish and other wild life resources available may affect costs decisively.

It is apparent, then, that the differentiation between exhaustible and inexhaustible resources indicates at best a difference of degree. Even if changes of wants and of the state of the arts are excluded by definition, such a differentiation involves knowledge of the time pattern of revenues and costs under the influence of cumulative use and of physical variations of resources without use. It is more helpful for the further analysis to make the differentiation on a somewhat different basis. The main classes of resources will be called "stock resources" and "flow resources," each of which in turn may be divided into a number of subclasses. The two sets of terms are by no means identical. Many flow resources are exhaustible and many stock resources are practically inexhaustible.

Resources are defined as stock resources if, without use, the resource does not vary significantly[5] with time or decreases with time. Each rate of use then diminishes some future rate of use. It is impossible to maintain use permanently.[6] With some stock

4. Theoretically, these effects disappear in "long-run" equilibrium. After oscillations depending on rates of deterioration, demand schedules will become stable over time.

5. Strictly speaking, some stock resources increase over time at a rate which is too small to be considered from the economic viewpoint. From the point of view of the geologist, for instance, the formation of coal and ore may still continue, and the quality improving or deteriorating. But such variations may be legitimately disregarded from the point of view of the economist. Existence of uncertainty imposes comparatively narrow limits on the economic "horizon," as explained below.

6. "Permanently" implies an infinite number of instants. It is a fundamental theorem in mathematics that any class of elements which can be

resources, on the other hand, it may be possible to maintain use over so many instants that the physical limitations on their quantity are economically irrelevant. This possibility, of course, depends on the quantitative relation between rate of use and stock, which in turn depends on revenue and cost conditions. But ordinarily the physical limitations on the quantity of stock resources are highly relevant economically. The ends-means scheme which defines a resource usually implies strict locational and qualitative characteristics. For such a resource as coal, it is not the stock in the whole cosmos which is of interest in economics, but the coal in a certain locality, at a certain depth, with certain width of seam, caloric value, and so on.

It is convenient to differentiate between two major sub-classes of stock resources, namely, those the quantity and quality of which do not vary at all with time (without use), and those which show deterioration. Ores, coal, clays, and stones are examples for the first sub-class. Oil and gas belong to the second sub-class in cases in which quantitative and qualitative changes take place through seepage and blow off.

Resources are defined as flow resources if different units of a resource become available at different instants of time. Rates of flow may increase or decrease over time without use. No rate of flow diminishes a future rate of flow, and it is possible to maintain use permanently, provided the flow is permanent. Two distinct sub-classes may be differentiated, according to whether the flow is or is not independent of human action.[7] The first class comprises mostly resources the flow of which is permanent and characterized by constant cyclical variations.[8] From cycle to cycle, variations in rates of use are determined by changes in wants and in the state of the arts, not by the effects of human action upon the flow. Examples are solar radiation, precipitation, tides, and winds. In the second class, human action in one instant may

exhausted by taking elements away one by one cannot be an infinite class. Cf. Edward V. Huntington, The Continuum. Cambridge, 1929. Section 27, pp. 22–23.

7. The term "human action" is employed here instead of the term "use," because flow may be increased by positive conservation efforts, as explained later. Decreases in rates of flow caused by human action are the effects of use in the widest sense, including the effects of carelessness and intentional destruction.

8. Here again "permanent" and "constant" have meaning only in the economic sense.

decrease or increase some or all rates of flow in future instants. This, in turn, affects the time pattern of revenues and costs and, therefore, of rates of use.

The second class of flow resources may be divided into two sub-groups, according to the existence or nonexistence of a "critical zone" in the decrease of rates of flow. By this is meant a rate of flow or range of rates below which a decrease becomes technically irreversible or economic obstacles against reversibility grow very rapidly. The decrease in the flow of animal and plant life, for instance, becomes technically irreversible within a certain species if the rate of flow reaches zero, that is, if the breeding stock is destroyed. Even if the rate of flow has not reached zero, economic possibilities of reversibility may rapidly decrease if highly complex biological and technical relations are affected. A decrease in the flow of crop yields, for instance, can be relatively cheaply reversed as long as the decrease is due to depletion of plant nutrients, increase in soil acidity, deterioration of soil structure, or deposition by floods; but if the *A*-horizon has suffered greatly because of sheet erosion, or if deep gullies have been formed which interfere with farm operations, or if all soil has been destroyed through erosion to bedrock or hydraulic mining, economic possibilities for reversibility are greatly diminished.

Durable producer and consumer goods may be regarded as stock resources, if attention is focused on them as use bearers, and as flow resources of the second sub-class, if the productive services obtained from the use bearers are considered. The latter is usually more realistic, because for technical reasons productive services become available only over a number of instants and the flow can often be increased or kept constant, as well as decreased, through human action.

The use of different resources is, of course, interrelated. This means that revenues and costs are functionally related, not only to the rates of use of the resource which is considered, but also to the rates of use of a great number of other resources — strictly speaking, the use of all resources. Use of natural resources is obviously related to that of tangible cultural resources, and many relations exist among natural resources themselves. For example, the use of coal is related to the use of iron ore, because they complement each other in a quantitatively dominant use, the production of pig iron and steel; the use of coal is related to that of water power and other fuels, because they compete with each other in important uses; the use of silver ore is related to that of copper ore, because both are yielded by joint productive processes.

For economic analysis, interrelations between rates of use of different resources have in principle the same significance, and can be studied with the same theoretical tools, as interrelations of differently dated rates of use of the same resource. As shown later,[9] both types of interrelations can be classified as complementary, competitive, and independent, on the basis of the same criteria. This study is concerned with the influence of taxation upon the interrelations of differently dated rates of use. Rates of use of other resources except the taxed resource will be considered explicitly only in a few places where the effects of taxation make this desirable. Otherwise the relations between taxation and conservation would be submerged by the interrelations of different resources. Implicitly, these interrelations are taken into account in cost and revenue functions, and the following argument can without difficulty be extended to them.

III. The Meaning of Conservation and the Optimum State of Conservation

Before the term "conservation" was deliberately adopted in 1907[1] by the conservation movement in the United States, it was used mainly in the spiritual meaning, with positive or negative moral connotation, of keeping institutions, prerogatives, ideals, and the like "unimpaired," in the "status quo," on the "present level." In relation to resources, terms like "preservation," "protection," and "wise use" appeared in the literature on conservation during the last quarter of the nineteenth and the beginning of the present century. Since its adoption, however, the term "conservation" has been used, not only for flow resources, which, as we have seen, may under certain conditions be kept on the present level of productivity in spite of use, but also for stock resources, which are necessarily impaired through use. To cite only a few examples, people commonly speak of the conservation of oil, natural gas, coal, ores, and already produced strategic metals. For this reason alone it is inadequate to define the mean-

9. Section IV and Mathematical Appendix, § 3.

1. Cf. Pinchot, Gifford, "How Conservation Began in the United States." Agricultural History 11 (4): 255–265.

ing of conservation as maintenance of the status quo. Furthermore, such a definition is highly misleading for flow resources. Conservation of crop yields, of timber growth, of fisheries, of stream flows, if defined as maintenance of an original or present level of productivity, not only contradicts common usage but is economically inadequate. Actions aimed at slowing down a decrease in the rate of flow or at increasing rates of flow are economically as important as actions aimed at keeping a flow constant. With respect to rates of use, we have already emphasized that they are rarely constant over time; first, because of changes in wants and in the state of the arts; second, because of the effects of cumulative use and of qualitative and quantitative variations in resources without use; and third, because of competitive and complementary relations between different resources. Constancy of rates of use is merely a hypothetical special case in resource utilization, which has neither been adopted as criterion in the common use of the term "conservation" nor has significance for scientific terminology. Such a restriction of the concept makes it possible to use the somewhat crude tools of instantaneous analysis at the expense of oversimplifying or evading the essential theoretical problems and of preventing practical use of the theory in more complex economic problems; for instance, in analyzing the effects of taxation.

As used in this study "conservation" and its corollary "depletion" refer to actions that result in changes in the distribution of rates of use of resources over instants of time. The subjects of these actions, the planning agents, are defined by the "ends-means scheme" implied in the concept "resource," as indicated above. In "conservation" the redistribution of rates of use is in the direction of the future; in "depletion," in the direction of the present.

