The Henry George rule, optimal population, and interregional equity

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INTRODUCTION

An economic model of a federation involving a mobile homogeneous population, fixed local natural factors (say land) and local pure public goods was investigated by Flatters, Henderson, and Mieszkowski (1974) and analysed further by Stiglitz (1977). The size of a region defined by population involves a nice trade-off: for a fixed level of public goods, larger size results in lower per capita taxes but on the fixed resource base, larger size results in a lower marginal product per worker. Flatters et al. examined this trade-off first in a single region model. One arrives at an optimal population. The trade-off was examined in a two-region model with a fixed population. An interregional transfer was introduced. We consider an optimal population in this federal or two-region model. We observe that at the optimal population, the interregional transfer results in the aggregate value of resources in public goods production in the federation being equal to the aggregate value of Ricardian natural resource rents in the federation – an extension of a single region result emphasized by Stiglitz and labelled a Henry George rule.[1] We also observe that this Henry George rule depends on individual utilities being made equal across regions – a Rawlsian situation. For non-Rawlsian social welfare functions defined on individuals in different regions, the precise Henry George relationship – land rents equal the value of resources in the public sector – fails to hold. Furthermore the utility of an individual in region i should be set for social optimality, at a different level that the utility of an identical individual in region j – another occurrence of a seeming paradox first analysed by Mirrlees (1972).

OPTIMUM POPULATION AND THE HENRY GEORGE RULE FOR A TWO-REGION ECONOMY

We proceed under the assumption of a Rawlsian social welfare function across individuals and regions. To this end, we require that the welfare

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1 Henry George [1839–97] was born in Philadelphia and became a well-known journalist and editor in his thirties in Sacramento and San Francisco. He published his famous economics treatise *Progress and Poverty* in 1879 and became a celebrated speaker in England and Scotland shortly after. Labour groups persuaded him to run for mayor of New York in 1886 and his fame was established in the United States, although he was not elected. His writings centred on issues of land ownership, immigration, and free trade. In the 1890s his name became firmly linked with the single-tax movement. See Barker (1955).

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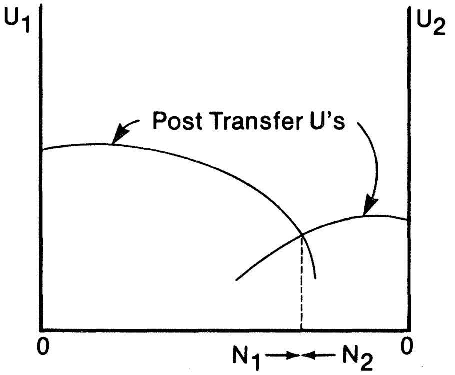

FIGURE 1a Arbitrary population

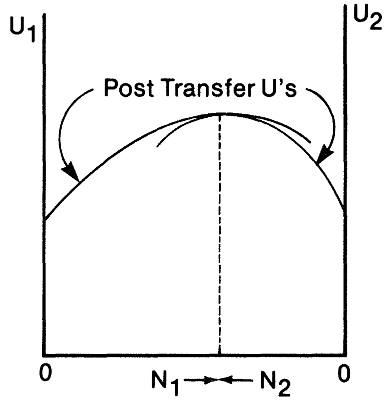

FIGURE 1b Optimal population

attained by all individuals is the same. *Within* each region, we require that resources are allocated so as to maximize an individual's welfare or utility level. Thus maximizing $u^i((F(N_i, L_i) - G_i - S/N_i), G_i)$ by choice of G_i yields the first-order condition

$$u_1{}^i = N_i u_2{}^i \quad (i = 1, 2), \tag{1}$$

where $u^i(\cdot\,, \cdot)$ is the utility function of an individual in region i,

$u_1{}^i \triangleq \partial u^i/\partial c_i$ and $u_2{}^i \triangleq \partial u^i/\partial G_i$ are assumed positive, and $u^i(0, G_i) = u^i(c_i, 0) = 0$, and $u(\cdot\,, \cdot)$ is assumed to be strictly concave;

$c_i \triangleq (F(N_i, L_i) - G_i - S/N_i)$ is per capita consumption of the private good by an individual in region i;

$F^i(\cdot\,, \cdot)$ is the production function in region i with inputs of labour, N_i and land L_i. $F_1{}^i \triangleq \partial F/\partial N_i$ and $F_2{}^i \triangleq \partial F/\partial L_i$ are assumed positive, $F^i(0, L_i) = F^i(N_i, 0) = 0$, and $F^i(\cdot\,, \cdot)$ is assumed to be strictly concave; (at no loss of generality we let $F^i(\cdot\,, \cdot)$ be the same across regions leaving $F(\cdot\,, \cdot)$ the production function)

G_i is the public good in region i, and

S is the transfer of grant from region i.

