Methods of Capital Gains Taxation and the Impact on Asset Prices and Welfare

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Abstract - *Taxation of capital gains upon realization instead of accrual provides incentives to hold winners as long as possible and sell losers immediately. This so–called lock–in effect possibly distorts the liquidation and investment decision and, hence, is usually regarded as harmful. This paper analyzes the impact the method of taxation has on asset prices and welfare within a simple general equilibrium model of an exchange economy with heterogeneous agents. It is shown that asset prices are higher under a realization–based tax system than under an accrual one. However, due to distributional effects, total welfare is not necessarily lower.*

INTRODUCTION

In most of the world's economies, changes in the value of an investor's asset (or entire portfolio) are subject to a tax, the so–called capital gains tax. From a theoretic point of view, there are basically two different methods of collecting this tax: taxation of capital gains upon *accrual* or upon *realization*.

Under an accrual system—sometimes also referred to as "yield–to–maturity" approach—the tax is payable, in theory, as soon as there is a change in the value of an asset (e.g., by a change in the asset price) or, in practice, periodically. Among others, the most severe problems that arise under an accrual tax are those of liquidity and valuation. Some investors might be forced to sell some of their assets, which they would keep hold of otherwise, just in order to pay the tax. For some assets that are not frequently or not publicly traded, it can be very costly if not impossible to permanently or periodically assess their value.

For those practical reasons, assets for which such problems arise are mostly taxed upon realization. Under a realization system—sometimes also referred to as the "wait and see" approach—the tax is payable only when the investor sells the asset, thereby realizing a gain or a loss. Solving the problems of liquidity and valuation, the realization tax creates a new problem of its own by distorting the investor's optimal liquidation policy and, hence, possibly his investment decision: It equips the investor with a timing option that enables him to realize capital losses immediately and defer capital gains in order to save taxes. To see this, look at the following example.

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The owner of an asset with basis P_0, actual price P_1 and final payout P_2 decides on either selling and repurchasing the asset in period 1 or holding the asset until period 2 in order to maximize his period-2 payout after taxation at the constant rate $0 < \tau < 1$.

Under an accrual tax the investor obviously is indifferent between the two strategies. Both of them leave him with the same after-tax payout in period 2 equal to

$$W^{acc} = \frac{P_1 - \tau(P_1 - P_0)}{P_1}[P_2 - \tau(P_2 - P_1)].$$

Note that under the *hold strategy* he still has to liquidate part of the asset in period 1 in order to fulfill his tax liability. Under a realization system the after-tax payout in period 2 following the "hold" strategy is

$$W_H^{real} = P_2 - \tau(P_2 - P_0),$$

whereas the *sell–and–repurchase strategy* yields $W_R^{real} = W^{acc}$, the same payout as under an accrual tax. A comparison of the two strategies shows that the hold strategy is superior under a monotone price path:

[1] $W_H^{real} \geq W_R^{real} \Leftrightarrow$

$(P_0 \geq P_1 \geq P_2) \vee (P_0 \leq P_1 \leq P_2).$

Assuming that, as in most oft he relevant cases, the investor expected the asset to appreciate when he purchased it and still does in period 1 ($P_0, P_1 \leq P_2$), the optimal liquidation policy according to [1] suggests to choose W_H^{real} if $P_0 \leq P_1$ and W_R^{real} otherwise, i.e., defer gains (as long as possible) and realize losses (immediately).

The same result is derived by Constantinides (1983) from a similar situation, but where firstly the asset is risky, secondly there is an alternative investment opportunity, which thirdly is taxed upon accrual, i.e., different taxation methods coexist.[1] The above analysis shows that the result hinges on neither of those additional assumptions.[2]

However, additionally assuming the existence of an alternative investment opportunity, Auerbach (1991) proves that the investor finds it optimal to keep hold of an asset with accrued capital gains instead of selling it and buying the alternative one even for some (expected) pre–tax rates of return smaller than the alternative pre–tax rate. This indicates that, besides the distortion of the optimal liquidation policy, a realization–based tax possibly leads to inefficient portfolio selection and a distortion of the investment decision.

Usually both distorting effects arising from taxing capital gains upon realization are summarized and labeled the lock–in effect. Nevertheless, for analytical purposes it is worthwhile distinguishing between one and the other: The effect on the optimal liquidation policy is always present and referred to as the *primary lock–in effect* for the remainder of this paper. The effect on the investment decision arises only in the presence of alternative investment opportunities and is referred to as the *secondary lock–in effect*. Note again that the assumptions of uncertainty and the coexistence of different taxation methods are not necessary for those effects to occur. Hence, the analysis stated below surrenders these assumptions in order to isolate the lock–in effect from possibly additional effects due to

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[1]   The assumption of a riskless, tax–exempt bond by Constantinides (1983) is equivalent to the assumption of a riskless bond taxed upon accrual, which yields the same after tax rate.

[2]   However, the above analysis requires the assumption that the investor expects the asset to appreciate ($P_0, P_1 \leq P_2$), which is not necessary for the result in Constantinides (1983) to hold.