The terms "in the direction of the future" and "in the direction of the present" need explanation. They could be defined simply on the basis of the time sequence of increases and decreases in rates of use. This would be satisfactory if mere qualitative differentiation between conservation and depletion were sufficient, and if increases and decreases were contiguous (that is, without alternations between the two) or, as a special case, if all changes were of the same sign. Although these conditions are fulfilled for many practical problems, there are others that involve further complications. A satisfactory definition of conservation and depletion must be applicable to all conceivable alternations between positive and negative changes in rates of use, and must provide for comparing different degrees, that is, for quantitative measurement of conservation and depletion. That can only be done by considering the aggregate of all changes, having due regard to their sign, and using as weights their distances from the instant in which the action of planning agents takes place, that is, "the present" in order to avoid cumbrous language. In this study the first power of distance rather than higher powers or more complicated functions of distance is used for weighting. This is admittedly arbitrary. But the particular function of time which is used has no effect upon the subsequent argument. Theoretically, the function used here is in no way inferior to any possible alternative. It has the practical merit of being the simplest and of being identical with that function of time which is generally used in economic problems; for instance, in compounding and discounting.

We may then define as conservation, changes in the utilization plan brought about by actions of planning agents, if the aggregate weighted change in rates of use is greater than zero. Correspondingly, we have depletion if the aggregate weighted change is smaller than zero. If the aggregate weighted change is equal to zero, there is neither conservation nor depletion. We may define the degree of conservation or depletion as the ratio of the aggregate weighted change in rates of use to the existing aggregate weighted rates of use expressed in per cent.[2]

It may be well to note the differences between conservation and investment, and between depletion and disinvestment. Conservation and depletion refer to *physical* changes in the time distribution of rates of use of individual resources. Investment and disinvestment refer to *value* changes of total capital of persons, firms, or whole social groups as a result of differences between income and consumption (in value terms) in the same instant (Keynes) or in the previous instant (Robertson).[3] Investment and

2. In the Mathematical Appendix, § 1, these definitions are mathematically formulated, and their differences from related concepts, for instance, "average weighted change" in rates of use and "average weighted distance of changes" in rates of use are discussed.

3. Although the terms "investment" and "disinvestment" are used ambiguously by many authors, all agree that value changes of total capital and not physical changes in the use of individual resources are under consideration. Space does not permit discussing this point fully. Abundant references are found in G. von Haberler, Prosperity and Depression, pp. 170–254. Geneva, 1937.

disinvestment have no necessary relation to rates of use of individual resources. It often happens that investment results in depletion and disinvestment in conservation. Even if "investment" and "disinvestment" are used in the popular sense as referring to individual resources, they are not identical with conservation and depletion. For example, investment in oil fields, timberlands, or mines through purchase may be accompanied by depletion and disinvestment through sale by conservation.

It follows from our definitions that conservation and depletion have no necessary relation to changes in the unweighted aggregate of rates of use in all instants, that is, to changes in ultimate cumulative use. This is in conformity with common usage. Conservation can be obtained through decrease in rates of use and through increase of positive efforts. Positive efforts may be directed at reducing deterioration of stock resources, at increasing the flow in case of flow resources, and at recovering stock and flow resources after they have been used. Depletion can be obtained, *mutatis mutandis*, in the same way. Technically, different ways of accomplishing conservation and depletion are closely intertwined. For economic theory these different ways represent merely different combinations of productive services to obtain a desired time pattern of rates of use. It is also apparent that the definitions refer to the direction of change, not to a certain state in the time distribution of rates of use. The latter will be called the "state of conservation," which is identical with the "state of depletion."

It may now be asked what state of conservation is the goal of the planning agent. It is assumed here that the goal is that distribution of rates of use over time which maximizes the present value of net revenues. This assumption is sufficiently realistic if expected revenues and costs, which, of course, must be measurable in comparable units, are interpreted in accordance with the problems confronting the planning agent. The resulting distribution of rates of use will be called "optimum state of conservation."

The optimum state of conservation can be determined through various approaches. The approach through joint revenues and joint costs is the most suitable one from the standpoint of analyzing changes in the optimum state of conservation brought about by taxation. As already mentioned, revenues and costs of rates of use in each instant are in principle non-additive functions of rates of use planned in *all* instants; technical causes of the interrelation of rates of use in different instants through revenues and costs are the effects of cumulative use and of positive conservation efforts, the durability of producer as well as consumer goods, and the recoverability of scrap. The maximization of present net revenues, considering more than one instant, can thus be treated in the same way as the maximization of net revenues considering more than one product (resource) in instantaneous production theory. Marginal revenues and costs employed for this purpose are partial derivatives of *present* — in the sense that all contributing value elements occurring in different instants are properly discounted (or compounded) — total revenue and cost functions with respect to rates of use in a certain instant. The undiscounted value elements occurring in different instants which contribute to present revenues and costs will be called, for brevity, "current" revenues and costs, and their difference in the same instant "current" net revenues. Current revenues, costs, and net revenues are of particular interest for this study, because taxes may be expressed in terms of present or current revenues, costs, and net revenues. Equality of current marginal revenues and corresponding costs is not a necessary condition for maximizing present net revenues. On the other hand, equality of present marginal revenues and corresponding costs is a necessary condition for maximum present net revenues. For sufficiency, of course, well-known auxiliary requirements must be met.[4]

The costs referred to are "short run" costs, in the sense that fixed costs may exist. But Marshall's differentiation between "short run" and "long run" has little significance in time economics. All productive services are dated with the instant in which they are acquired. A durable factor constitutes, as we have seen, a stock of productive services which are dated with the instant in which the durable factor is acquired. Depreciation is taken into account through current revenues and costs. Depreciation accounting is merely a technical necessity of bookkeeping required by the difference between accounting period and planning period. Salvage value is regarded as part of revenues of the last instant. All revenues are dated with the instant in which they are realized. Risk and uncertainty are taken into account through current reve-

4. Maximization of net revenues over time and the relation of present to current revenues, costs, and net revenues are more precisely formulated in the Mathematical Appendix, § 2.

nues and costs, either as a deduction from current revenues and an addition to current costs on the basis of the planning agent's probability appraisal and his safety preference, or as the effect of provision for flexibility in the utilization plan.[5]

The effects of risk and uncertainty upon current revenues and costs make it possible to regard the number of instants included in the utilization plan as finite instead of infinite or indefinite. The decrease of expected current revenues and the increase in expected current costs due to risk and uncertainty increase with time, until from a certain instant onward no positive rate of use adds to present net revenues. When this occurs depends in principle on all previous rates of use in the utilization plan. The number of instants included in the utilization plan is, therefore, not fixed but is itself an unknown in the maximization problem. In the joint production approach this is taken into account through including as variables a number of future rates which in the solution become zero.[6] The effects of risk and uncertainty are accentuated through a positive interest rate. Theoretically the mere existence of a positive interest rate does not terminate the utilization plan, but it weakens progressively the influence of more distant current net revenues. The effects of risk, uncertainty, and interest rate upon the extent of the utilization plan should not be confused with the effects of their changes upon conservation and depletion. These latter effects were discussed in detail elsewhere.[7] They need consideration in this study only as they are directly related to taxation.

For economic theory and for our present purposes this sketch of the meaning of the optimum state of conservation is sufficient. For determining the optimum state of conservation in actuality, approximations to partial derivatives and in many other ways become necessary. These approximations are similar to those

5. For details see, among others, Hicks, J. R., op cit., pp. 124–126; Hart, A. G., Anticipations, Uncertainty and Dynamic Planning, Studies in Business Administration 11 (1), The Journal of Business of the University of Chicago 13 (4): 1–98, 1940, Makower, H. and J. Marschak, "Assets, Prices and Monetary Theory," Economica, August, 1938; Ciriacy-Wantrup, S. v., "Private Enterprise and Conservation," Journal Farm Economics, February, 1942.

6. Compare the Mathematical Appendix, § 2. As an alternative to this discontinuous solution the maximization problem may be approached by formulating present net revenues as an integral over a variable period of time. Use of the calculus of variations is necessary in this case.

7. Ciriacy-Wantrup, S. v., op. cit., pp. 84–94.

which apply to the optimum product vector in joint instantaneous production.[8]

IV. Some General Problems of Taxation Theory Encountered in the Economics of Conservation

Definition of the economic meaning of conservation and its corollary, depletion, makes it possible to study their relation to taxation. This will be done, first, by considering some general problems of taxation theory encountered in the economics of conservation, and second, by analyzing individually the effects of the most important types of taxes on resources.

