URBAN DENSITY FUNCTIONS

EDWIN S. MILLS

[Received October 1969]

The causes and consequences of suburbanisation are—almost literally as well as figuratively—burning issues of our time. Suburbanisation enters directly or indirectly into the discussion of almost every urban problem. But it is amazing how little we know about the subject. To what extent has suburbanisation of population caused suburbanisation of employment and vice versa? To what extent has public policy—zoning, lower taxes in suburbs than in central cities, federal mortgage insurance, etc.—accelerated the process of suburbanisation? To what extent is suburbanisation the consequence of higher incomes, improved transportation and of the mere growth of urban areas? Indeed, how much suburbanisation has occurred, and has it been proceeding more or less rapidly since World War II than before?

Despite the thousands of pages that have been written on these and other issues related to suburbanisation, we have remarkably few hard facts or verified hypotheses. The purpose of this paper is to explore carefully a narrow range of issues regarding suburbanisation. Specifically, two questions will be asked. First, how much suburbanisation has occurred? Second, how much of the observed suburbanisation can be explained by the simplest and most obvious economic and demographic variables?

Before proceeding to the major subject of the paper, a few words are in order concerning the measurement of suburbanisation. By far the most common measures of suburbanisation are the numbers or percentage of people living or working within central cities and in the surrounding suburbs. Although such measures can reveal the broad outlines of changes, they are subject to severe limitations.

First, and most important, the central city-suburb dichotomy does not provide a fixed measure of suburbanisation, since the part of the metropolitan area that is included in the central city differs greatly from one metropolitan area to another. A five-point change in the percentage of the area's residents living in the central city has a different meaning in a metropolitan area in which the central city contains one-third of the residents than in one in which it contains three quarters. It is desirable to have a measure of suburbanisation that does not depend on the historical accidents of locations of central city boundaries.

Second, and closely related, some central city boundaries change through time, mainly because central cities annex parts of contiguous suburbs. Although corrections can be made for boundary changes, they are laborious and approximate at best, and cannot be made at all for periods prior to World War II. Again, a measure of suburbanisation not dependent on changing locations of city boundaries is needed.

Third, census data are highly aggregative across space (and, in the case of employment, across industries), providing only two observations on each variable for a given metropolitan area at a given point in time. It is possible, at some cost of time and effort, to obtain population data on a

Professor Mills is in the Department of Political Economy, Johns Hopkins University, Maryland U.S.A.

much less aggregative basis, since most population data are published for Census Tracts, of which there are several dozen in a large metropolitan area. But employment is a different matter. Disclosure rules prevent the Census Bureau from publishing employment data on a more detailed basis than the city-suburb dichotomy except in very few metropolitan areas. For some metropolitan areas it is possible to obtain detailed employment data either from local directories of manufacturers or from surveys made for local land use and transportation planning studies. And for some purposes these data have proved very useful to urban researchers. But the diversity of detail, coverage, definition and timing from one of these sources to another makes it virtually impossible to obtain comparable comprehensive data for a large number of metropolitan areas.

All three of the above limitations can be surmounted if one knows or can assume something about the pattern of density in relation to distance from the city centre. Indeed, several studies, to be surveyed in the next section, have provided strong evidence that the density of population and economic activity falls off smoothly and at a decreasing rate as one moves out from the city centre. These studies have found that the negative exponential density function provides a good approximation. It can be written as

$$D(u) = De^{-\gamma u} \qquad \ldots [1]$$

where $D(u)$ is the density u miles from the centre, e is the base of the natural logarithm, and D and γ are parameters to be estimated. D is the measure of density at the city centre, and γ, which is positive, is a measure of the rate at which density declines as one moves out from the centre. If γ is large, density falls off rapidly; if it is small, density falls off slowly.

The basic insight in the present context is that, if [*1*] is an accurate representation of the density function, its estimation does not depend on where the city boundary is drawn or on whether its location changes from one time to another. Furthermore, since [*1*] represents a two-parameter family of curves, it can be estimated with the two observations provided by the city-suburb data.

What is not possible with city-suburb data is to test goodness-of-fit of [*1*], or of any two-parameter function, since there are no degrees of freedom. In fact, what the city-suburb data provide is two exhaustive and non-overlapping areas under the density function, and this permits a much better estimate of [*1*] than would a sample of densities in randomly selected Census Tracts. At least for population, previous studies suggest that the negative exponential density function provides a good fit. If that is accepted, the city-suburb data provide a perfectly acceptable way of estimating its parameters.[1] The great advantage of estimating [*1*] from city-suburb data is that the estimation procedure is simple and the data are available for a large variety of times, places and employment categories.

Previous studies of urban density functions

There have been many studies of the suburbanisation of population and employment in U.S. cities. A sample of recent and relatively high quality studies is Kain (1968), Katagawa and Bogue (1955), Moses and Williamson (1967). And there have been a few careful studies of urban density functions, which it is the purpose of this section to survey. But there have been no systematic comparisons among density functions for population and the various employment categories. Filling that gap is the major purpose of the present paper.

The first extensive study of population density functions of which I am aware is that by Colin Clark (1951). Clark presents estimates of [*1*] for a large number of European, U.S. and Australian cities for a variety of years in the nineteenth and twentieth centuries, apparently using all data that were readily available. For each city, he drew a series of concentric rings, spaced at intervals of one mile, centred on the city centre. Using Census Tract data, and excluding the

[1] The details of the procedure by which [*1*] is estimated from city-suburb data are described on p. 8.

central business districts, he calculates the average density at each concentric circle, and regresses the natural log of density on distance from the city centre. He concludes that density falls off exponentially in all cities at all times and that the density functions become flatter through time. The latter observation is attributed to declining real cost of transportation through time.

Clark's study is deficient in several ways. He does not discuss the characteristics of his data, such as whether his densities are net or gross, how he handles bodies of water and other topographical irregularities, and how he identifies central business districts. His statistical procedure leaves something to be desired in that he presents no multiple correlation coefficients, significance tests of his regression coefficients, or tests for the linearity of his logarithmic regression equations. Finally, his statement that declining transportation costs cause density functions to flatten needs clarification and analysis. He apparently refers to the money costs of transportation. But opportunity cost of time spent travelling is a large part of commuting costs and if opportunity cost rises with income, transportation cost may increase through time. Furthermore, the relationship between transportation cost and the density function is complex in some models and Clark's statement of causality may or may not hold (Mills 1967). Nevertheless, Clark's historical generalisation has been borne out by subsequent studies, and he was among the first to perceive the pattern.