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risk[3] and the concomitance of different taxation methods. Moreover, to keep things simple, this paper focuses on the primary lock–in effect and, hence, a single investment opportunity.

Considering the distortions caused by the lock–in effect, there is a natural question arising: Does taxation of capital gains upon realization do harm creating a welfare loss? Put differently: Is social welfare smaller under a realization tax than under an accrual tax? The answer usually given in the economic literature is in the affirmative, but the reasoning is rather based on heuristic considerations than proper analysis in a formal model (e.g., Kovenock and Rothschild, 1987). The present paper tries to fill the gap and examines the question more closely.

Of course a welfare analysis within the framework used in the above example, where asset prices are exogenously given, is not very fruitful as it neglects the impact a specific method of capital gains taxation has on asset prices. To take this price effect into account but still keep the analysis tractable, a simple general equilibrium model of an exchange economy with heterogeneous agents is investigated. It is shown that in the presence of accrued capital gains, asset prices are higher under a realization tax than under an accrual system. Since the realization system creates incentives to defer accrued gains to later periods, actual total demand for the asset and, thus, its price increase. These results are in line with prevailing empirical findings reporting the price–enhancing impact of the lock–in effect (e.g., Blouin, Raedy, and Shackelford, 2003; Jin, 2006).

However, the impact the method of taxation has on welfare is ambiguous in terms of the Pareto–criterion. Though a realization system distorts the individual saving decisions, the elimination of these distortions by an accrual system causes distributional effects, impeding a clear–cut welfare improvement. While a realization system discriminates agents without accrued capital gains, it is in favor of individuals holding assets with such gains. This result may explain why attempts to implement reforms of capital gains taxation towards an accrual system consistently fail. A recent prominent example is the case of the Italian 1998 tax reform, where some important elements of capital gains taxation designed in order to put the system on an accrual–like basis were abolished only a few months after their introduction.[4]

The remainder of this paper is structured as follows. The next section offers a short review of the related literature. The third section specifies the model and establishes the consumer's problem of utility maximization under different regimes of capital gains taxation. The analysis shows that comparative statics results within the existing literature often are due to special assumptions, mostly with respect to the consumers' utilities. The details of this analysis are provided in Appendix A. In the fourth section the impact of the method of taxation on asset prices and welfare is analyzed by comparing an accrual–based system with a realization tax. The results are illustrated in Appendix B considering the example of quasi–linear logarithmic preferences within a slightly extended version of the model. The fifth section discusses some possible extensions of the model, and the sixth section concludes.

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[3] For the relation between risk taking and capital gains taxation see, e.g., Sandmo (1985). The role of risk within the model developed here is briefly discussed later on in an extension.

[4] Maybe not surprisingly, between introduction and abolishment of these elements, the government changed from left–wing to conservative; see Alworth, Arachi, and Hamaui (2003) for a detailed study.

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REVIEW OF THE LITERATURE

One branch of the literature on realization–based taxation of capital gains consists of a series of articles that consider the lock–in effect to be harmful and, hence, search for tax systems avoiding or, at least, reducing the problem. Such proposals are mostly based upon the idea of imitating an accrual tax by retrospective taxation on a realization basis, first expressed in Vickrey (1939), and are found in the economic (e.g., Meade, 1978; Auerbach, 1991; Bradford, 1995; Auerbach and Bradford, 2004; Boadway and Keen, 2003; Alworth et al., 2003) as well as in the tax law literature (e.g., Shuldiner, 1992; Cunningham and Schenk 1992; Warren, 1993; Land, 1996); Warren (2004) and Sahm (2007) provide comprehensive surveys of this topic.

Though these papers show in theory what a realization–based tax system would have to look like in order to circumvent the distortions raised by the lock–in effect without running into problems of liquidity and valuation, such schemes are hardly ever used in practice. As Alworth et al. (2003) report, this might be due to a lack of transparency or an inconsistency with respect to the underlying concept of (ex–ante vs. ex–post) fairness. However, investigating the impact that the method of taxation has on welfare, the paper at hand argues that distributional aspects, i.e., tax–clientele concerns, might play a decisive role for the political reluctance. Tracing this idea, it is convenient to review another branch of the literature on capital gains taxation that puts a different question: How are portfolio selection, asset prices, and tax incidence affected by the realization requirement?

As in Auerbach (1991), the impact on portfolio selection can be analyzed within a partial equilibrium framework, where (expected) asset prices or, equivalently, (expected) pre–tax rates of return are exogenously given. Balcer and Judd (1987)

show that the method of capital gains taxation as well as the investors' individual horizons for saving will affect the optimal portfolio composition. Similarly, in a simulation model, Dammon, Spatt, and Zhang (2001) show that the optimal dynamic consumption and portfolio decision is a function of the investor's age, initial portfolio holdings, and tax basis. Kovenock and Rothschild (1987) compute the effective tax rates under a realization system and compare the net returns of different portfolio strategies. However, if one wants to take price effects into account, a general equilibrium model has to be engaged.