The effect of all types of resource taxes upon conservation and depletion can be thought to operate through changes in the optimum state of conservation, that is, through changes in present marginal revenues and costs of differently dated rates of use. In order to restore equality of present marginal revenues and costs, when new taxes are levied or existing ones changed, changes in the pre-tax[9] time distribution of rates of use become necessary.

Changes in present marginal revenues and costs brought about by taxation in a certain instant are not determined solely by tax changes in this instant but by tax changes in all other instants and by all technological and economic conditions which influence the interrelation of differently dated rates of use of the taxed resource considered and of all other resources.[1] This follows from the joint-product character of rates of use. We may say that the relation of taxation to conservation and depletion depends on the complementarity, competitiveness, and independence in present marginal revenues and costs of differently dated rates of use. Rates of use in two different instants may be defined as complementary, competitive, or independent in revenues (or net revenues) according to whether a small increase in one rate increases, decreases, or leaves unchanged the present marginal revenues (or present marginal net revenues) of the other rate; similarly, complementarity,

8. Ciriacy-Wantrup, S. v., "Economics of Joint Costs in Agriculture," Journal Farm Economics, November, 1941.

9. The term "pre-tax" always refers to conditions before the *change* in taxes which is considered took place.

1. Obviously, the use that is made of the proceeds of the tax by the tax authority is also of great importance; for instance, if the proceeds are used to subsidize conservation. It is appropriate, however, to regard such effects as a separate influence, which should be analyzed as such, and should not be confused with the direct effects of taxation.

competitiveness, and independence in costs exist if a small increase in one rate of use decreases, increases, and leaves unchanged the present marginal costs of the other rate.[2] The same definitions apply to the concepts of complementarity, competitiveness, and independence of rates of use of different resources. As emphasized above, interrelations of the latter type and those of the former are in principle of equal importance.

This situation renders it impossible to make general statements about the effects of taxation upon conservation and depletion without assumptions concerning the time distribution of tax changes and the interrelations between differently dated rates of use of the same and of different resources. This will be done below when the effects of individual taxes are analyzed. The assumptions will be made, first, that taxes are expected to be constant over time with respect to rates, methods of assessment, modes of payment, and in other respects; second, that the time distribution of rates of use of other resources than of the taxed resource needs no explicit consideration; and third, that certain known conditions with respect to complementarity, competitiveness, and independence of differently dated rates of use of the same resource exist. Before proceeding, however, those general problems will be considered which arise if the first two of these assumptions are not fulfilled.

There are four major factors that may lead to expectations of changes in taxation over time; first, expectations of political changes, types and rates of taxes being frequently connected with the policy of a government or a political party; second, expectations on the basis of tax changes that have been experienced by planning agents in the past; third, expectations that important present events which are known to affect public expenditures decisively — for instance, wars — will influence taxation policy; fourth, and most important, if the planning agent considered is the government, taxes can never be regarded as constant, that is, the tax authority may employ planned tax changes over time to influence the time distribution of rates of use in private enterprises.

If a given change in taxes is not constant, there is a tendency for rates of use to be redistributed over time in such a way that the tax base is increased in those instants in which taxes are more favorable. Redistribution goes on as long as the discounted sav-

2. These concepts are formulated more precisely in the Mathematical Appendix, § 3.

ings in taxes are greater than the decrease in present net revenues which would have been caused by such a redistribution under pre-tax conditions. Conservation or depletion of any degree may result according to the time distribution of tax changes and to the interrelation of differently dated rates of use in revenues and costs.

A special case of such a redistribution is of some practical importance in public conservation policy. The levying or increasing of taxes on individual resources followed by a decrease in a properly chosen later instant may be employed to bring about conservation, if taxes decrease unweighted aggregate use up to the instant in which taxes are decreased, and if rates of use previous to that instant are competitive in cost and independent (or competitive) in revenues with later rates of use. These two conditions are frequently fulfilled in actuality, because taxes on individual resources usually lead to some shifts of productive services into other employments, as explained below, and because with most stock resources the familiar effects of cumulative use bring about the above interrelations between differently dated rates of use. Thus intelligent variation of tax rates over time may be used as a tool of public conservation policy.

The general effects of progression and regression in taxation upon the time distribution of rates of use are similar to the effects of non-constancy of taxes. The effects of regression can be omitted because they are symmetrical with the effects of progression and are in practice less important for conservation policy.

Most pre-tax utilization plans are characterized by variations of the tax base over time. Under these conditions imposition of a progressive tax or an increase in progression of existing taxes must lead to a redistribution of rates of use in such a way that the tax base in different instants becomes more equal. Again, redistribution proceeds as long as the discounted savings in taxes are greater than the decrease in present net revenues which would have been caused by such a redistribution under pre-tax conditions. Conservation or depletion of any degree may result according to the pre-tax variations of the tax base over time and to the interrelation of differently dated rates of use in revenues and costs.

A special case may also be mentioned, which is of practical importance for public conservation policy. In resource utilization the tax base is very often much smaller in the more remote future than nearer the present. This is caused by the effects of cumulative use and uncertainty upon current revenues and costs. The effects of cumulative use are also responsible for the fact that rates of use nearer the present are often competitive in cost with rates more distant in the future. If this is not offset by complementarity in revenues — which scarcely ever happens in reality — imposition of progressive taxes or increase in the progression of taxes must lead to conservation. Progression in taxation may therefore be regarded as a tool of conservation policy. More will be said about this when individual types of taxes are discussed.

Our second assumption, namely, that changes in rates of use in other resources need no explicit consideration, becomes inadequate if the effects of inequality of taxation between different resources upon conservation and depletion are to be studied. The increase in inequality of taxation between different resources induces private planning agents to transfer productive services to the less heavily taxed resource. If taxes are regarded as constant, the result of such a transfer for the more heavily taxed resource is depletion from the standpoint of the private planning agent. From the standpoint of the tax authority, for which taxes are never constant, the result is conservation, if adjustments in the way just discussed are planned for a later period. For the less heavily taxed resource the result may be conservation or depletion, depending on the interrelation of differently dated rates of use in revenues and costs. For most stock resources the result will be depletion; for many flow resources, conservation.

One type of shift of productive services into other employments may be especially mentioned. This is the shift of services of the planning agent himself to "leisure." No theoretical difficulties are involved, because "leisure" may be regarded as "negative employment," may be evaluated subjectively, and may be compared with monetary revenues. The importance of the shift of the services of the planning agent to leisure upon cumulative use, and therefore upon the time distribution of rates of use, is often considerable, but it is generally overlooked in taxation theory. The direction of the redistribution of rates of use, that is, conservation or depletion, is similar to the effects of shifts in other productive services.

In actuality the transfer of productive services from the more heavily to the less heavily taxed resource will be slow and limited, because of imperfections in the adaptability of productive services, particularly of capital and labor and managerial ability of planning agents themselves. Under these conditions the result for the more heavily taxed resource is likely to be depletion from the standpoint of individual planning agents as well as from the social point of view.

This point leads to another general problem of taxation encountered frequently in conservation economics. Taxation decreases money incomes from the taxed resource, if the effects of government spending are neglected.[3] Such a decrease, in turn, may lead to increases in the rate of time preference[4] of individual planning agents. Under the imperfections mentioned, an increase in time preference causes liquidation of assets through depletion, not through sale or credit operation as under perfect markets for capital, labor, and managerial ability. The practical significance of this in conservation economics has been discussed elsewhere.[5] It follows from the functional relation[6] between marginal utility of money and money income that this effect of taxation is larger in the low income levels and becomes progressively smaller in the high income levels. Thus these effects are of interest mainly with respect to the tax burden on resource users with small incomes, particularly when incomes decline; for instance, in the course of economic fluctuations. In other words, the problems of regression and of flexibility in resource taxation need special attention. It may be noted that this is a second reason why progressive taxes are more desirable from the standpoint of conservation policy.

3. Footnote 1, page 170 above.

4. The rate of time preference may be defined as the ratio between the present marginal utility of money income in more distant future instants and the present marginal utility of the same money income in instants nearer to the present, reduced by unity and expressed in per cent and per instant. If it is desired to avoid reference to marginal utilities — only their ratios are under discussion — the same concept may be defined as the numerical gradient of the indifference curves of money incomes in two different instants measured at points of equal money income, reduced by unity and expressed in per cent.

5. Ciriacy-Wantrup, S. v., Private Enterprise and Conservation, Journal Farm Economics, February, 1942.

6. With respect to marginal utility schedules, we may make the usual assumptions that they decrease monotonically without inflexion point (without change in sign of the second derivative) with increasing income, and approach both axes asymptotically. Under these assumptions, the ratios between corresponding points on schedules in different instants must be large at low incomes, and must decrease progressively with increasing incomes until they become very small.