By far the most careful and sophisticated estimation and analysis of urban population density functions is that of Richard Muth (1961, 1969). Muth selected for study the central cities of 46 large urbanised areas, eliminating those with two or more central cities and those whose CBD's could not be identified. Within each of the 46 central cities he selected 25 Census Tracts at random, and determined their gross population densities for 1950 and the distances from the centre of the CBD to the centres of the Census Tracts.

For each city, Muth regressed the natural log of Census Tract population density on distance from the CBD centre. The correlation coefficient between log of density and distance is significant for 40 of the 46 cities, and the median of the squared correlation coefficients is nearly one-half. A quadratic term in distance proved significant at the 10% level in 12 of the 46 cities.

Muth's estimated density gradients vary from 0·07 to 1·20, but most fall between 0·20 and 0·50. He believed that differences in density gradients among metropolitan areas were to be explained by three sets of factors: the nature and cost of commuting transportation available to CBD workers; the spatial distribution of employment and shopping centres; and preferences for housing in various parts of the city.

He estimated and tested the importance of these factors by regressing the density gradients on several variables believed to be measures of the three sets of factors. Among the variables found to be significant in explaining the density gradients were car registrations per capita, the proportion of the metropolitan area's manufacturing employment located in the central city, the proportion of the area's urbanised population living in the central city, and the proportion of the central city's population that is Negro. Using these and similar variables, Muth was able to explain about 70% of the variance of the log of the density gradients.

Opinions may of course differ as to what variables it is appropriate to use to explain urban population density. I have commented elsewhere (1969) that a major deficiency of many studies of urban land use and land value is that many explanatory variables in these studies are really endogenous to the urban economy. The basic problem is of course that whether a variable is endogenous or predetermined depends on the details of a simultaneous equation system, and few urban economists are accustomed to thinking in terms of simultaneous equation systems. Here, as elsewhere, Muth is ahead of most researchers. He tests whether some variables are exogenous by comparing his single equation estimates with simultaneous equation estimates.

Mills (1969) formulated a small simultaneous equation model of land values and land uses in a metropolitan area. Within the framework of his model, he showed that the negative exponential function can be used to approximate the decline of both land values and the density of land uses as one moves out from the city centre. Using data from Chicago, he presented a detailed analysis of both land values and land uses in that city. The measure of density is floor space per acre of land in several use categories: residential, manufacturing, commercial and public. Regression of log density on distance from the city centre provides a good fit, but with uniformly lower R^2's than the regression of log of density on log of distance.

Estimation procedure

The purpose of this section is to describe in detail how city-suburb data can be used to estimate the negative exponential density function [*1*].

Suppose that [*1*] accurately represents the density of population or a certain employment category in a metropolitan area. Suppose further that the metropolitan area is circular in shape except that a pie slice of $2\pi - \theta$ radians has been taken out. (For Chicago, for example, θ is about equal to π. Extremely irregular metropolitan areas, such as San Francisco, probably cannot be approximated by this model.) Then the number of people $n(u)$ in the category within a ring of width du, centred u miles from the city centre, is

$$n(u) = D(u)\theta u du,$$

and the total number of people in the category within k miles of the city centre, $N(k)$, is

$$N(k) = \int_0^k n(u)du.$$

Substituting and integrating by parts, we get

$$N(k) = \frac{\theta D}{\gamma^2}\left[1-(1+k\gamma)e^{-\gamma k}\right]. \quad \ldots [2]$$

Letting k go to infinity, we get the total number of people in the category in question in the entire metropolitan area, N, where

$$N = \frac{\theta D}{\gamma^2}. \quad \ldots [3]$$

We are now in a position to describe the estimation procedure. Let k be the radius of the metropolitan area's central city. k was estimated by drawing on a map a semi-circle whose centre was at the city's centre and whose boundary approximated as closely as possible the city's boundary. Cities with extremely irregular boundaries were excluded from the sample. k is then the radius of the semi-circle. θ was calculated as the value it would have to be if a semi-circular city of radius k were to have the area that the Census gave for the city in question.

Armed with these estimates of k and θ, the number of people in the appropriate category in the central city and in the entire metropolitan area are obtained from census data. These two numbers are the left hand sides of [*2*] and [*3*]. At this point, all the terms appearing in [*2*] and [*3*] are known except γ and D. Hence all that remains is to solve [*2*] and [*3*] simultaneously for γ and D. To do this, substitute N for the term outside the square brackets in [*2*]. Then γ is the only unknown in [*2*]. It was estimated iteratively by the Newton-Raphson method (Hildebrand 1961). Having thus calculated γ, it is substituted into [*3*], which permits D to be calculated.

Empirical density functions

In this section, estimates of [*1*] are presented and analysed for population and several employment categories for a sample of United States metropolitan areas. The employment categories are those for which census data are widely available. Employment data pertain to the location of the place of employment, whereas population data pertain to the location of place of residence.

The sample of metropolitan areas was chosen purposively rather than randomly. First, the shape of the central city and urbanised area had to be reasonably similar to semi-circles. Although areas were not excluded just because political boundaries were irregular, they were excluded because of major topographical irregularities. San Francisco is an example of such an exclusion. More important, semi-circular shape is precluded if two or more large cities are too close to each other. The New York-Northern New Jersey, Chicago-Gary and Los Angeles-Long Beach areas were excluded for this reason. Second, an attempt was made to include within the sample metropolitan areas with a large range of sizes. Third, an attempt was made to choose areas from different regions of the country. Fourth, areas with a wide range of historical growth rates were chosen. Finally, however, cities were chosen without any knowledge of their historical patterns of density changes. And all the calculations that were undertaken are reported here.

The sample consists of the 18 SMSA's listed in Table 1. Density functions were calculated for population and each of four employment categories (manufacturing, retailing, services and wholesaling) for 1948, 1954, 1958 and 1963. (Population data were interpolated between census years.) In total, we have the 360 density functions ($18 \times 5 \times 4$) shown in Table 1.

The calculated density functions display a remarkably consistent pattern of flattening through time. There are only 15 cases in which a gradient becomes steeper between successive years for which it was calculated. Of the 15 increases, 4 are by only ·01. Five are in Albuquerque, three in Pittsburgh, two each in Rochester and San Antonio, and one each in Boston, Philadelphia and Wichita. By sector, there are five increases each in manufacturing and services, two each in population and retailing, and one in wholesaling.

It is reasonable to say that the larger is γ the more centralised or less suburbanised is the sector. Unweighted average values of the γ's for the five sectors and the four years are shown in Table 2. By this measure, population is the least centralised, or most suburbanised, sector, followed by manufacturing, retailing, services and wholesaling in that order. Furthermore, although all sectors have suburbanised, their ranking by degree of suburbanisation has remained unchanged during the 15-year period. Table 2 also shows that the degree of suburbanisation has become more uniform among the sectors during the postwar period. The difference between the largest and smallest of the averages of the γ's is smaller both absolutely and relatively in 1963 than in 1948. In 1948, the largest average γ was almost three-fourths larger than the smallest, whereas in 1963 it was just less than half larger.