As pointed out, for example, by Lang and Shackelford (2000), whenever investigating price effects of capital gains taxation, one has to be aware of an impact that arises independently from the method of taxation, be it accrual or realization based: A higher tax rate lowers the after–tax return of an asset, which in turn results in a lower demand for the assets and, hence, given a fixed supply, a lower asset price. This so–called *capitalization effect* is opposed to the lock–in effect, which occurs only under a realization–based system: A higher tax rate induces bigger incentives to postpone the realization of accrued capital gains resulting in higher demand for those assets and, hence, given a fixed supply, higher asset prices. The intuition behind the lock–in effect can also be stated as follows: The owner of an asset with accrued capital gains will sell it only if he is compensated for the tax advantage he foregoes by selling it. Thus, prices must rise. Empirical studies draw a mixed picture concerning the question whether the capitalization effect (see, e.g., Lang and Shackelford, 2000; Rendleman and Shackelford, 2003) or the lock–in effect (see, e.g., Landsman and Shackelford, 1995, Klein, 2001, Blouin et al., 2003, Jin, 2006) is prevailing. Dai, Maydew, Shackelford, and Zhang (2008) employ a reduced form model of the stock

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market to demonstrate that the equilibrium impact of capital gains taxes reflects both effects. They show and empirically confirm that—depending on time periods and asset characteristics—either one may dominate.

There is a broad literature modeling the capitalization effect in different settings under the assumption of an accrual tax (e.g., Auerbach, 1979; Gordon and Bradford, 1980; Collins and Kemsley, 2000). In contrast, so far only few articles exist that explicitly account for the fact that capital gains are usually taxed upon realization and, hence, are able to incorporate the lock–in effect.

Constantinides (1983) develops a capital asset pricing model under the assumptions of a realization tax and perfect capital markets. However, using short–selling strategies, in his model investors are able to separate their liquidation decision from their consumption and saving decision and, thus, to defer tax payments until so–called "events of forced liquidation" (e.g., death). Consequently, the lock–in effect is capitalized in the asset prices only to the extent that such events occur.

Stiglitz (1983) shows that under realistic assumptions, by applying sophisticated trading strategies, investors on perfect capital markets can avoid not only the payment of realization–based capital gains taxes but all income taxes. This provides an indication and Poterba (1987) supports empirical evidence that the assumptions of perfect capital markets, especially the one of unlimited short–selling, are not sustainable if one wants to describe a reality in which investors pay a considerable amount of capital gains taxes.

Klein (1999) engages a general equilibrium model to study the impact of capital gains taxation on asset prices and portfolio selection under the assumptions of imperfect capital markets where short–selling is not allowed. In a multi–period setting, finitely many individuals maximize their utility from consumption by periodically deciding on how much to consume and save given their initial endowments. The investment opportunities are exogenously given and consist in a riskless asset taxed upon accrual and finitely many risky assets taxed upon realization. His findings can be summarized as follows: The pre–tax returns for assets with accrued capital gains are smaller, i.e., their prices are higher, than for assets without accrued gains. The lock–in effect is capitalized in asset prices and might overcompensate for the capitalization effect. Put differently, asset prices may increase by higher taxes. Moreover, the optimal portfolio selection depends not only on one's own amount of accrued capital gains and saving horizon, but also on the amounts of accrued capital gains and the saving horizons of all other investors.

Viard (2000) employs a setup similar to that of Klein (1999) in order to model an infinite–horizon exchange economy and investigate the dynamic asset pricing effects and incidence of realization–based capital gains taxation.[5] He finds that, in contrast to accrual taxation, asset prices are increased by an increase in the current realization tax rate. Therefore, the resulting tax burden is borne not only by the sellers but—similar to the incidence of an excise tax—divided between the buyers and sellers of an asset.

The complexity of Klein's framework makes it very hard if not impossible to use it for a welfare analysis. His model is rich in the sense that it does not only account for the pure effects of capital gains taxation, i.e., the capitalization effect and the lock–in effect, but also for possibly additional effects arising from uncertainty and the coexistence of different taxation

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[5] Whereas Viard (2000) compares the *incidence of the tax rate* for different methods of taxation, the welfare analysis of this paper examines the *incidence of the taxation method* itself given the tax rate.

methods. The idea of the analysis presented below is to simplify the model in order to separate the different effects from each other. The aim is to remain in the position to analyze the impact the method of capital gains taxation has on asset prices but, in addition, to get into a position that allows for undertaking a welfare analysis. The model described in the following sections accomplishes that by renouncing risk and the coexistence of different tax systems. Moreover, only the impact of the primary lock–in effect will be analyzed, i.e., there is a single saving opportunity. As compared to the framework of Klein (1999), the model uses stronger assumptions concerning the initial endowments of the agents, whereas it gets by with much weaker assumptions on consumers' preferences.

A SIMPLE GENERAL EQUILIBRIUM MODEL

In this section, the model outlined above is developed more formally and used to derive some comparative statics results for different regimes of capital gains taxation.