Finally, before individual types of resource taxes are taken up, a few words concerning the general problem of capitalization should be added. Capitalization of taxes is indeed of great interest from the standpoint of the distribution of the tax burden among taxpayers, but capitalization of taxes in itself does not affect the present marginal costs and revenues of differently dated rates of use. Its influence upon the distribution of rates of use over time, if there is any, is indirect and small. The problems of capitalization, therefore, need be considered only in passing in this study.[7]

V. The Effects of Particular Taxes

Taxes on Current Net Revenues. Taxes on income and profits from resource utilization are taxes on current, not on present net revenues of the utilization plan. Rarely, however, does the legal definition of "income" and "profits" coincide with the meaning of net revenues in economic theory. The discrepancies, especially with respect to the treatment of investment and of income from services rendered by the planning agent, are well known.[8] Nevertheless, from the standpoint of changes in the pre-tax state of conservation, these and similar taxes may be discussed as a homogeneous group and compared with present value taxes, death taxes, yield taxes, and lump-sum taxes.

In studies of taxation the view is expressed again and again that income and profit taxes will not be shifted because the planning agent has no inducement to change the pre-tax system of production.[9] In production over time this is correct only if income and profit taxes are proportional — under the assumptions that they are also constant, that no shifts of productive services into "leisure" or into less heavily taxed employments take place, and that the effects through changes in individual time preference rates can be disregarded. Under these assumptions, proportional taxes

7. See below, p. 180.

8. Examples are tax regulations concerning the necessary "depletion allowance" in the utilization of stock resources, the use of "depletion accounts" in the utilization of flow resources with cyclical variation in rates of use, the segregation of current costs and investments when the flow of flow resources is increased, and the legal treatment of depreciation of buildings and equipment. The planning agent and his family frequently render labor and managerial services, particularly in agriculture. The remunerations of these services are costs for economic theory and income for the purpose of the income tax.

9. For a recent example cf. Taxation of Corporate Enterprise. Temporary National Economic Committee, Monograph No. 9, Washington, 1941.

on current net revenues do not offer any incentive to redistribute current net revenues and — as a necessary prerequisite — rates of use over time. The tax discount, that is, the preference for deferred taxation, cannot have this effect because any gain in present net revenues through postponement of tax payments would be more than offset by loss in present net revenues through postponement of current net revenues. Only in cases in which the tax on current net revenues is higher than one hundred per cent, that is, if taxation renders current net revenues negative, would this be different. A constant tax of this type is inconceivable.

Progressive or regressive taxes on current net revenues, on the other hand, usually provide an incentive to redistribute rates of use over time. The general conclusions reached above may be repeated as applied to taxes on current net revenues, because they are so often overlooked. A progressive tax in combination with lower current net revenues in later instants gives an incentive to redistribute current net revenues and rates of use in the direction of the future, as long as discounted savings in tax payments are greater than the decrease in present net revenues which would have occurred under pre-tax conditions. This means conservation except under rare conditions of competitiveness of differently dated rates of use. The degree of conservation depends on the interrelation of differently dated rates of use, on the steepness of progression in taxation, and on the differentials between instants of pre-tax current net revenues. This example is of great practical significance, because current net revenues planned for different instants decrease generally with remoteness of instants as a result of cumulative use and uncertainty, as pointed out above. On the other hand, progressive taxation may lead to depletion if for technological or other reasons current net revenues expected toward the end of the utilization plan are large relative to earlier ones. Regressive taxes have a tendency to accentuate instead of level differentials between instants in current net revenues. Accordingly, their effect upon conservation and depletion tends in the opposite direction from that of progressive taxes.

Taxes which are in effect progressive with respect to income — the tax base may be net revenues or any other — are preferable to proportional and especially to regressive ones, if "neutrality" of taxation with respect to the state of conservation is desired.[1] Pro-

1. When the effects of different types of taxes upon the state of conserva-

gressive taxes increase individual time preference less than proportional and regressive taxes. The reason for this and the relation of time preference to changes in the state of conservation under impure and imperfect markets were explained above. Tax exemptions in low-income groups, a feature of progressive taxation, are also desirable from the standpoint of tax "neutrality" with respect to the pre-tax state of conservation. These conclusions are independent of generally accepted standards of social "justice" and "equity" in taxation, although they happen to be in agreement with these standards.

Present Value Taxes. Taxes on present value of resources payable recurrently or, for brevity "present-value taxes," are at present by far the most important type of resource taxes.[2] Besides the general property tax, many special taxes on the value of land, mines, buildings, and equipment are of this type. In the United States the early adoption of the general property tax left little room for these special taxes, which since earliest times have been the backbone of resource taxes in European countries; but with the differentiation of property and the growing evasion of taxes on personal property, the general property tax has become to a large extent a special tax on natural and tangible cultural resources.

It will be assumed that, for the purpose of taxation, present value is identical with the sum of discounted current net revenues of economic theory. This is in conformity with the goal of most tax authorities, although it is only approximated in actuality. The effects of deviation from this goal because of inadequate methods of assessment will be indicated later. There are also the same discrepancies between concepts as used by tax authorities and concepts as used in economic theory which were mentioned in the case of taxes on current net revenues.

Recurrent (annual) taxes on present value of resources may be regarded as a special type of taxes on current net revenues. If present value is the sum of discounted current net revenues, then in each instant (year) to which the tax applies, current net revenues

tion are compared in this paper, it is assumed that tax rates are adjusted in such a way as to make total tax receipts identical for different types of taxes.

2. In the United States, for instance, it is estimated (1927) that 83.8 per cent of all federal, state, and local taxes paid by agriculture were general property taxes. During the depression years in the 'thirties this percentage was probably even higher. Coombs, Whitney, Taxation of Farm Property. U. S. Dept. Agric. Tech. Bul. 172. Washington, 1930.

of all future instants are taxed. The farther, therefore, current net revenues are removed from the present, the more often they are subject to the tax. This provides an incentive to redistribute current net revenues in the direction of the present, in order to reduce the number of times they are taxed. This process continues as long as discounted savings in tax payments are larger than the decrease in present net revenues which would have occurred under pre-tax condition. Redistribution of current net revenues in the direction of the present can be accomplished only through redistribution of rates of use in the same direction.[3] This means depletion. The degree of depletion and the new time pattern of rates of use depend on the latter's interrelation in revenues and costs. The effect of present-value taxes upon the utilization plan is thus similar to the effects of the interest rate discussed elsewhere.[4]

A simple numerical example will illustrate this practically important effect of present-value taxes. Let us assume two alternative time distributions of current net revenues (A and B) for a utilization plan extending over four years (in dollars):

	t_1	t_2	t_3	t_4
A	1,000	1,000	1,000	1,000
B	1,500	1,500	500	400

At an interest rate of four per cent, the present value of series A is $3,775.09; that of series B, $3,760.18.[5] The planning agent will, therefore, choose series A, which, if rare cases of competitiveness between differently dated rates of use are disregarded, must result from a more conservative time distribution of rates of use. If the two underlying time distributions of rates of use are the only alternatives, that distribution on which series A is based may be called the optimum state of conservation. Now, let us assume that a property tax of three per cent is imposed. To obtain the expected value of the property tax in each year during the whole period of

3. Storage may be regarded as part of the productive process. Anticipation and postponement of revenues and costs through credit operations may bring about obvious modifications. The extent of these modifications, however, is limited.

4. Ciriacy-Wantrup, S. v., op. cit., pp. 88ff. The tax discount need not be considered in the case of present value taxes for the same reasons which were discussed in the case of taxes on current net revenues.

5. For simplicity's sake all current net revenues are discounted as of the first day of each year.

utilization, present values as of the first day of each year are computed (in dollars):

	t_1	t_2	t_3	t_4
A	3,775.09	2,886.09	1,961.54	1,000.00
B	3,760.18	2,350.59	884.62	400 00

The expected property tax for each year is then (in dollars):

	t_1	t_2	t_3	t_4
A	113.25	86.58	58.85	30.00
B	112.81	70.52	26.54	12.00

The sum of the expected property taxes discounted at four per cent to the present is $277.58 for series A and $215.82 for series B. Deducting these values from the pre-tax present value of the two net revenue series, we obtain $3,497.51 for series A and $3,544.36 for series B. Through imposition of the property tax, therefore, the lower state of conservation which underlies series B becomes the optimum state of conservation. The planning agent will change his utilization plan accordingly. This change means depletion.