The parameters themselves vary a great deal from city to city. For population, D varies by a factor of 10, from just less than 6,000 to about 60,000. There is an obvious and expected tendency for D to be large in large metropolitan areas. γ varies much less. Again taking population as an example, it ranges from a low of ·2 to a high of nearly 1·0. There appears to be some tendency for cities that are large and that have had rapid recent growth to have small γ's. There are, however, many exceptions to these tendencies, and a detailed analysis of the determinants of density function parameters is presented in the next section.

All the data analysed so far pertain to the period following World War II. The striking pattern of flattening density functions in almost all cities and sectors raises the question whether this is a continuation of a prewar pattern or whether there has been a break in the historical pattern. Much popular literature is written as though suburbanisation were mainly a postwar phenomenon, induced by the peculiar circumstances of urban life in that period. For example, it is sometimes claimed that home mortgage insurance by the federal government has been mainly responsible for postwar suburbanisation. Or, it is claimed, postwar suburbanisation has resulted mainly from the attempt of whites to flee from the increasing numbers of Negroes in central cities. Finally, postwar suburbanisation is sometimes attributed to the rapid growth of automobile ownership during that period. All three of these factors have operated somewhat differently in the postwar period than in earlier times. If they are the major factors responsible, we should expect postwar suburbanisation to have been faster than prewar.

Of course, the answer one gets depends on the measure of suburbanisation one uses. Frequently, the measure used is the extent to which population and economic activity have moved from central cities into surrounding suburbs. But by this measure suburbanisation is an inevitable result of growth, since under any reasonable set of assumptions the number of people living and working beyond the edge of the central city will grow as the metropolitan area's population and employment grow (unless the city's boundaries are moved out as the area grows).[2] A better way to test the hypo-

[2] A property of the negative exponential density function is that if the urban area grows by increasing D, with γ constant, the fraction of people living beyond any fixed distance from the city centre will be unchanged.

Table 1

Density Functions for 18 U.S. Metropolitan Areas

		Population				*Manufacturing*			
		1948	*1954*	*1958*	*1963*	*1948*	*1954*	*1958*	*1963*
Albuquerque	γ	·56	·61	·61	·62	·71	·32	·49	·61
	D	5,748	11,387	14,148	18,180	157	151	253	483
Baltimore	γ	·48	·40	·36	·33	·48	·42	·37	·35
	D	51,159	42,693	37,481	34,541	6,815	5,575	4,665	4,059
Boston	γ	·27	·25	·23	·21	·29	·27	·25	·23
	D	35,473	32,629	28,630	24,922	4,853	4,336	3,788	3,292
Canton	γ	·69	·62	·58	·54	·94	·84	·80	·70
	D	19,994	18,724	17,610	16,591	7,814	6,374	5,143	4,130
Columbus	γ	·78	·65	·58	·52	·77	·76	·69	·63
	D	44,303	38,680	34,643	31,710	5,178	6,705	5,170	5,069
Denver	γ	·59	·45	·38	·33	·85	·64	·46	·36
	D	27,779	22,884	19,678	18,008	3,938	2,658	1,754	1,434
Houston	γ	·37	·28	·24	·21	·34	·30	·27	·23
	D	15,156	13,118	11,881	11,243	1,078	1,114	1,036	914
Milwaukee	γ	·47	·37	·32	·27	·48	·40	·35	·29
	D	58,318	44,262	37,823	31,123	12,996	8,954	7,048	5,189
Philadelphia	γ	·31	·27	·25	·23	·33	·30	·29	·26
	D	53,264	45,714	41,868	38,268	9,229	7,836	6,896	5,765
Phoenix	γ	·51	·39	·33	·28	·60	·52	·38	·31
	D	11,324	11,244	10,350	9,521	427	676	594	627
Pittsburgh	γ	·27	·25	·24	·22	·23	·22	·22	·26
	D	25,072	22,780	21,699	18,974	2,846	2,337	2,079	2,928
Rochester	γ	·73	·55	·47	·40	1·41	1·34	1·27	·89
	D	39,682	28,194	24,033	20,527	33,223	31,831	25,895	15,297
Sacramento	γ	·77	·56	·48	·41	1·03	·73	·44	·27
	D	22,120	18,337	16,782	15,262	1,405	925	634	409
San Antonio	γ	·63	·56	·50	·45	·97	·80	·48	·49
	D	27,513	28,705	25,855	23,951	2,293	1,823	732	902
San Diego	γ	·27	·23	·21	·20	·48	·36	·32	·30
	D	10,438	12,583	13,164	14,972	1,524	1,969	2,310	1,727
Toledo	γ	·83	·72	·67	·61	·98	·93	·85	·70
	D	41,123	34,661	31,768	28,151	10,638	8,414	6,223	5,517
Tulsa	γ	·89	·63	·50	·40	·62	·44	·43	·42
	D	28,788	20,126	15,339	11,947	905	890	892	839
Wichita	γ	·98	·74	·63	·54	·67	·37	·31	·33
	D	29,149	23,589	20,153	17,613	1,211	1,159	797	751

thesis that different forces have been at work in the postwar period is to see whether the pattern of shifting density functions has been different from that in the prewar period. If special forces have been at work in the postwar period, the rate of flattening of density functions should be greater than in the prewar period.

Available census data make it possible to estimate some density functions by the method outlined in section 3 back as far as the nineteenth century. It is not possible to adjust the data for changes in city boundaries, but the method employed here does not require adjustment. The further back one goes, the fewer sectors and cities for which data are available, and the more difficult it is to assemble and evaluate data. Furthermore, at any point in time, most data are collected for relatively large cities. Since rela-