Basic Framework and Specific Assumptions

Consider a two–period ($t \in \{1,2\}$) exchange economy with two agents ($i \in \{1,2\}$), who are price takers and trade only at equilibrium prices. Given their initial endowments, they maximize their utility $U^i(c_1^i, c_2^i)$ from private consumption in period 1, c_1^i, and 2, c_2^i, deciding on how much to save in period 1. To guarantee the existence of a Walrasian equilibrium, assume the preferences to be continuous, strictly convex, strongly monotone, and, moreover, such that consumption in period t is a normal commodity, as it is

common for large aggregates. For analytical convenience, assume that preferences can be expressed by twice continuously differentiable utility functions

$$U^i : (\mathbb{R}_0^+)^2 \to \mathbb{R} \quad \text{such that} \quad \frac{\partial^2 U^i}{\partial c_t^{i2}} < 0 < \frac{\partial U^i}{\partial c_t^i}.$$

For the reasons mentioned above, let there be a single saving opportunity: one arbitrarily divisible share of an asset with basis P_0 and safe payout P_2 in period 2. The asset possesses the characteristics of a zero–coupon bond and may be interpreted as a real investment in the following way: At some prior time[6] the amount of P_0 consumption goods has been invested in a project that yields a safe payout of P_2 consumption goods in period 2 but cannot be liquidated before (in period 1). The analysis aims at finding the asset price P_1, i.e., the price a share of this investment is traded at in period 1.

The most general framework would allow for an arbitrary division of the share in period 1 as part of the consumers' initial endowments. However, such a setting would create the problem of identifying the seller and the buyer of the asset respectively. As in Klein (1999), one would have to separately look at the cases where consumer 1 is either the buyer or the seller or no trade takes place. To avoid this problem the following assumption is made.

Assumption 1 *At the beginning of period 1, consumer 1 holds one unit of the asset with basis* P_0, *whereas consumer 2 has no shares. Moreover, in period 1 consumer i has income $W^i \geq 0$ measured in consumption goods. Besides the payout of the asset, none of them has any additional income in period 2.*

Therefore, at any equilibrium consumer 1 sells shares and consumer 2 buys them in order to consume in period 2. In particu-

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[6] For the analysis of the accrual tax regime, it is convenient to assume that this time lies in period 0 in order to guarantee that no capital gains taxes have been paid for the asset until period 1.

lar, by Assumption 1, short–selling never takes place.[7] Admittedly this situation is an extreme scenario but an economically interesting and relevant one.

From an abstract point of view, if one aims at analyzing distributional effects of the taxation method, one has to introduce some source of heterogeneity between the agents. The easiest way to do so is by different initial endowments. Remember that the model considers a pure exchange economy in which production is exogenous and real investment has taken place in some prior period.[8] In such a reduced framework, different individuals' endowments at a certain point in time might be interpreted also as the result of the agents' heterogeneity with respect to production possibilities (e.g., different skills) or investment capacities (e.g., different credit ratings) in the past.

Reading the model more literally, Assumption 1 can be thought of as describing a situation in which both consumers enter the economy at the beginning of period 1 and live for two periods. While both consumers are equipped with inherited skills enabling them to earn income W^i in their youth (period 1), only consumer 1 has inherited wealth (in the form of share ownership) as well. This outline fits pretty well the situation at the housing market in the UK. In Great Britain a relatively huge amount of total wealth is held in the form of housing equity (see, e.g., Banks, Blundell, and Smith, 2002). Accordingly, like in Assumption 1, on the one hand there is the group of house owners endowed with a considerable amount of that asset and, on the other hand, a group of people without housing wealth. Capital gains on that market have a strik-

ing influence on wealth distribution and consumption in the UK (see, e.g., Henley, 1998; Disney, Henley, and Jevons, 2003), which explains the vivid discussion about how to tax them.

Depending on the equilibrium asset price P_1, the agents' different initial endowments imply heterogeneity with respect to their accrued capital gains in period 1 as well. To abbreviate the analysis and avoid case differentiation, the model focuses on a (weakly) increasing price path.

Assumption 2 *In equilibrium the asset price P_1 in period 1 satisfies $P_0 \leq P_1 \leq P_2$.*

This is not an assumption on the input but the outcome of the model and, thus, seems to be quite strong at first glance. However, for the purpose of a comparison between accrual and realization taxation respectively, a (weakly) increasing price path describes the only interesting and relevant outcome among the possible equilibria. This can be seen within two steps.

First note that $P_0, P_1 \leq P_2$ may be regarded as reflecting the implicit assumption that the consumers could—in addition to holding some share of the asset—simply stockpile consumption goods in order to transfer utility between periods. If so, $P_2 < P_1$ could never be part of an equilibrium, since then neither consumer would like to use the asset as a saving vehicle.[9] Similarly, for $P_2 < P_0$ there would have been no investment in the asset in the first place.

Second, remember that in case of a capital loss, i.e., for $P_1 < P_0$, because of inequality [1] consumer 1 would always choose the sell–and–repurchase strat-

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[7]   In other words, here the assumption of no short–selling explicitly made by Klein (1999) or Dammon et al. (2001) is implicit in the model.

[8]   The case of a production economy is discussed below in an extension.