The illustration is chosen in such a way that the lower state of conservation becomes the optimum state of conservation through imposition of a property tax. Other examples could be used which would show merely that a property tax decreases the present value of a less conservative utilization plan relatively less than that of a more conservative plan.[6] This, however, is sufficient to bring about a change in the optimum state of conservation, if the alternatives in the time distribution of rates of use and the resulting present values are very numerous, as can be assumed in economic theory. In theory, a certain state of conservation is always "marginal," in the sense that alternatives with a higher or lower degree of conservation are close at hand.[7] In economic reality the alterna-

6. If, for instance, series B is chosen so as to give ultimate cumulative current net revenues equal to series A, (t_1, $1,500.00; t_2, $1,500.00, t_3, $500.00, t_4, $500.00), the less conservative distribution of rates of use is the optimum one before imposition of the tax, as long as a positive interest rate is used. Through imposition of the tax its superiority becomes greater. The ratios of present value of the two series (series A = 100) are 101.96 before imposition of the tax and 103.58 after imposition of the tax.

7. Fairchild's statement on this point is confined to a special case: "From its very nature, the property tax favors a use which yields an early income. Of course, this effect is controlling only in the case of those properties which are on the margin between use for a deferred yield and use for annual

tives confronting the planning agent are discontinuous, as in our example.

In some cases, the inherent effects of present-value taxes are desired by tax authorities. The two most important examples are attempts to check speculation in resources withheld in the expectation of later monopoly profits, and efforts to induce better methods and practices of use. Both cases need some clarification in the light of the previous analysis.

Under monopoly conditions the optimum state of conservation is frequently characterized by extreme curtailment of immediate use in the expectation of higher profits later. In such cases, taxes based on current utilization — net revenue taxes or yield taxes — would produce no, or only low, tax receipts in the beginning, and would not interfere with the time distribution of rates of use which may be regarded by tax authorities as socially undesirable. Vacant city lots, large land holdings in young, rapidly growing countries, and deposits of scarce minerals are examples. Present-value taxes, on the other hand, produce tax receipts immediately and change the time distribution of rates of use in the direction desired. The latter effect is sometimes denied on the ground that a present-value tax will be capitalized and merely induce the sale of the taxed property to financially stronger speculators, without affecting the time pattern of use.[8] There is no doubt that present-value taxes are easily capitalized when the taxed property is sold; such a sale may or may not be forced by the tax. But the new owner feels the same economic inducement under the tax to change the pre-tax time pattern of use as the old owner. Here again a sharp differentiation must be made between the effects of the tax with respect to the tax burden and with respect to the distribution of rates of use

return." Op. cit., p. 46. Fairchild compares two time patterns of use characterized by deferred yield (forestry) and annual yield (grazing). The effect of the property tax "is controlling," not only if deferred yield and annual yield are close alternatives, but also if different time patterns of deferred yield or different time patterns of annual yield are alternatives. In forestry a choice between numerous different deferred yield cycles has to be made, and the cycles planned need by no means be constant over time. In grazing many time patterns of use between reckless overgrazing and extreme restrictionism are of economic importance. Confinement to constant rates of use or to strictly periodic variations of rates of use over time oversimplifies the economic problems involved.

8. Sargant, C. H., Urban Rating, pp. 145ff. London and New York, 1890; Smart, W., Taxation of Land Values and the Single Tax, pp. 95ff. Glasgow, 1900.

over time. In this paper interest is focused on the latter effects, which are not altered by the capitalization of taxes. On the other hand, it may be observed that the objective of the tax may often be more effectively obtained through direct public action aimed at the elimination of private monopolies, because of the difficulties in assessing future monopoly profits for present-value taxes. If monopoly cannot be eliminated, a confiscatory tax on unearned increments of value payable periodically is more effective than taxes on total present value. The usual difficulties of differentiating earned from unearned increments are small in the cases mentioned.

Present-value taxes on resources already used, but not used in conformity with existing economic possibilities, are frequently recommended because of their alleged "educational" value in forcing planning agents to adopt better methods and practices or to sell their properties to others who will.[9] Imperfections of knowledge, however, and individual non-monetary goals of utilization, such as prestige, class distinction, seclusion, and simplicity of management, are not altered through taxation. Present-value taxes change the utilization plan of *all* planning agents in the direction of depletion, but the same imperfections in knowledge and the same differences in goals persist. The burden of present-value taxes imposed on the basis of market prices for such properties, that is, on the basis of potential monetary net revenues under "normal" management, is, of course, greater for properties the utilization of which falls short of realizing such net revenues. Whether this leads eventually to a sale of these properties depends on the financial strength of their owners. Other owners, who use resources in conformity with economic possibilities, but who are financially weaker, may be affected first. At best, the present-value tax is a slow, uncertain, and costly instrument for bringing about the adoption of better methods and practices in resource utilization. If taxation is to be used for this purpose, a tax on potential rates of net revenues would be preferable to a present-value tax. Such a tax would not cause a general tendency towards depletion, and would still impose an especially heavy burden on those utilization plans which are based on imperfect knowledge or are not oriented towards maximization of present monetary net revenues. Here, also, more direct public action through education

9. This is the basis for Aereboe's often repeated doctrine of the social benefits from land taxes. Agrarpolitik, particularly pp. 315–327. Berlin, 1928.

or regulation usually reaches results more quickly, surely, and economically than does taxation.

In most cases the inherent effects of present-value taxes are not desired by tax authorities. They are tolerated because they are not recognized or because their importance is underestimated. An outstanding example is the effect of present-value taxes in forestry.[1] Similar effects can be observed in grazing, agriculture, and mining. Besides these natural resources, certain tangible cultural resources, such as buildings and equipment, may feel the characteristic effects of present-value taxes. The economic and social causes of city slums are too complex to be discussed in this connection,[2] but there seems to be no doubt that the increase of present-value taxes in centers of big cities influences the utilization plan within the limits set by building ordinances. Depletion in this case appears in decrease of expenditures for renovation and upkeep. In order to utilize such buildings, rent inducements have to be offered to tenants. The production plan, on the other hand, calls for high immediate rates of utilization. Crowding of tenants is the result. Thus the three economic characteristics of slums — blighted buildings, low-income tenants, and crowding, in combination with high real estate values — can be traced to the effects of present-value taxes. This, of course, does not mean that such taxes are the only or even the most important cause of slums. It means merely that the effects of present-value taxes upon the time distribution of rates of use warrant consideration in efforts directed at slum clearance.

No argument is needed to prove that present-value taxes inherently show less automatic flexibility during economic fluctuations than net-revenue or other taxes based on current utilization of resources. In practice present-value taxes frequently increase in periods of economic depression, in spite of some decrease in the base. Rates are increased because public expenditures are swelled by relief and public works and because the tax base for net revenue and other taxes connected with current production decreases rapidly or disappears entirely. The experience of American agriculture during the depression following World War I is a well-known example. The reason why inflexibility of taxes leads to depletion

1. Fairchild, F. R., op. cit., and literature cited in this study.

2. See, for instance, Ford, James, Katherine Morrow, and J. N. Thompson. Slums and Housing. Cambridge, 1936.

under imperfect or impure markets has already been pointed out.

The depleting effects of present-value taxes are accentuated in the United States by certain aspects of assessment. It has been found by numerous studies of the general property tax that the ratio of assessed value to present value tends to decrease as present value increases.[3] The tendency of regression in taxation — in this case in assessment — to favor depletion under imperfect or impure markets, and why such effects take place, has been explained above.

Another inequality in assessment which is also of great importance in the economics of conservation is the overassessing of properties in a low state of conservation relative to properties in a high state of conservation. This has been studied particularly with respect to cut-over forest land with poor natural regeneration,[4] but it applies also to eroded farm land and overgrazed ranges. A vicious circle is set in motion: inequalities of assessment encourage depletion most on those properties which are already relatively more depleted. This effect is often increased by tax regression.