Table 1

DENSITY FUNCTIONS FOR 18 U.S. METROPOLITAN AREAS

Retailing				*Services*				*Wholesaling*			
1948	*1954*	*1958*	*1963*	*1948*	*1954*	*1958*	*1963*	*1948*	*1954*	*1958*	*1963*
1·26	1·16	·91	·77	1·49	1·20	·93	·80	1·02	1·14	·88	·72
1,691	1,971	1,611	1,376	757	742	618	542	355	700	520	450
·72	·60	·50	·40	·76	·67	·62	·48	·91	·87	·76	·63
7,029	5,086	4,073	2,587	2,300	1,955	2,326	1,400	3,974	3,555	2,878	2,188
·38	·37	·33	·28	·44	·43	·44	·39	·57	·52	·46	·38
4,500	4,315	3,622	2,564	1,758	1,771	2,193	1,980	4,390	3,537	2,807	1,986
·94	·91	·86	·71	1·20	1·05	·95	·89	1·26	1·25	1·08	·98
2,146	1,965	1,848	1,256	817	647	599	538	908	1,030	752	722
1·08	·90	·78	·53	1·42	1·00	·80	·64	1·03	·84	·77	·63
6,017	4,601	3,516	1,849	3,287	1,809	1,254	1,064	1,881	1,394	1,253	967
·83	·65	·52	·39	1·12	·77	·61	·52	1·25	·89	·75	·62
3,876	2,617	2,094	1,366	2,203	1,272	1,112	1,019	4,886	2,315	2,007	1,561
·46	·41	·33	·27	·55	·45	·36	·30	·58	·50	·39	·32
1,593	1,431	1,069	778	755	642	558	485	1,267	1,129	824	635
·63	·53	·46	·30	·72	·57	·54	·37	·75	·59	·49	·36
6,951	4,666	3,857	1,877	2,443	1,758	1,955	1,001	4,042	2,528	1,947	1,035
·37	·44	·30	·26	·43	·42	·39	·36	·59	·49	·44	·37
4,182	5,797	2,855	2,889	1,604	1,685	1,720	1,710	4,139	3,058	2,529	1,891
1·04	·69	·44	·31	·97	·71	·45	·31	1·07	·71	·46	·34
2,643	1,477	909	627	741	567	396	268	1,096	726	404	277
·41	·37	·35	·33	·52	·53	·49	·50	·74	·68	·62	·50
3,323	2,260	2,142	1,809	1,306	1,435	1,334	1,437	3,284	1,820	1,510	1,495
1·00	1·12	·90	·54	·93	1·24	1·05	·82	1·42	1·39	1·22	·84
4,672	5,519	3,811	1,941	1,393	2,413	1,967	1,350	2,770	2,955	2,434	1,338
1·47	1·09	·72	·45	1·47	1·19	·83	·48	1·36	1·07	·89	·61
4,876	3,227	1,868	1,137	1,368	1,240	819	500	1,882	1,080	863	662
1·03	·75	·60	·47	·55	·76	·59	·51	1·08	·83	·62	·54
4,248	2,614	1,805	1,160	422	1,006	737	615	1,757	1,178	720	611
·34	·29	·26	·21	·35	·31	·30	·25	·43	·33	·30	·28
916	859	877	671	347	345	480	417	327	245	276	258
1·03	1·03	·80	·56	1·34	1·18	·96	·74	1·14	·111	·95	·73
4,325	3,894	2,396	1,482	1,964	1,765	1,252	856	1,866	1,714	1,367	977
1·35	·97	·68	·51	1·57	1·01	·72	·57	1·46	1·03	·75	·57
4,302	2,363	1,459	841	1,978	1,065	656	444	2,230	1,160	777	488
1·42	1·13	·83	·62	1·58	1·09	·89	·68	1·42	1·15	·84	·70
4,538	3,506	1,975	1,144	2,031	1,005	683	524	2,005	1,318	821	589

Table 2

AVERAGES OF GRADIENTS BY SECTOR AND YEAR, 18 CITIES

	1948	*1954*	*1958*	*1963*
Population	·58	·47	·42	·38
Manufacturing	·68	·55	·48	·42
Retailing	·88	·75	·59	·44
Services	·97	·81	·66	·53
Wholesaling	1·00	·86	·70	·56

tively large cities tend to grow less rapidly than relatively small cities, extending the analysis back in time tends to bias the sample toward slowly growing areas. It was decided to select six of the cities studied previously and extend the estimates back to as near the turn of the century as possible. The major criterion was data availability, but an effort was made to choose urban areas with a variety of sizes and geographical locations. Inevitably, however, data tend to be most available in the prewar period for cities in the north-eastern part of the country. All the data are from censuses of population, manufacturers and business.

Density functions for the six cities are presented in Table 3. For convenience of reference, the more recent data for the six cities are repeated from Table 1. The obvious implication of the estimates in Table 3 is that the process of flattening of density functions started long before World War II. The relative frequency of increases in density gradients is about as great in Table 3 as it was in Table 1.

To facilitate comparison with the data in Table 2, average density gradients by sector and year are presented for the six cities in Table 4. The last four columns of Table 4 show the six-city averages for the years and sectors for which Table 2 shows 18-city averages. Comparison between corresponding entries shows that the six cities are by no means entirely representative of the 18 cities.

Within the population sector, the 1910-1920 period saw the smallest decrease in γ, whereas the 1920-1930 decade showed the largest decrease. Presumably the former reflects the influence of World War I and the latter the prosperity of the 1920's and to some extent the growing use of motor vehicles. There was a small decrease during the 1930's presumably owing to the effects of the depression, and a surprisingly large decrease from 1940 to 1948, considering that the period contained World War II. In absolute terms the decrease from 1948 to 1958 was considerably larger than the decrease from 1954 to 1963.

It is clear from Table 4 that other sectors were suburbanising steadily before World War II. Table 4 also shows the convergence in degree of suburbanisation among sectors that was observed in Table 2. In 1929 the largest entry is almost twice as large as the smallest (assuming that the unavailable average for services is less than the average for wholesaling; otherwise, the convergence is even greater), whereas in 1963 the largest is less than two-thirds larger than the smallest. It is interesting to observe that the rate of flattening of the density function for manufacturing employment was small relative to that for population in the early period, but has become large in recent years. The gradients for the two sectors were virtually identical in 1920, whereas in the early post-World War II years, the gradient for manufacturing employment exceeded that for population by about ·20. By 1963, the difference had fallen to ·12. This suggests, but does not prove, that the movement of people to the suburbs has attracted manufacturing employment rather than vice versa.