[9]   In the case without taxation (cf. next subsection), for example, the intertemporal budget constraint for consumer $i \in \{1, 2\}$ would then be given by $c_2^1 = W^1 + P_1 - c_1^1 + S^1(P_2 - P_1)$ and $c_2^2 = W^2 - c_1^2 + S^2(P_2 - P_1)$ respectively, indicating $S^i = 0$ for any $P_2 < P_1$.

egy under realization taxation, thereby inducing the same tax treatment as under accrual taxation. Put differently, the existence of such a strategy[10] implies that both methods of taxation are equivalent in case of a capital loss. Particularly, in this case, their impact on the asset price and welfare is the same. Hence, with Assumption 2 implicitly supposing the availability of a sell–and–repurchase strategy, one can restrict the analysis to the case of capital gains without further loss of generality.

In period 1 the asset is traded. Let S^i be the fraction of the asset consumer i possesses after trade has taken place. The market clearing condition requires

[2] $S^1 + S^2 = 1$.

Capital gains are taxed at the constant rate $0 \leq \tau < 1$ in each period after trade or rather payout and before consumption have taken place. Now the consumer's problem can be stated and analyzed under different tax regimes.

The Problem without Taxation

To highlight some basic equilibrium properties, first look at the problem without taxation. By the choice of $S^i \in [0, 1]$, consumer i maximizes $U^i(c_1^i, c_2^i)$ such that the budget constraints

[3] $c_1^1 = W^1 + (1 - S^1)P_1$

$c_2^1 = S^1 P_2$

and

[4] $c_1^2 = W^2 - S^2 P_1$

$c_2^2 = S^2 P_2$

respectively hold. From the first order conditions

[5] $\dfrac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i} = \dfrac{P_2}{P_1}$

price dependent demand functions $S^i(P_1)$ for the asset can be derived, and the market clearing condition [2] delivers an equilibrium price P_1^*. Applying the implicit function theorem on the first order conditions yields the following comparative statics results (cf. Appendix A):

[6] $\dfrac{dS^1}{dP_1} \substack{\geq \\ <} 0, \quad \dfrac{dS^2}{dP_1} < 0.$

An increasing asset price P_1 makes consumption in period 1 relatively cheaper, i.e., has a negative substitution effect on consumption in period 2 and, hence, decreases savings S^i for both consumers. While the income effect is also negative for agent 2, it is positive for agent 1. Therefore, the asset demand of consumer 2 is decreasing in the asset price, whereas the effect is not clear cut for consumer 1.

Under the assumptions made, there is—as mentioned above—always an equilibrium price, but it is not necessarily unique. However, in the case of multiple equilibria for almost every combination of values P_0, P_2, W^1, and W^2 a locally isolated equilibrium price P_1^* exists such that aggregated asset demand is decreasing in P_1 (see, e.g., Mas–Colell, Whinston, and Green, 1995, Section 17.D.):

[7] $\dfrac{d(S^1 + S^2)}{dP_1} < 0.$

The following analysis refers always to this type of equilibrium. Note, in particu-

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[10] This assumption is reasonable even though so–called *wash sales* usually are excluded from deduction by the tax code. In practice, this exclusion is easily circumvented, e.g., by switching to another asset with similar characteristics (Constantinides, 1983).

lar, that inequality [7] is satisfied if the equilibrium price P_1 is unique.

The Problem under Accrual Taxation

Under an accrual tax the consumer's budget constraints are given by

[8]　$c_1^1 = W^1 + (1 - S^1)P_1 - \tau(P_1 - P_0)$

$c_2^1 = S^1[P_2 - \tau(P_2 - P_1)]$

and

[9]　$c_1^2 = W^2 - S^2 P_1$

$c_2^2 = S^2[P_2 - \tau(P_2 - P_1)]$

respectively. Note that the taxable base of consumer 1 in period 1 is not only the realized part but his entire capital gain. The first order conditions for a maximum become

[10]　$\dfrac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i} = \dfrac{P_2 - \tau(P_2 - P_1)}{P_1}.$

Now the consumers' asset demand depends not only on the price P_1 but also on the tax rate τ. Additionally, consumer 1's consumption and, hence, asset demand respond to changes in accrued capital gains, i.e., changes in the basis P_0. Consequently, in general one has $S^1 = S^1 (P_1, \tau, P_0)$ and $S^2 = S^2 (P_1, \tau)$ respectively. Hence, the equilibrium price $P_1^* = P_1^*(\tau, P_0)$ is also a function of the tax rate and accrued capital gains.

With the same reasoning as before, the comparative statics results of expression [6] hold in the presence of an accrual tax (cf. Appendix A). Investigating changes in the tax rate, one gets a somewhat surprising result:

Because income and substitution effect work in opposite directions with respect to consumption in period 1, in the absence of additional assumptions, a change in the tax rate has no clear–cut effect on the consumers' saving decision and, hence, the effect on the asset price is ambiguous as well. This is often overlooked by the literature describing the depressing nature of the capitalization effect (e.g., Lang, and Shackelford, 2000): Even under an accrual system, an increasing tax rate does not necessarily result in decreasing asset prices. These observations are summarized in the following.