Inequalities of assessment are largely the result of inexpert judgment concerning the future net-revenue flow. Assessment of the general property tax is frequently entrusted to locally elected, temporarily employed, and not thoroughly trained assessors. They are more familiar with properties of low present value because they are more numerous. For these properties market prices as substitutes for an appraisal of future net revenues can be more easily used because there are more market transactions. In agriculture and other resource industries, properties of low present value have usually a higher ratio of market price to present value than properties of high present value; market demand for the former is relatively greater because of imperfect markets for loans

3. Englund, E, Assessment and Equalization of Farm and City Real Estate in Kansas, Kansas Agr. Exp. Sta. Bul. 232, Manhattan, 1924, Yount, H. W., Farm Taxes and Assessments in Massachusetts, Mass. Agr. Exp. Sta. Bul. 235, Amherst, 1927, Dreesen, W. H., A Study in the Ratios of Assessed Values to Sale Values of Real Property in Oregon, Oregon Agr. Exp. Sta. Bul. 233, Corvallis, 1928; Simpson, H. D., The Tax Situation in Illinois, Northwestern University Institute for Research in Land Economics and Public Utilities, Studies in public finance, Research Monograph No. 1, Chicago, 1929; Coombs, Whitney, Taxation of Farm Property, U. S. Dept. Agr. Tech. Bul. 172, Washington, 1930; Gabbard, L. P., Inequalities in Taxation of Farm Lands and City Property Due to Scope and Method of Assessment, Texas Agr. Exp. Sta. Bul. 458, College Station, 1932.

4. Fairchild, F. R., op. cit. Part 4.

and for family labor. Furthermore, owners of properties with high present value are in a better position to fight overassessment inside and outside of the courts because of better knowledge, better legal advice, and greater political influence in the community.

It is difficult for the average assessor to appraise the effects of differences in the state of conservation upon the value of properties. Market prices of the past are not sufficiently indicative of these differences in the present, and may not be applicable at all to non-fungible objects such as land, mines, and buildings. In many localities assessors regard it as politically expedient to assess all properties below their present value, usually from forty to sixty per cent below. This makes it difficult to discover inequalities and to prove them in court. This practice also slackens demand for periodic reassessment, which is essential in order to take into account changes in present value caused by changes in the state of conservation. This situation may be improved through appointment, rather than election, of assessors who are professional specialists trained in appraising the existing state of conservation and its relation to present value. Assessment should always be one hundred per cent of present value, and periodic reassessment should become a rule. Desired variations in total tax receipts should be brought about through variation of the tax rate rather than the assessment ratio. Even if these proposals are carried out, equality of assessment will be more difficult to obtain for present-value taxes than for taxes based on current utilization.

Death Taxes. In modern times death taxes on resources are theoretically present-value taxes, in the sense that they are based on expected future net revenues discounted to the instant in which death of the owner occurs.[5] They are called estate taxes if the tax is imposed regardless of the number of beneficiaries or their relationship to the testator. They are called inheritance taxes if taxes are imposed on the shares of the beneficiaries; in this case exemptions and tax rates usually differ with the relationship between each beneficiary and the testator.

If the utilization plan extended over several generations, expectation of death taxes would have the same inherent tendency to cause depletion as recurrently payable present-value taxes.

5. In some parts of the world death taxes are lump-sum taxes, or they are based on some physical expression of present wealth, such as the number of livestock owned.

It may be assumed, however, that usually only one payment of death taxes is taken into account. If this assumption is granted, death taxes have the same effect upon the optimum state of conservation as a non-recurrent tax on present net revenues or a proportional tax on current net revenues.[6] It follows from the previous analysis of net-revenue taxes that death taxes are neutral with respect to the optimum state of conservation under perfect markets for services. In economic reality, markets for loans and for other productive services are imperfect (Section IV). Under these conditions death taxes may cause depletion, because of their influence upon the time preference of testator and beneficiaries. If the testator expects that a portion of his estate will be turned over to the government and not (directly) benefit his heirs, his motives for investment may be weakened or he may be induced to disinvest.[7] Under imperfect markets for loans and for other productive services, investment and disinvestment may take place in the form of conservation and depletion.[8] The beneficiaries, in turn, are faced with payment of a tax which usually far exceeds current net revenues from the inherited resources. If beneficiaries have no liquid reserves, and if markets for loans and other important productive services are imperfect, they may be forced to liquidate a portion of the inherited resource through depletion in order to pay the tax. This type of depletion is a familiar occurrence in forestry and agriculture.

In order to avoid these effects of death taxes, payments should be extended in installments over such a period of time that current net revenues are sufficient to meet current payments.[9] In Great Britain, for instance, death taxes on forest holdings need be paid only at the time the timber is actually cut. For the sake of formal completeness, lowering of death taxes and avoidance of progression

6. Proportional with respect to current net revenues in different instants for the same estate. With respect to present value of different estates, death taxes are usually strongly progressive.

7. The opposite effect may happen, but so rarely, especially under progressive death tax rates, that it may be neglected.

8. Cf Section III above.

9. The extreme in this respect would be a transformation of death taxes into income taxes. On this point, cf. Barna, Tibor, "The Burden of Death Duties in Terms of an Annual Tax," Review of Economic Studies, November, 1941, and Kaldor, N., "The income burden of capital taxes," Review of Economic Studies, Summer, 1942.

may be mentioned.[1] This remedy appears socially questionable on grounds not connected with the economics of conservation.

Yield Taxes. Under the term "yield taxes" a great variety of resource taxes are grouped. Some of them are imposed on units of physical production, regardless of sales. Severance taxes are frequently of this type. Others are imposed on the physical product sold, regardless of price. Still others are imposed on the value of product sold (ad valorem taxes, gross-revenue taxes).

Taxes on physical yield are an addition to current costs proportional to rates of use. Ad valorem taxes are a proportional deduction from current revenues by a percentage equal to the rate of the tax. Both types of yield taxes affect present marginal revenues and costs, and therefore lead to a redistribution of rates of use over time. The direction of this redistribution, that is, conservation and depletion, is analytically highly complex and great care is necessary in stating the assumptions made.

If yield taxes are assumed to be constant, and shifts of productive services from the taxed resource into other employments are assumed to take place, yield taxes must result, according to our formal definition, in depletion from the standpoint of private planning agents. As we have already seen, taxes are never constant from the standpoint of the tax authority. Planned changes of yield taxes over time in combination with (assumed) shifts of productive services may, therefore, be used to bring about conservation from the standpoint of the public.

If it is assumed that no shifts of productive services to other employment take place, the result may be privately and socially conservation or depletion, depending on the interrelation of differently dated rates of use in revenues and costs and on the effects of the tax upon the rate of time preference of private planning agents. In other words, only shifts over time need be considered. If the familiar effects of changes in time preference rates are excluded by assumption, such shifts may be in the direction of the future, or in the direction of the present. They will be in the direction of the future, and yield taxes will therefore result in conservation, if rates of use near the present are competitive in

1. The National Committee on Inheritance Taxation recommends a maximum of 15 per cent of present value Cf. National Committee on Inheritance Taxation, Report to the National Conference on Estate and Inheritance Taxation held at New Orleans, Louisiana, November 10, 1925. Washington, 1925.

costs and independent (or competitive) in revenues with later rates, or if competitiveness in costs is more decisive than complementarity in revenues. In the opposite event the result will be depletion. For stock resources, as we know, the former will usually be the case. In this special case, therefore, Hotelling's statement that a severance tax "tends to conservation" is correct.[2] In his study the necessary assumptions are mentioned, but this statement is repeated by other authors less carefully and applied to yield taxes in general. This is theoretically incorrect and practically highly dangerous. Yield taxes cannot be recommended as a reliable tool of conservation policy, especially since tax authorities usually do not have the research facilities to appraise the effects upon the state of conservation.

It may be asked whether the tax discount must be considered as an incentive to redistribute rates of use in the direction of the future. In the case of ad valorem taxes any gain in present net revenues through postponement of tax payments would be more than offset by loss through postponement of current revenues. In the case of taxes on physical yield the tax discount means merely that taxes enter into the calculations of the planning agent, that is, into present marginal costs, with their discounted values in the same way as all other current costs. The tax discount itself cannot be separated from other effects of the tax and cannot be regarded as a special incentive to conservation.[3]

Although taxes on the physical product are generally proportional with respect to rates of use and ad valorem taxes with respect to current revenues, both may be progressive or regressive with respect to income. If the planning agent's family is a consumer of his own products, as in agriculture, yield taxes on the physical product are usually regressive and yield taxes on sales are progressive. This is due to the higher ratio of home consumption to sales in smaller enterprises with lower incomes. It scarcely needs to be mentioned that yield taxes are not necessarily proportional with respect to rates of use or current revenues; nor do we need to

2 Hotelling, Harold, op cit., p. 165. Hotelling does not define conservation. In connection with the above quotation he refers to extension in time of the period of utilization and to ultimately greater rates of production and lower prices than if there had been no tax. Such a change in the utilization plan would be conservation in the terminology of the present paper.