Has postwar suburbanisation been faster than prewar? From 1910 to 1940, the population density gradient fell by about ·01 per year. For the period from either 1940 or 1948 to 1963, the average annual fall was about 0·13. In manufacturing, the average annual rate of decrease in γ was less than ·01 from 1920 to 1939, and since 1948 it has been nearly ·02. The gradients for retailing and wholesaling also show somewhat more rapid rates of flattening in the postwar than in the prewar period. Thus the evidence from these six metropolitan areas seems to be that postwar density functions have flattened faster than prewar. However, a large part of the prewar evidence is from the decade of the 1930's, and it can hardly be doubted that the depression slowed up

Table 3

DENSITY FUNCTIONS FOR SIX U.S. METROPOLITAN AREAS

		Population							
		1910	*1920*	*1930*	*1940*	*1948*	*1954*	*1958*	*1963*
Baltimore	γ	·97	·75	·64	·60	·48	·40	·36	·33
	D	111,230	79,681	67,630	65,542	51,159	42,693	37,481	34,541
Denver	γ	·87	·87	·83	·76	·59	·45	·38	·33
	D	28,291	34,870	36,265	35,334	27,779	22,884	19,678	18,008
Milwaukee	γ	·88	·81	·56	·51	·47	·37	·32	·27
	D	108,510	114,200	74,209	65,434	58,318	44,262	37,823	31,123
Philadelphia	γ	·45	·43	·37	·36	·31	·27	·25	·23
	D	63,566	70,839	62,034	59,789	53,264	45,714	41,868	38,268
Rochester	γ	1·44	1·37	·96	·88	·73	·55	·47	·40
	D	82,015	95,878	58,464	50,775	39,682	28,194	24,033	20,527
Toledo	γ	1·13	1·43	1·01	·93	·83	·72	·67	·61
	D	41,407	85,828	56,260	47,031	41,123	34,661	31,768	28,151

		Manufacturing						
		1920	*1929*	*1939*	*1948*	*1954*	*1958*	*1963*
Baltimore	γ	·70	·66	·49	·48	·42	·37	·35
	D	9,478	7,547	4,416	6,815	5,575	4,665	4,059
Denver	γ	1·07	·94	·92	·85	·64	·46	·36
	D	3,215	2,506	1,710	3,938	2,658	1,754	1,434
Milwaukee	γ	·52	·44	·40	·48	·40	·35	·29
	D	11,713	8,921	5,012	12,996	8,954	7,048	5,189
Philadelphia	γ	·32	·35	·32	·33	·30	·29	·26
	D	7,586	7,332	5,243	9,229	7,836	6,896	5,765
Rochester	γ	1·51	1·28	1·32	1·41	1·34	1·27	·89
	D	24,514	16,493	14,235	33,223	31,831	25,895	15,297
Toledo	γ	1·55	1·24	1·16	·98	·93	·85	·70
	D	17,097	13,214	6,570	10,638	8,414	6,223	5,517

		Retailing					
		1929	*1939*	*1948*	*1954*	*1958*	*1963*
Baltimore	γ	1·02	·88	·72	·60	·50	·40
	D	7,257	6,592	7,029	5,086	4,073	2,587
Denver	γ	1·10	1·00	·83	·76	·52	·39
	D	3,933	3,697	3,876	2,617	2,094	1,366
Milwaukee	γ	·59	·56	·63	·53	·46	·30
	D	4,039	4,074	6,951	4,666	3,857	1,877
Philadelphia	γ	·47	·39	·37	·44	·30	·26
	D	4,493	3,118	4,182	5,797	2,855	2,229
Rochester	γ	1·35	1·24	1·00	1·12	·90	·54
	D	5,685	5,135	4,672	5,519	3,811	1,941
Toledo	γ	1·61	1·30	1·03	1·03	·80	·56
	D	6,501	4,486	4,325	3,894	2,396	1,482

2

the rate of suburbanisation. If one considers population and compares the pre-1954 period with the 1954-1963 decade, or the 1910-1930 period with the 1948-1963 period, the rates of suburbanisation are about the same. A fair conclusion from these data seems to be that most of

Table 3 contd.

		Wholesaling					
		1929	*1939*	*1948*	*1954*	*1958*	*1963*
Baltimore	γ	1·54	1·49	·91	·87	·76	·63
	D	7,639	6,866	3,974	3,555	2,878	2,188
Denver	γ	1·61	1·51	1·25	·89	·75	·62
	D	4,393	3,645	4,856	2,315	2,007	1,561
Milwaukee	γ	·78	·77	·75	·59	·49	·36
	D	3,045	2,609	4,042	2,528	1,947	1,035
Philadelphia	γ	·70	·63	·59	·49	·44	·37
	D	4,384	2,934	4,139	3,058	2,529	1,891
Rochester	γ	1·63	1·40	1·42	1·39	1·22	·84
	D	2,697	1,732	2,770	2,955	2,434	1,338
Toledo	γ	2·29	1·63	1·14	1·11	·95	·73
	D	6,393	2,405	1,866	1,714	1,367	977

		Services				
		1939	*1948*	*1954*	*1958*	*1963*
Baltimore	γ	1·07	·76	·67	·62	·48
	D	2,777	2,300	1,955	2,326	1,400
Denver	γ	1·24	1·12	·77	·61	·52
	D	1,494	2,203	1,272	1,112	1,019
Milwaukee	γ	·68	·72	·57	·54	·37
	D	1,323	2,443	1,758	1,955	1,001
Philadelphia	γ	·49	·43	·42	·39	·36
	D	1,243	1,604	1,685	1,720	1,710
Rochester	γ	1·47	·93	1·24	1·05	·82
	D	1,144	1,393	2,413	1,967	1,350
Toledo	γ	1·76	1·34	1·18	·96	·74
	D	1,660	1,964	1,765	1,252	856

Table 4

Averages of Gradients by Sector and Year, 6 Cities

	1910	*1920*	*1929*	*1939*	*1948*	*1954*	*1958*	*1963*
Population*	·96	·94	·73	·67	·57	·46	·41	·36
Manufacturing	—	·95	·82	·77	·76	·67	·60	·48
Retailing	—	—	1·02	·90	·76	·73	·58	·41
Services	—	—	—	1·12	·88	·81	·70	·55
Wholesaling	—	—	1·43	1·24	1·01	·89	·77	·59

* Figures in columns headed 1929 and 1939 are for 1930 and 1940 respectively.

the rapid suburbanisation just after World War II was the result of the war and the preceding depression, rather than of basically new forces.

Determinants of density functions

The relationship between intensity of land use for any purpose and distance from the centre of a metropolitan area is complex and can only be explored adequately within the framework of a detailed model of urban structure. All theoretical models of urban structure suggest that the negative exponential density function is at best a rough approximation to reality and that any exogenous change is likely to affect land use intensity in complex ways.

The justification for the use of the negative exponential density function in this and other studies is its computational convenience, its approximate accuracy at cetain levels of aggregation and its value as an easily understood descriptive summary. But since the negative exponential density function is not closely related to detailed theoretical models, it follows that attempts to specify the determinants of its parameters are somewhat intuitive and unrigorous. The major problems are to decide which variables should be taken to be exogenous determinants of the density function parameters, the direction of the effect, and the form of the relationship. Ideally, all these questions should be answered with the help of a general equilibrium model. Nevertheless, it is possible to overstress this point. Many empirical studies in economics are less closely related to theoretical models than would be ideal. Most empirical studies of consumer demand, for example, are not much more closely related to the theory of consumer demand than density function studies are to theories of urban structure.