Remark 1 *Under accrual taxation, a marginal increase in the tax rate τ marginally decreases the equilibrium asset price P_1^*, i.e., the capitalization effect of capital gains taxation is negative if and only if the substitution effect dominates the income effect.*

Models that neglect the income effect using a reduced form description of the stock market (e.g., Dai et al., 2008) or specific consumer preferences (e.g., Viard, 2000) may be justified by empirical evidence. However, caution is necessary when interpreting the reactions of asset prices to changes in the (realization–based) tax rate. While a decrease in asset prices unambiguously shows the prevalence of the (negative) capitalization effect, an increase in asset prices is a priori not necessarily a consequence of the realization requirement and the associated lock–in effect. As Remark 1 makes clear, this might be simply due to a strong income effect.

Moreover, in the existing literature, the amount of capital gains is often thought to be neutral under an accrual tax in the

[11]　$\dfrac{dS^1}{d\tau} \gtrless 0, \quad \dfrac{dS^2}{d\tau} \gtrless 0 \quad$ and, thus, $\quad \dfrac{dP_1^*}{d\tau} \gtrless 0.$

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sense that it does not affect the saving and investment decision (e.g., Auerbach, 1991). However, this is not true if income effects play a role like in the model presented here. To see this, first note that consumer 2's problem and, thus, asset demand are not altered by a change in the accrued capital gain of consumer 1, i.e., by a change in P_0. In contrast, a lower accrued gain, i.e., higher P_0, has a pure income effect, increasing consumer 1's consumption in both periods. Since his period 2 consumption does not depend directly on P_0, it can only be augmented by saving more (cf. Appendix A):

[12] $\quad \dfrac{dS^1}{dP_0} = \geq 0.$

Applying the implicit function theorem on the market–clearing condition [2], and employing inequalities [7] and [12] yield the following result:

[13] $\quad \dfrac{dP_1^*}{dP_0} = -\dfrac{\dfrac{\partial S^1}{\partial P_0}}{\dfrac{\partial (S^1 + S^2)}{\partial P_1}} \geq 0.$

Put differently, as a reaction to higher accrued gains, total asset demand and, hence, the equilibrium price may decrease. This means, in particular, that in the presence of capital gains taxes, asset prices do not only depend on (expected) future payoffs but possibly also on past prices, even if the tax is levied upon accrual.

The Problem under Realization Taxation

While consumer 2's budget constraints [9] and first order condition [10] do not alter under a realization system, consumer 1 now pays taxes in period 1

only for the realized part of his capital gains and, hence, faces the following constraints:

[14] $\quad c_1^1 = W^1 + (1 - S^1)[P_1 - \tau(P_1 - P_0)]$

$\qquad c_2^1 = S^1[P_2 - \tau(P_2 - P_0)]$

resulting in the first order condition

[15] $\quad \dfrac{\partial U^1 / \partial c_1^1}{\partial U^1 / \partial c_2^1} = \dfrac{P_2 - \tau(P_2 - P_0)}{P_1 - \tau(P_1 - P_0)}.$

As a consequence, if there is heterogeneity among the agents with respect to their accrued capital gains, i.e., if $P_0 < P_1$, consumers no longer face the same relative prices for consumption in period 1 and 2 respectively.[11] Put differently: Whereas the marginal rates of substitution for consumer 1 and 2 coincide under an accrual tax, they differ under a realization tax. Hence, the resulting equilibrium allocation under taxation upon realization cannot be Pareto efficient. This result may be seen as a formal justification for the branch of literature surveyed in Warren (2004) or Sahm (2007) that tries to find a way of circumventing the lock–in effect by simulating an accrual system on a realization basis. However, as Proposition 2 will show, the result does not imply that the equilibrium allocation resulting from an accrual tax Pareto dominates the equilibrium allocation under a realization tax.

Comparative statics show that relations [6] and [11] still hold under a realization tax (cf. Appendix A). This means in particular that, as in other models incorporating both the capitalization and lock–in effect (e.g., Klein, 1999; Viard, 2000; Dai et al., 2008), an increasing tax rate may possibly lead to higher asset prices.

However, the prediction of Klein (1999) that the pre–tax returns for assets with

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[11] For $P_0 = P_1$ the FOCs [10] and [15] coincide. For $P_0 < P_1$, by inequality [1] consumption in period 2 is relatively cheaper for consumer 1 than 2.

accrued capital gains are smaller, i.e., their prices are higher, than for assets without such accrued gains cannot be verified in this setting. Yet, in contrast to the case of an accrual system—remember inequality [13]—under a realization tax, higher accrued capital gains can possibly increase asset prices as the following comparative statics show (cf. Appendix A):

will show that comparing the impact of taxation methods on asset prices leads to clear–cut results.