In keeping with the spirit of this model representing a federal state, we assume that the utility functions are the same in the two regions. The essential feature is a difference in resource endowments represented by 'land' between regions.

Equation (1) is the 'Samuelson condition' for public goods in an economy. This equation also permits us to obtain an expression for dG_i/dN_i which we require below. The planning problem is to allocate a fixed amount of labour N

($= N_1 + N_2$) between regions and to set a transfer S so as to maximize per capita utility. Within each region, equation (1) must be satisfied. That is maximize, by choice of N_1 and S,

$$u^1\left(\frac{F(N_1, L_1) - G_1 - S}{N_1}, G_1\right) \tag{2}$$

subject to

$$u^1\left(\frac{F(N_1, L_1) - G_1 - S}{N_1}, G_1\right) = u^2\left(\frac{F(N - N_1, L_2) - G_2 + S}{N - N_1}, G_2\right). \tag{3}$$

The two equilibrium conditions[2] are (3) and

$$\frac{G_1 - R_1 + S}{N_1} = \frac{G_2 - R_2 - S}{N_2}, \tag{4}$$

where $R_i \triangleq F - (\partial F/\partial N_i)N_i$. Condition (4) yields a formula (Flatters et al., 1974) for the optimal transfer between regions,

$$S = \frac{N_1 N_2}{N}\left[\left(\frac{G_2 - R_2}{N_2}\right) - \left(\frac{G_1 - R_1}{N_1}\right)\right]. \tag{5}$$

We now consider our two-region economy in which the population is chosen to maximize per capita utility. At the optimal population, S and N_1 will

2 Let λ be the Lagrangian multiplier for the constraint in (3). Then the first-order conditions are

$$(1 - \lambda)\left[u_1{}^1\left[\frac{-1 - \dfrac{dG_1}{dS}}{N_1}\right] + u_2{}^1\frac{dG_1}{dS}\right] = \lambda\left[u_1{}^2\left[\frac{1 - \dfrac{dG_2}{dS}}{N_2}\right] + u_2{}^2\frac{dG_2}{dS}\right] \tag{1F}$$

$$-(1 - \lambda)\left[u_1{}^1\left[\frac{\dfrac{\partial F^1}{\partial N_1} - \dfrac{dG_1}{dN_1}}{N_1} - \frac{c_1}{N_1{}^2}\right] + u_2{}^1\frac{dG_1}{dN_1}\right]$$

$$= \lambda\left[u_1{}^2\left[\frac{\dfrac{\partial F^2}{\partial N_2} - \dfrac{dG_2}{dN_2}}{N_2} - \frac{c_2}{N_2{}^2}\right] + u_2{}^2\frac{dG_2}{dN_2}\right]. \tag{2F}$$

Observe that the public good is chosen optimally *within* each region (i.e., using (1), terms cancel each other in (1F) and (2F) respectively). Note the structure of optimal allocation. The region's problem: given N_i and S, a region has an optimal division of output into private and public goods (the Samuelson condition). Thus G_i is implicitly a function of N_i and S. The federation problem: choose N_i and S optimally, given total population N and the social welfare function. Now N_i and S are implicit functions of N and the Lagrangian multiplier λ. The optimal population problem: choose N optimally, given the optimality of the region's problem and the federations problem. Each 'region' treats S and N_i as parametric. One should entertain the possibility of a 'region' reacting to changes in N_i and S and other forms of nonparametric behaviour. This will be taken up elsewhere. I am indebted to the Editor for questioning the 'parametric behaviour' assumption.

satisfy (3) and (4) and (1) will be satisfied within each region. The optimal population problem is to maximize $u^1(\cdot, \cdot)$ by choice of N.

The envelope theorem can provide us with our result straightaway. By the envelope theorem (differentiating at the maximum of problem (2), (3) with respect to N and equating this expression to zero), we have

$$u_1{}^2\left\{\frac{F_{N_2}}{N_2} - \frac{F(N_2, L_2) - G_2 + S}{N_2{}^2}\right\} = 0,$$

or

$$R_2 - G_2 + S = 0,$$

which with (4) yields our principal result, namely $R_1 + R_2 = G_1 + G_2$. Aggregate land rent after the transfer has been made equals aggregate government expenditure.[3] We illustrate in figure 1a an optimal solution with an arbitrary population and in figure 1b an optimal solution with an optimal population.