Changes in urban density functions involve the erection, alteration and demolition of structures, which have notoriously long lives. It is therefore important to distinguish carefully between the determinants of equilibrium values of parameters of density functions and the process of adjustment from one equilibrium value to another. In view of the widespread recognition that structures have very long lives, it is surprising that studies of density functions have paid no attention to the disequilibrium adjustment process.

It is assumed here that the equilibrium values of the parameters of the density functions are determined as follows:

a. Size of SMSA. It seems clear that in equilibrium D should be an increasing function of the size of the SMSA. Almost any conceivable model would imply that large metropolitan areas would extend both upward and outward further than small ones. γ should be a decreasing function of SMSA size, since large SMSA's can support sub-centres for shopping and employment, and are therefore less dependent on the city centre than are small SMSA's. The most obvious measure of size is the SMSA's population. For the employment categories, however, total SMSA employment in the category may also be a determinant of D and γ

b. Income. There is a high income elasticity of demand for high quality and low density housing. It follows that in the residential sector both D and γ should be decreasing functions of income per family in equilibrium. There does not, however, appear to be any reason to believe that family income should affect the density functions in the employment categories.

c. Transportation prices. A decrease in the relative price of transportation per passenger mile will presumably lead to an increase in passenger miles travelled. For given amounts and densities of land devoted to other purposes, the result will be an increase in land used as an input in the transportation sector. Thus, more land will be used by the SMSA, and population and employment will be spread more thinly over the larger amount of land. This argument implies that D and γ should be decreasing functions of transportation prices in equilibrium. The difficulty with the foregoing argument is that a change in the relative price of transportation does affect the amount of non-transportation activities in the metropolitan area and the intensity of their land uses. The result is that it is difficult to predict the effect of transportation prices on density functions. A practical difficulty is that it is hard to get data on transportation prices. Real operating costs per passenger mile of transportation vehicles have probably fallen during the postwar period, but a large part of commuting cost is the opportunity cost of time spent travelling, which presumably rises with income. The temptation is to introduce time as a proxy for changes in the price and technology of transportation in the explanation of variations in density function parameters. Although the tradition of using time as a proxy for the state of technology is well established in economics, it may be less clear just what the result means in this application than in others.

There is little theoretical basis for choosing among alternative forms of the relationships between parameters of the density functions and the explanatory variables, but linear and log linear forms have been most successful in previous studies.

The foregoing suggests estimation of the following relationships:

$$D^*_{it} = \alpha^*_0+\alpha^*_1 P_{it}+\alpha^*_2 Y_{it}+\alpha^*_3 t+\eta_{it} \qquad \ldots [4]$$

and

$$\gamma^*_{it} = \beta^*_0+\beta^*_1 P_{it}+\beta^*_2 Y_{it}+\beta^*_3 t+\varepsilon_{it} \qquad \ldots [5]$$

for the population sector, and

$$D^*_{it} = \alpha^*_0+\alpha^*_1 P_{it}+\alpha^*_2 N_{it}+\alpha^*_3 t+\eta_{it} \qquad \ldots [6]$$

and

$$\gamma^*_{it} = \beta^*_0+\beta^*_1 P_{it}+\beta^*_2 N_{it}+\beta^*_3 t+\varepsilon_{it} \qquad \ldots [7]$$

separately for each employment category; and the logarithmic counterparts of these equations. In these equations, D^*_{it} and γ^*_{it} are the equilibrium values of D and γ in the ith SMSA and the tth time period. P stands for the population of the SMSA, Y for median family income and N for employment in the SMSA in the relevant category.

Starred values of the γ's and D's represent equilibrium values and are unobserved except in the unlikely event the system is in equilibrium. It is assumed that the adjustment of the γ's and D's to equilibrium can be approximated by the well known distributed lag adjustment process, which assumes that between successive observations the variable adjusts by a constant fraction of its deviation from equilibrium in the earlier period. This process can be represented by

$$D_{it}-D_{it-1} = \mu(D^*_{it}-D_{it-1}) \qquad \ldots [8]$$

and

$$\gamma_{it}-\gamma_{it-1} = \lambda(\gamma^*_{it}-\gamma_{it-1}) \qquad \ldots [9]$$

where μ and λ are the adjustment coefficients.

Substituting the right hand sides of [4]-[7] for D^* and γ^* in [8] and [9] gives

$$D_{it} = \alpha_0+\alpha_1 P_{it}+\alpha_2 Y_{it}+\alpha_3 t +(1-\mu)D_{it-1}+\eta_{it} \qquad \ldots [10]$$

and

$$\gamma_{it} = \beta_0+\beta_1 P_{it}+\beta_2 Y_{it}+\beta_3 t +(1-\lambda)\gamma_{it-1}+\varepsilon_{it} \qquad \ldots [11]$$

for the household sector, and

$$D_{it} = \alpha_0+\alpha_1 P_{it}+\alpha_2 N_{it}+\alpha_3 t +(1-\mu)D_{it-1}+\eta_{it} \qquad \ldots [12]$$

and

$$\gamma_{it} = \beta_0+\beta_1 D_{it}+\beta_2 N_{it}+\beta_3 t +(1-\lambda)\gamma_{it-1}+\varepsilon_{it} \qquad \ldots [13]$$

for the employment sectors. In these equations, $\alpha_i = \mu\alpha^*_i$ and $\beta_i = \lambda\beta^*_i$. (The logarithmic analogues to [4]-[13] are obtained by writing the logarithms of the variables instead of the variables themselves. The logarithmic versions of [8] and [9] assume that D and γ change by a constant fraction of the percentage deviation from equilibrium each period, rather than by a constant fraction of the numerical deviation, as in [8] and [9].) All the variables in [10]-[13] are observable, and from estimates of [10]-[13] it is easy to derive estimates of the starred parameters and of λ and μ.

Estimates of [10]-[13] are presented in Table 5; the derived estimates of [4]-[7] are in Table 6. Estimates of the logarithmic analogues to [10]-[13] are in Table 7; the derived estimates of the logarithmic analogues to [4]-[7] are in Table 8. Sample points are the D's and γ's shown in

Table 1.[3] Figures in parentheses below the coefficients are t-values.

The R^2's in Tables 5 and 7 are consistently large. None is below 0·65, and only those for retailing are below 0·70. The R^2's in the equations for γ are not very different from those in the equations for D. Nor are the R^2's for the logarithmic equations very different from those for the linear equations. (Of course, a different sum of squares is minimised in the logarithmic equations than in the linear equations.)