3. For a contrary argument with respect to the effects of the tax discount, see Fagan, op. cit., p. 75.

repeat the conditions under which progression and regression affect conservation and depletion. Progression and regression of yield taxes with respect to the income of planning agents should clearly be differentiated from their usual regression with respect to the incomes of consumers to the extent that shifting of yield taxes takes place. In this study we are interested in the latter type of progression and regression only insofar as the planning agent's family consumes his own product.

With respect to tax flexibility in the course of economic fluctuations, yield taxes are preferable to present-value taxes. They are inferior to net-revenue taxes, because changes in costs are not taken into account. Yield taxes are superior to all other types of resource taxes with respect to economy and accuracy of administration. For this reason, yield taxes in the form of ad valorem taxes may be temporarily expedient under war conditions. The characteristics which make them undesirable in peacetime, namely, their highly complex and uncertain effects upon the state of conservation and their regressiveness with respect to consumers' incomes, may be overlooked or may even be desirable in wartime; for instance, their relatively higher burden on taxpayers with higher propensity to consume.[4]

Lump-Sum Taxes. Lump-sum taxes are levies of fixed[5] amount on individual enterprises payable currently without regard to rates of use, or payable only in instants in which rates of use are greater than zero. The pre-tax optimum state of conservation is, therefore, altered if it is economical to rearrange the utilization plan in such a way as to evade lump-sum taxes in certain instants altogether. This is the case as long as discounted savings in tax payments are greater than the decrease of present net revenues which would have been caused by such a rearrangement under pre-tax conditions.

If lump-sum taxes are imposed without regard to rates of use, evasion is possible only by shortening the period of utilization and liquidation of the enterprise. On the other hand, if lump-sum taxes are imposed with regard to rates of use, it may be economical to let rates drop to zero in certain instants but to stay in business.

4. The case of sales taxes as a part of war taxation is well stated by C. O. Hardy in Do We Want a Federal Sales Tax. Washington, 1943.

5. Lump-sum taxes are, of course, discounted at the same rate as revenues and costs. The term "fixed" refers to changes in current net revenues, revenues, costs, and rates of use over time.

The first type causes depletion, if taxes are regarded as permanent and constant and if rare cases of competitiveness between differently dated rates of use are excluded. If these assumptions are not fulfilled, the effect may be conservation. Tax authorities, for example, may use lump-sum taxes for conservation, if the rearrangement of the utilization plan by private enterprises decreases cumulative use. The second type may cause either depletion or conservation, according to the location in time of instants in which use is zero and in the interrelation of rates of use in different instants. In forestry, for example, various combinations of these conditions are possible. In exploitative forestry — that is, if only one harvest is planned — lump-sum taxes of the second type favor conservation on cut-over forest lands, and favor depletion of mature, even-aged stands. In sustained-yield forestry, lump-sum taxes affect the length of the cutting cycle or cycles. This may be defined as depletion or conservation according to the way the volume of timber harvested per instant is affected. In sustained-yield forestry, a change in the state of conservation means, by definition, a change in a constant rate of use. In practice the post-tax rate of use becomes constant only after some oscillations.

Lump-sum taxes tend obviously to depletion on account of inflexibility and regression. In general, lump-sum taxes have the same advantages and disadvantages as yield taxes. They are easily assessed and administered; but they are not neutral with respect to the state of conservation, and the direction and the degree of the change caused by them is difficult to appraise with sufficient certainty for tax authorities. Moreover, the extreme inflexibility and regression of lump-sum taxes make them socially undesirable for reasons outside the field of conservation economics.

VI. Conclusions

Analysis of the most important types of resource taxes has shown that only two types are approximately neutral with respect to the pre-tax state of conservation, if constancy of taxes is assumed. These two types are proportional current-net-revenue taxes and death taxes, provided that the latter conform in mode of payment as far as possible to taxes on current net revenues. Progressive and regressive taxes on current net revenues may likewise be neutral, if the tax base does not vary over time. In actuality the tax base varies over time in such a way that progressive taxes on current net revenues usually encourage conservation and regressive ones encourage depletion.

Taxation may be used as a tool of conservation policy to change the pre-tax state of conservation. Progression in taxation generally favors conservation. Present-value taxes have the tendency to change the state of conservation in the direction of depletion. If this direction is desired by tax authorities, confiscatory taxes on unearned increment of present value payable periodically, or tools other than taxation, are more effective than present-value taxes. The alleged "educational" value of present-value taxes in resource utilization does not justify their use. Yield taxes and lump-sum taxes are highly uncertain tools of conservation policy, and cannot be recommended for the purpose of changing the pre-tax state of conservation in a desired direction to a desired degree. If they are employed for this purpose, it should be in the form of planned tax changes over time. The desired objective is obtained more surely, quickly, and economically through planned changes of current-net-revenue taxes.

It follows that, from the standpoint of public conservation policy, emphasis should be placed in resource taxation on proportional, or better still on progressive, taxes on current net revenues in the form of the individual income tax, corporation income tax, and excess profits tax. In the United States the bulk of resource taxes are present-value taxes, in the form of the general property tax or, to a lesser degree, yield taxes in the form of sales taxes and excises. There is danger that World War II will accentuate this lop-sided tax structure. Tendencies towards an increase of sales taxes are already noticeable. On the basis of past experience, relative or even absolute increases of the general property tax can be expected during the post-war period. This situation may cause undesirable effects on the utilization of resources. To the necessary depleting tendencies brought about by the changed time pattern of revenues and costs incident to the war, is added unnecessarily an institutional factor operating in the same direction. Every effort should be made to decrease these institutional effects in a period of strain on the national resources. This means that war taxes should be imposed in the form of income taxes, rather than property taxes and sales taxes. If sales taxes are to be used in war taxation, they should be imposed as retail sales taxes, not in the form of yield taxes on resources. Furthermore, there should be an attempt at structural tax reform; the general property tax should be gradually decreased in importance in favor of income taxes. These proposals stem from an analysis of the relation between taxation and resource conservation. They are not influenced by considerations of inflation control during the war and social equity during the ensuing peace. Fortunately these latter considerations lead to similar conclusions.

There are some obstacles to overcome in making the proposed changes. A shift from sales taxes to income taxes payable, as far as possible, at the source may not be too difficult. A structural change impairing the dominant status of the general property tax among resource taxes involves greater difficulties. Support of local governments (counties, cities, special districts, in some cases states) depends upon the general property tax. Administratively it would be a great economy to levy local taxes simply as a percentage of federal income taxes; from the standpoint of overall tax economy it would thus become worthwhile to refine administration of the latter to a very high degree. Such a tax reform, which, of course, could be brought about only gradually, would result in greater fluctuations of local tax receipts, because it is impossible and undesirable to balance the decrease of the tax base during economic depressions through increases in tax rates. One remedy would be to use reserve funds to stabilize revenues available to local governments for expenditure.[6] Another would be to facilitate borrowing and debt retirement by local governments during economic fluctuations. Both remedies require new social institutions which can scarcely come into existence and effective operation without federal assistance and regulation. At present this may seem a politically insurmountable obstacle. The jurisdictional boundaries of governments at different levels are in flux, however, and the war and its aftermath will almost certainly bring about further decisive changes in this respect. Institutions with the function just referred to would not only be beneficial through facilitating a tax reform desirable on conservation grounds; they may also become useful instruments in a fiscal policy aimed at reducing the amplitude of economic fluctuations themselves.

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6. Cf. Shere, L. and C. Shoup, Use of Reserve Funds to Stabilize Revenue Available for Expenditure. New York State Commission for the Revision of the Tax Laws. Memorandum 11. Albany, 1932. (In Report of the New York State Commission for the Revision of the Tax Laws. Legislative Document No. 77.)

MATHEMATICAL APPENDIX

1. Definition of Conservation, Depletion, and Degree of Conservation (Depletion)

If $x_1, x_2, x_3 \cdots x_n$ are rates of use of the resource X in instants $t_1, t_2, t_3 \cdots t_n$, we have conservation if $\Delta x_1 + 2\Delta x_2 + 3\Delta x_3 \cdots + n\Delta x_n > 0$; we have depletion if $\Delta x_1 + 2\Delta x_2 + 3\Delta x_3 \cdots + n\Delta x_n < 0$; we have neutrality if $\Delta x_1 + 2\Delta x_2 + 3\Delta x_3 \cdots + n\Delta x_n = 0$.