PRICE AND WELFARE EFFECTS

In this section, the impact that the method of capital gains taxation has on asset prices and welfare is investigated.

$$[16] \quad \frac{dS^1}{dP_0} \gtrless 0 \quad \text{and, hence,} \quad \frac{dP_1^*}{dP_0} = -\frac{\frac{\partial S^1}{\partial P_0}}{\frac{\partial(S^1 + S^2)}{\partial P_1}} \gtrless 0.$$

The intuition behind this result is as follows: Now, for consumer 1, a smaller accrued gain, i.e., higher P_0, does not only result in a positive income effect on consumption in both periods but also in a substitution effect such that consumption in period 2 becomes more expensive. Thus, the overall effect on consumption in period 2 and, hence, on saving is ambiguous.

Comparative Statics Summary

The results derived so far can be summarized in the following way: In general the optimal saving decision of an investor depends on his accrued capital gain, even under an accrual tax. Under an accrual tax, P_1 is increasing in P_0, i.e., the higher the accrued capital gain is, the lower is the asset price. Under a realization tax, in general the impact of a change in P_0 on P_1 is ambiguous, i.e., the effect can possibly but does not necessarily have to be reverted. Moreover, due to opposing income and substitution effects, the influence of a change in the tax rate on asset prices is not clear cut either, not even under an accrual system. In contrast to that somehow unsatisfactory ambiguity with respect to the tax rate as an instrument of public policy, the next section

To this end, the equilibrium allocation under an accrual–based taxation system is compared to the equilibrium outcome under a realization–based one.

The Impact on Asset Prices

To carry out a welfare analysis, one first has to learn how equilibrium prices are affected by a certain method of taxation.

Proposition 1 (Price effect) *Given a fixed tax rate, $0 < \tau < 1$, in the presence of accrued capital gains ($P_0 < P_1$), the equilibrium asset price P_1^* is higher under a realization tax than under an accrual system.*

The result is intuitive: Compared to accrual taxation, the realization system creates an incentive for agent 1 to defer part of his accrued gains until period 2 in order to save taxes, while consumer 2 is not directly affected by the method of taxation. Thus, total demand for the asset and its price are higher; more formally:

Proof

Consumer 2's problem and, hence, asset demand $S^2(P_1)$ are the same under both taxation methods.[12] For any given $P_0 < P_1 \leq P_2$, inequality [1] ensures

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[12] This observation, of course, is due to the special structure of the model, where consumer 2 holds the asset for exactly one period and, hence, the methods of taxation are equivalent.

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$$\frac{P_2 - \tau(P_2 - P_1)}{P_1} \le \frac{P_2 - \tau(P_2 - P_0)}{P_1 - \tau(P_1 - P_0)};$$

hence, consumer 1's problem under a realization tax differs from the one under an accrual tax in two ways: As one can see from the intertemporal budget constraints resulting from [8] and [14] respectively,

$$c_2^{1acc} = \frac{P_2 - \tau(P_2 - P_1)}{P_1}[W^1 + P_1 - \tau(P_1 - P_0)]$$

$$- \frac{P_2 - \tau(P_2 - P_1)}{P_1}c_1^{1acc},$$

$$c_2^{1real} = \frac{P_2 - \tau(P_2 - P_0)}{P_1 - \tau(P_1 - P_0)}[W^1 + P_1 - \tau(P_1 - P_0)]$$

$$- \frac{P_2 - \tau(P_2 - P_0)}{P_1 - \tau(P_1 - P_0)}c_1^{1real},$$

first his budget set is larger. Secondly consumption in period 2 is relatively cheaper. Since consumption in period 2 is a normal commodity, both income

and substitution effect are positive with respect to period–2 consumption of consumer 1 and, hence, $c_2^{1real} > c_2^{1acc}$. The situation is illustrated by the solid lines in Figure 1. However, this is only possible by higher savings, because for any given S^1 and $P_0 < P_1 \le P_2$ by the constraints [8] and [14]

$$c_2^{1acc} = S^1[P_2 - \tau(P_2 - P_1)]$$

$$> S^1[P_2 - \tau(P_2 - P_0)] = c_2^{1real}$$

holds, i.e., ceteris paribus consumer 1's consumption in period 2 is higher under an accrual than a realization system. Therefore, consumer 1's demand $S^1(P_1)$ and, thus, total demand for the asset are higher under a realization tax. By inequality [7], that results in a higher equilibrium price P_1^*. QED

The Impact on Welfare

In order to investigate welfare effects in a framework with taxes, one generally

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Figure 1. Consumer 1's Problems
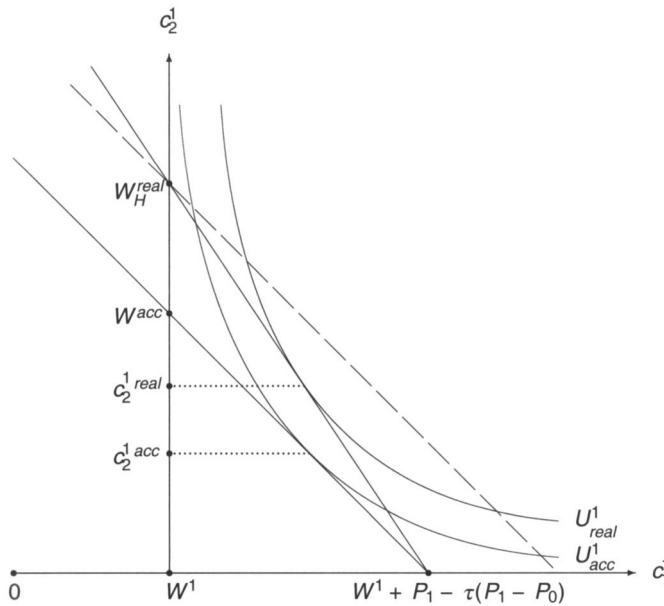
has to take into account how tax revenues are spent in the public sector. A trivial way to circumvent this problem is to assume that the tax revenues are used for expenditures that do not create any surplus for consumers, e.g., transfers to third parties or a Leviathan government. Within the present setting, an alternative solution to the problem is the assumption of no intertemporal discounting by the authorties.