As should be expected, the adjustment process is quite slow for both γ and D. In the linear equations for γ, for example, the coefficients of lagged γ average about 0·75, indicating that only about one-fourth of any deviation from equilibrium is corrected during a five-year period. And it is not surprising to learn that service employment adjusts faster than other employment categories and than population. Retailing is the next fastest category to adjust, and manufacturing and wholesaling are slowest. Population adjusts about as slowly to disequilibrium as manufacturing. The logarithmic equations show much the same ranking of sectors by speed of adjustment to disequilibrium, except that wholesaling adjusts much more slowly than any other sector. The expectation that service and retail employment might adjust most rapidly is based on the belief that less construction or movement of capital is involved in their movement than in other sectors. However, it is not clear why wholesaling should be particularly slow to adjust to disequilibrium.

The striking implication of both the linear and the logarithmic equations for population is that the cause of the historical flattening of density functions has been growth of population and income, rather than the passage of time (or the cheapening of transportation, if time is assumed to be a proxy for transportation cost). These equations imply that if population and income remained constant, cities would gradually become more, rather than less, centralised. Indeed, Table 6 shows that the equilibrium value of γ would increase by about 0·25, or half its average value in the sample, per decade at constant population and income. Evaluated at the sample means of the variables, the logarithmic equation for population in Table 8 implies that γ would increase by about 30%, about 0·15, per decade at constant population and income.

In the employment sector equations for γ, there is a striking sign pattern among the coefficients, which is entirely consistent between the linear and logarithmic equations. The coefficient of population is negative for manufacturing and positive for all other employment sectors. The coefficients for SMSA sector employment and time are positive for manufacturing and negative for all other employment sectors. Signs of lagged dependent variables are all positive. The implication of these equations is that manufacturing employment would become less, and other employment sectors more, suburbanised through time if SMSA population and sector employment were constant. The former observation, plus the fact that the coefficient of SMSA population is negative in the manufacturing employment equation, confirms the suggestion made above that postwar flattening of manufacturing employment density functions has resulted from the growth and suburbanisation of population rather than from the growth of sector employment or the passage of time. This conclusion will be surprising to some, but is not really implausible. But what about the other employment sectors? The equations in Tables 5 and 7 indicate that postwar flattening of employment density functions in retailing, services and wholesaling has been caused by the growth of SMSA sector employment and the passage of time rather than by the growth of SMSA population. Why should coefficients have opposite signs in the manufacturing sector from those in the other sectors? And how does one rationalise the sign pattern in the non-manufacturing sectors? I have no satisfactory answer to these questions. That the pass-

[3] The prewar data in Table 3 could not be used because the interval between observations was longer than for the postwar data.

age of time should result in flattening of density functions is not surprising, but it is not clear why the effect is in the opposite direction in manufacturing. That the growth of sector employment causes flattening can also be rationalised, but why should population growth have the opposite effect? In fact, the only substantial correlations among the independent variables are those between SMSA employment in the various sectors and population. These are all in excess of 0·95, and the coefficients of SMSA population and sector employment are therefore unreliable. But it is intriguing that the signs are all the same in the non-manufacturing employment sectors and, except for that the lagged dependent variable, exactly the opposite in the manufacturing sector.

The sign pattern among coefficients in the equations for *D* is much more complex. Signs of coefficients differ from one non-manufacturing employment sector to another. More important, in some cases the sign of a particular coefficient differs as between the linear and logarithmic equations. Of course, the intuitive appeal of the model is somewhat less strong as an explanation of *D* than of γ. Indeed, some researchers have not even included *D* in their attempts to explain shifts in density functions. But the question remains why the R^2's are so large for the *D* equations.

Table 5

Linear Regressions for Parameters of Density Functions

Dependent Variable		*Constant Term*	*Population*	*SMSA Median Family Income*	*SMSA Sector Employment*	*Time*	*Lagged Dependent Variable*	R^2
Population	γ	$\cdot1635(10)^{-1}$ (·3448)	$-\cdot4453(10)^{-8}$ (−·5696)	$-\cdot8827(10)^{-5}$ (−·8353)		$\cdot2531(10)^{-1}$ (1·6307)	·7952 (18·5037)	·937
	D	$\cdot3209(10)^{4}$ (1·8989)	$\cdot6710(10)^{-3}$ (2·1342)	−·5439 (−1·0244		$\cdot8863(10)^{3}$ (1·1193)	·7651 (28·4906)	·961
Manu-facturing	γ	$-\cdot2070(10)^{-1}$ (−·2848)	$-\cdot7805(10)^{-7}$ −(1·3396)		$\cdot6411(10)^{-6}$ (1·5059)	$\cdot2038(10)^{-1}$ (1·2125)	·7997 (14·2689)	·874
	D	$\cdot8525(10)^{3}$ (1·3189)	$\cdot4850(10)^{-3}$ (·6320)		$-\cdot3086(10)^{-2}$ (−·5130)	$-\cdot3125(10)^{3}$ (−1·5155)	·8123 (28·6586)	·960
Retailing	γ	·2063 (2·4802)	$\cdot2797(10)^{-7}$ (·4106)		$-\cdot5479(10)^{-6}$ (−·4000)	$-\cdot4932(10)^{-1}$ (−3·1767)	·7225 (13·6722)	·906
	D	$\cdot1015(10)^{4}$ (1·9254)	$-\cdot4692(10)^{-3}$ (−·8103)		$\cdot1135(10)^{-1}$ (·9647)	$-\cdot2070(10)^{3}$ (−1·5946)	·5598 (8·2627)	·692
Services	γ	·2310 (2·3179)	$\cdot5840(10)^{-7}$ (·7959)		$-\cdot3641(10)^{-5}$ (−·8708)	$-3048(10)^{-1}$ (−1·5101)	·6607 (11·5080)	·856
	D	$\cdot5342(10)^{3}$ (2·5511)	$-\cdot5904(10)^{-4}$ (−·2798)		$\cdot1438(10)^{-1}$ (1·1848)	$-\cdot1308(10)^{3}$ (−2·2839)	·5640 (8·2627)	·744
Wholesaling	γ	$\cdot8749(10)^{-1}$ (·9473)	$\cdot5533(10)^{-7}$ (1·0653)		$-\cdot2350(10)^{-5}$ (−·9426)	$-\cdot2731(10)^{-1}$ (1·5915)	·8092 (14·8281)	·895
	D	$\cdot2792(10)^{3}$ (1·2488)	$\cdot7225(10)^{-4}$ (·4024)		$\cdot1421(10)^{-2}$ (·1581)	$-\cdot5938(10)^{2}$ (−·9722)	·6337 (12·1947)	·877