MULTI-REGION OUTCOMES UNDER ALTERNATIVE SOCIAL WELFARE FUNCTIONS

The outcome in section 2 with the Rawlsian social welfare function was striking in its simplicity. We defend the approach by suggesting that such a social welfare function seems to reflect the goals of federalism. A tradition in federalism is that resource-rich regions share their rents with poor regions. There is a philsophy of equalizing individual welfare levels.[4] However, we look at alternative cases in this section in order to develop the analysis more fully. We forgo the easy, general, two-region case and consider the special (and also not difficult) case of a Benthamite social welfare function. We observe that under the Benthamite function, the 'equal treatment of equals' does not in general result in equal individual welfare across regions.

The social welfare is

$$B = N_1 u^1 + N_2 u^2, \tag{6}$$

and we obtain the basic optimality condition for arbitrary population size

$$\frac{G_1 - R_1 + S}{N_1} + \frac{u^1}{u_1{}^1} = \frac{G_2 - R_2 - S}{N_2} + \frac{u^2}{u_1{}^2}. \tag{7}$$

For the case of N chosen optimally in addition to having (1) and (7) satisfied, we obtain the conditions

3 Existence of solutions in these models requires that certain conditions on utility functions and production functions be satisfied. These conditions can readily be met in theory. One has to rule out having all people in one region or having infinitely many in the economy. See Stiglitz (1977) and Helpman (1978).

4 See the exchanges between Buchanan (1950) and Scott (1952).

$$G_1 - R_1 + S + \frac{N_1 u^1}{u_1{}^1} = 0 \qquad (8)$$

and

$$G_2 - R_2 - S + \frac{N_2 u^2}{u_1{}^2} = 0. \qquad (9)$$

Recall that for the Rawlsian case above, we obtained $G_1 - R_1 + S = 0 = G_2 - R_2 - S$. Since $N_i u^i / u_1{}^i = 0$ $(i = 1, 2)$ for the Rawlsian case, we can assert that, since the outcome under the Benthamite social welfare function differs from that under the Rawlsian social welfare function, $u^2 \neq u^1$ immediately above. Hence the Benthamite posture of 'treating each person as an equal' a priori fails to result in each person attaining the same utility level. This is an instance of Mirrlees's result (Mirrlees, 1972).

CONCLUDING REMARKS

The Henry George Rule seems to be central to problems with local public goods and optimal size but it may appear heavily disguised when heterogeneous populations, scale economies, many commodities, and public services (a private good produced by the public sector which is divided equally among similar households) are introduced.[5] For example, Bewley (1979) does not mention it in his penetrating survey. One would like to see explorations of the direction of transfers and of the existence and stability solutions. Preliminary results are found in Stiglitz (1977).

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5 Stiglitz (1977) labels certain results as 'Henry George.' Other key 'Henry George' papers are Mirrlees (1972), Starrett (1974), and Arnott and Stiglitz (1979). See also Hartwick (1979), (1980). With regard to the economics of federalism, Boadway and Flatters (1979) have considered a two-region model with rigid wages and moving costs under alternate subsidy regimes.

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Export-base and neoclassical type models of urban growth: a synthesis

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INTRODUCTION

There are essentially two economic models which try to explain urban growth. The first and most commonly used is the export-base model, EB hereinafter (see Hewings, 1977, 17–26). The second is a neoclassical type construction developed by Muth (1968) along lines suggested by Borts and Stein (1964) (BSM hereinafter).[1] The structure of both models can be briefly summarized as follows. In the EB model the local economy is divided into an export and a domestic sector. It is then assumed that a less than perfectly elastic demand for labour in the export sector is shifted by changes in the level of exports, which in turn are related to some exogenous non-local variable. Labour demand in the production of the domestic good is made proportional to demand in the export sector. The wage rate is exogenously fixed, and along with total demand determines the level of employment in the local economy. The supply curve is inelastic and shifts, via migration, in response to the unemployment rate. Thus the non-local variable drives total local employment and the labour force. The model is summarized graphically for a given instant of time, in figure 1. D^B represents demand for labour in the export sector, D is total demand, and the horizontal distance between them is proportional to D^B. $\overline{W}$ is the exogenously given wage rate and L is the inelastic supply. E^B and E are the resulting employments in the export sector and in

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1 The literature on urban growth contains, in general, two branches. One branch treats spatial patterns of urban economies, and some of the papers are formulated in a dynamic framework. However, these tend to be concerned with the consequences of urban growth rather than with its explanation. The other branch of the literature concentrates on the explanation of urban growth. For a brief but illuminating discussion of this classification see Rabenau and Hanson (1979). For a summary of urban growth models in the EB and BSM tradition, see the recent paper by Miron (1979).

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