The degree of conservation (depletion) is defined by the ratio:

$$\frac{\Delta x_1 + 2\Delta x_2 + 3\Delta x_3 \cdots n\Delta x_n}{x_1 + 2x_2 + 3x_3 \cdots nx_n}$$

The degree of conservation (depletion) should not be confused with average weighted change of rates of use

$$\frac{\Delta x_1 + 2\Delta x_2 + 3\Delta x_3 \cdots n\Delta x_n}{1 + 2 + 3 \cdots + n}$$

average weighted distance of changes in rates of use

$$\frac{\Delta x_1 + 2\Delta x_2 + 3\Delta x_3 \cdots + n\Delta x_n}{\Delta x_1 + \Delta x_2 + \Delta x_3 \cdots \Delta x_n},$$

average weighted rate of use $\dfrac{x_1 + 2x_2 + 3x_3 \cdots nx_n}{1 + 2 + 3 \cdots + n}$ and average weighted distance of rates of use $\dfrac{x_1 + 2x_2 + 3x_3 \cdots nx_n}{x_1 + x_2 + x_3 \cdots + x_n}$.

The concepts of average weighted distance and average weighted distance of changes in rates of use are important because they play a significant rôle in interest theory.[1] They may also be used for defining conservation and depletion. Such a definition, however, would not be useful when considering some important special cases; for instance, an increase or a decrease of a constant rate or a rate with constant fluctuations — problems that occur particularly in agriculture, forestry, and grazing. Conservation and depletion are not identical with "lengthening" and "shortening" the much disputed "average period of production," and are not subject to the theoretical objections which have been brought against Böhm-Bawerk's concept.[2]

1. Compare, for instance, Hicks, J. R., Value and Capital, p. 186. Oxford, 1939.

2. Cf. literature cited in Machlup, F., "Professor Knight and the period of production," Journal of Political Economy, October, 1935; And Kaldor

2. Maximization of Net Revenues Over Time and Relation Between Present and Current Revenues, Costs, and Net Revenues

If V stands for present net revenues, R for present revenues, and Q for present costs, we have:

$$V=R(x_1, x_2, x_3, \cdot\cdot x_n)-Q(x_1, x_2, x_3, \cdot\cdot x_n)$$

Differentiating, we obtain as necessary condition (for sufficiency, of course, well-known auxiliary requirements must be met) for the optimum state of conservation:

$$\frac{\partial}{\partial x_1}R(x_1, x_2, x_3, \cdot\cdot x_n)=\frac{\partial}{\partial x_1}Q(x_1, x_2, x_3, \cdot\cdot x_n)$$

$$\frac{\partial}{\partial x_2}R(x_1, x_2, x_3, \cdot\cdot x_n)=\frac{\partial}{\partial x_2}Q(x_1, x_2, x_3, \cdot\cdot x_n)$$

$$\begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

$$\frac{\partial}{\partial x_n}R(x_1, x_2, x_3, \cdot\cdot x_n)=\frac{\partial}{\partial x_n}Q(x_1, x_2, x_3, \cdot\cdot x_n)$$

The meaning of the cost function in joint production and its relation to the production function have been discussed elsewhere.[3]

If $v_1, v_2, \cdot\cdot v_n$ and $r_1, r_2, \cdot\cdot r_n$ and $q_1, q_2, \cdot\cdot q_n$ denote current net revenues, revenues and cost in instants $t_1, t_2, \cdot\cdot t_n$, and i the interest rate, we may write:

$$\begin{array}{l} v_1(x_1, x_2, \cdot\cdot x_n)=r_1(x_1, x_2, \cdot\cdot x_n)-q_1(x_1, x_2, \cdot\cdot x_n) \\ v_2(x_1, x_2, \cdot\cdot x_n)=r_2(x_1, x_2, \cdot\cdot x_n)-q_2(x_1, x_2, \cdot\cdot x_n) \\ \cdot \qquad\qquad \cdot \qquad\qquad \cdot \\ \cdot \qquad\qquad \cdot \qquad\qquad \cdot \\ v_n(x_1, x_2, \cdot\cdot x_n)=r_n(x_1, x_2, \cdot\cdot x_n)-q_n(x_1, x_2, \cdot\cdot x_n) \end{array}$$

Present total functions in terms of current total functions may be written:

$$\begin{array}{l} V=v_1(1+i)^{-1}+v_2(1+i)^{-2}\cdot\cdot +v_n(1+i)^{-n} \\ R=r_1(1+i)^{-1}+r_2(1+i)^{-2}\cdot\cdot +r_n(1+i)^{-n} \\ Q=q_1(1+i)^{-1}+q_2(1+i)^{-2}\cdot\cdot +q_n(1+i)^{-n} \end{array}$$

Nicholas, "The controversy on the theory of capital," Econometrica, July, 1937.

3. Cf. Ciriacy-Wantrup, S v., "Economics of Joint Costs in Agriculture," Journal of Farm Economics, November, 1941.

Present marginal functions in terms of current marginal functions, for example, with respect to x_1 may be written:

$$\frac{\partial}{\partial x_1}V=\frac{\partial}{\partial x_1}v_1(1+i)^{-1}+\frac{\partial}{\partial x_1}v_2(1+i)^{-2}\cdots+\frac{\partial}{\partial x_1}v_n(1+i)^{-n}$$

$$\frac{\partial}{\partial x_1}R=\frac{\partial}{\partial x_1}r_1(1+i)^{-1}+\frac{\partial}{\partial x_1}r_2(1+i)^{-2}\cdots+\frac{\partial}{\partial x_1}r_n(1+i)^{-n}$$

$$\frac{\partial}{\partial x_1}Q=\frac{\partial}{\partial x_1}q_1(1+i)^{-1}+\frac{\partial}{\partial x_1}q_2(1+i)^{-2}\cdots+\frac{\partial}{\partial x_1}q_n(1+i)^{-n}$$

Necessary conditions (see qualifications above) for the optimum state of conservation in terms of present as well as current net revenues are:

$$\frac{\partial}{\partial x_1}V=\frac{\partial}{\partial x_1}v_1\,(1+i)^{-1}+\frac{\partial}{\partial x_1}v_2\,(1+i)^{-2}\cdots+\frac{\partial}{\partial x_1}v_n(1+i)^{-n}=0$$

$$\frac{\partial}{\partial x_2}V=\frac{\partial}{\partial x_2}v_1\,(1+i)^{-1}+\frac{\partial}{\partial x_2}v_2\,(1+i)^{-2}\cdots+\frac{\partial}{\partial x_2}v_n(1+i)^{-n}=0$$

$$\vdots$$

$$\frac{\partial}{\partial x_n}V=\frac{\partial}{\partial x_n}v_1(1+i)^{-1}+\frac{\partial}{\partial x_n}v_2\,(1+i)^{-2}\cdots+\frac{\partial}{\partial x_n}v_n(1+i)^{-n}=0$$

3. Complementarity, Competitiveness and Independence of Differently Dated Rates of Use and of Rates of Use of Different Resources

As before, $V(x_1, x_2, x_3, \cdots x_n)$, $R(x_1, x_2, x_3, \cdots x_n)$ and $Q(x_1, x_2, x_3, \cdots x_n)$ are the total present net revenue, revenue and cost functions of rates of use in instants $t_1, t_2, t_3, \cdots t_n$.

Two rates, for instance x_1 and x_2, are defined as complementary in net revenues if $\frac{\partial^2 V}{\partial x_1 \partial x_2}>0$; they are competitive if $\frac{\partial^2 V}{\partial x_1 \partial x_2}<0$; they are independent if $\frac{\partial^2 V}{\partial x_1 \partial x_2}=0$. The same definitions are used in the case of revenues. In the case of costs, complementarity is defined as $\frac{\partial^2 Q}{\partial x_1 \partial x_2} < 0$; competitiveness as $\frac{\partial^2 Q}{\partial x_1 \partial x_2} > 0$; and independence as $\frac{\partial^2 Q}{\partial x_1 \partial x_2} = 0$.

The second "cross" partial derivatives are likewise used for defining complementarity, competitiveness and independence of rates of use of different resources in any instants which may be considered. This means that rates of use of other resources — for example, $y_1, y_2, y_3, \cdots y_n, z_1, z_2, z_3, \cdots z_n$ of two resources Y and Z — are introduced as additional variables in the total present revenue, cost and net revenue functions.