Table 6

LINEAR REGRESSIONS FOR PARAMETERS OF EQUILIBRIUM DENSITY FUNCTIONS

Dependent Variable		*Constant Term*	*Population*	*SMSA Median Family Income*	*SMSA Sector Employment*	*Time*
Population	γ^*	·7984(10)$^{-1}$	−·2174(10)$^{-7}$	−·4310(10)$^{-4}$		·1236
	D^*	·1366(10)^{5}	·2856(10)$^{-6}$	−·2315(10)^{1}		·3773(10)^{4}
Manufacturing	γ^*	−·1033	−·3893(10)$^{-6}$		·3199(10)$^{-5}$	·1017
	D^*	·4692(10)^{4}	·2669(10)$^{-2}$		−·1699(10)$^{-1}$	−·1721(10)^{4}
Retailing	γ^*	·7433	·1008(10)$^{-6}$		−·1975(10)$^{-5}$	−·1777
	D^*	·2612(10)^{4}	−·1066(10)$^{-2}$		·2577(10)$^{-1}$	−·4703(10)^{3}
Services	γ^*	·6809	·1646(10)$^{-6}$		−·1073(10)$^{-4}$	−·8983(10)$^{-1}$
	D^*	·1225(10)^{4}	−·1354(10)$^{-3}$		·3299(10)$^{-1}$	−·3000(10)^{3}
Wholesaling	γ^*	·4585	·2900(10)$^{-6}$		−·1232(10)$^{-4}$	−·1431
	D^*	·7621(10)^{3}	·1972(10)$^{-3}$		·3879(10)$^{-2}$	−·1621(10)$^{-1}$

Table 7

LOG LINEAR REGRESSIONS FOR PARAMETERS OF DENSITY FUNCTIONS

Dependent Variable		*Constant Term*	*Log Population*	*Log SMSA Median Family Income*	*Log SMSA Sector Employment*	*Log Time*	*Lagged Dependent Variable*	R^2
Population	log γ	·9069 (·9806)	−·2829(10)$^{-1}$ (−1·2017)	−·1021 (−·9073)		·1415 (1·8125)	·9046 (19·850)	·967
	log D	·1756(10)^{1} (1·1508)	·1679(10)$^{-1}$ (·6691)	·6223(10)$^{-2}$ (·0323)		−·4877(10)$^{-1}$ (−·3634)	·7939 (20·0988)	·919
Manufacturing	log γ	−·1380 (−·2182)	−·4706(10)$^{-1}$ (−·6185)		·3845(10)$^{-1}$ (·7928)	·1020 (1·0838)	·8676 (12·4121)	·850
	log D	·8332 (1·1807)	−·7051(10)$^{-1}$ (−·7355)		·1034 (1·1985)	−·6320(10)$^{-1}$ (−·5330)	·8621 (20·5795)	·959
Retailing	log γ	−·1460 (−·1893	·7614(10)$^{-1}$ (·4579)		−·8330(10)$^{-1}$ (−·5464)	−·2608 (−4·2090)	·8672 (12·4597)	·937
	log D	3·4309 (1·769)	−·4434 (−·9936)		·5069 (1·1295)	−·3362 (−1·8762	·6519 (6·6274)	·653
Services	log γ	−·1539(10)^{1} (−2·1154)	·3161 (2·4304)		−·2978 (−2·3830)	−·1240 (−1·7319)	·8554 (12·5529)	·903
	log D	−·1245(10)^{1} (−1·0741)	·3343 (1·4860)		−·1497 (−·6753)	−·3123 (−2·4748)	·7572 (12·0124)	·833
Wholesaling	log γ	−·9544 (−2·0377)	·1499 (2·1250)		−·1226 (−2·0460)	−·9226(10)$^{-1}$ (−1·6932)	·9969 (18·7097)	·939
	log D	−·2373 (−·2780)	·1518 (1·0848)		−·9531(10)$^{-1}$ (−·6707)	−·1583(10)$^{-1}$ (−1·3432)	·8610 (14·6125)	·909

Table 8

LOG LINEAR REGRESSIONS FOR PARAMETERS OF EQUILIBRIUM DENSITY FUNCTIONS

Dependent Variable		Constant Term	Log Population	Log SMSA Median Family Income	Log SMSA Sector Employment	Log Time
Population	log γ^*	·9508(10)[1]	–·2966	–·1070(10)[1]		·1483(10)[1]
	log D^*	·8519(10)[1]	·8143(10)[–1]	·3018(10)[–1]		–·2365
Manufacturing	log γ^*	–·1042(10)[1]	–·3554		·2904	·7706
	log D^*	·6043(10)[1]	–·5114		·7502	·4584
Retailing	log γ^*	–·1101(10)[1]	·5738		–·6277	–·1966(10)[1]
	log D^*	·9857(10)[1]	–·1274(10)[1]		·1456(10)[1]	–·9660
Services	log γ^*	–·1057(10)[2]	·2171(10)[1]		–·2046(10)[1]	–·8514
	log D^*	–·5128(10)[1]	·1377(10)[1]		–·6165	–·1286(10)[1]
Wholesaling	log γ^*	–·2998(10)[3]	·4708(10)[2]		–·3851(10)[2]	–·2898(10)[2]
	log D^*	–·1708(10)[1]	·1092(10)[1]		–·6857	–·1139

Acknowledgment: The research reported in this paper was supported by a grant from Resources for the Future Inc.

REFERENCES

CLARK, C. (1951). Urban Population Densities. *Journal of the Royal Statistical Society*. Series A. Vol. 114. Pp. 490-496.

HILDEBRAND, F. (1961). *Methods of Applied Mathematics*. Englewood Cliffs, Prentice-Hall.

KAIN, J. (1968). The Distribution and Movement of Jobs and Industry. Discussion Paper No. 8, Program on Regional and Urban Economics. Harvard University.

KITAGAWA, W. and BOGUE, E. and D. *Suburbanization of Manufacturing Activity within Standard Metropolitan Areas*. Oxford, Ohio, Scripps Foundation. 1955.

MILLS, E. (1967). An Aggregative Model of Resource Allocation in a Metropolitan Area. *American Economic Review*. Vol. LVII, No. 2. Pp. 197-210.

MILLS, E. (1969). The Value of Urban Land. In *The Quality of the Urban Environment*. Harvey Perloff (ed.). Pp. 231-253. Baltimore, Johns Hopkins.

MOSES, L. and WILLIAMSON, H. (1967). Location of Economic Activity in Cities. *American Economic Review*. Vol. LVII, No. 2. May. Pp. 211-222.

MUTH, R. (1961). The Spatial Structure of the Housing Market. *Papers and Proceedings of the Regional Science Association*. Vol. 7. Pp. 207-220.

——. (1968). Urban Residential Land and Housing Markets. In *Issues in Urban Economics*. Harvey Perloff and Lowdon Wingo (eds.). Baltimore, Johns Hopkins. Pp. 285-333.

——. (1969). *Cities and Housing*. Chicago. Chicago University Press.