Assumption 3 *Either the government's*
(a) expenditures do not create any surplus for the consumers or
(b) discount rate ρ equals zero.

As stated by the following lemma, assuming that the government does not discount,[13] total tax revenues and, hence, expenditures are not affected by the method of taxation.

Lemma 1 *Given a fixed tax rate $0 < \tau < 1$ and no intertemporal discounting by the public authorities ($\rho = 0$), the present value of total tax revenue is the same under an accrual–based tax system and a realization–based one respectively.*

Proof

Compare the present value of total tax revenue under an accrual system T_{acc} and a realization one T_{real} respectively:

$$T_{acc} = \tau(P_1 - P_0) + \frac{1}{1+\rho}[\tau S^1(P_2 - P_1)$$
$$+ \tau(1 - S^1)(P_2 - P_1)]$$
$$\overset{\rho=0}{=} \tau(P_2 - P_0),$$

$$T_{real} = \tau(1 - S^1)(P_1 - P_0) + \frac{1}{1+\rho}[\tau S^1(P_2 - P_0)$$
$$+ \tau(1 - S^1)(P_2 - P_1)]$$
$$\overset{\rho=0}{=} \tau(P_2 - P_0). \quad \text{QED}$$

Note in particular that under neither system does the revenue depend on the asset price P_1 in period 1. By means of Proposition 1 and Lemma 1, it is possible to prove the following statement concerning total welfare.

Proposition 2 (Welfare effect) *Under Assumption 3 and given a fixed tax rate $0 < \tau < 1$, the equilibrium allocation under neither of the taxation methods Pareto dominates the other. In the presence of accrued capital gains ($P_0 < P_1$) consumer 1's utility is higher and consumer 2's utility is lower under a realization tax than under an accrual system.*[14]

Proof

Under Assumption 3(a), government expenditure has no effect on consumers' utilities; under Assumption 3(b), by Lemma 1, total tax revenue and, thus, expenditure is equal under each of both methods. Put differently, there is no effect on utilities caused by different public spending. As seen in the proof of Proposition 1, the method of taxation has no direct effect on the problem of consumer 2 whereas the budget set and, hence, utility of consumer 1 is larger under a realization tax than an accrual one. Additionally, by Proposition 1 the equilibrium price P_1 is higher under a realization tax, which has, compared to the situation under an

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[13] This could be modeled explicitly by assuming that the government is bound to use its period i revenues in order to finance a public good g_i in period i, which enters the consumers' utility functions in a quasi–linear way (cf. the example of Appendix B).

[14] Using the continuity of the relevant functions, one easily verifies that Proposition 2 will still hold for sufficiently small discount rates $\rho > 0$ if the tax rate is adopted to the corresponding method of taxation in a way such that the present value of total tax revenue remains unchanged. On the one hand, as shown in the proof of Proposition 2, a change in the method of taxation causes a discrete change in consumers' utilities. On the other hand, starting from $\rho = 0$, a marginal increase in the discount rate ρ only leads to a marginal change in consumers' utilities.

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accrual tax, two opposed effects: On the one hand, for any $S_1 > 0$ this further relaxes the budget constraint [14] of consumer 1 and, thus, increases his relevant budget set and, hence, utility. The situation is illustrated by the dashed line in Figure 1. On the other hand, for any $S^2 > 0$, this analogically tightens the budget constraint [9] of consumer 2 and, thus, decreases his relevant budget set and utility. QED

As has been pointed out above, the equilibrium allocation under a realization tax cannot be Pareto efficient, while the one under an accrual tax may be. However, Proposition 2 shows that the Pareto welfare criteria are not able to give a political device on the preferability of one taxation method or the other.[15]

Distributional Aspects

In view of Proposition 2, without further assumptions, Pareto efficiency does not provide a valid justification for an accrual tax on normative grounds if the policy space is restricted to the choice of the taxation method. To decide on the method of taxation, a weaker concept of social welfare has to be employed, which involves the aggregation of individual utilities. Note that any such utility aggregation, for example, by a social welfare function, implicitly incorporates an interpersonal comparison, i.e., a certain ideal of how utility should be distributed within the economy. The example provided in Appendix B shows that the optimal method of taxation depends on the social welfare function employed. Hence, in the political process, the distributional norm is decisive for the method of taxation.

For example, if the norm is fairness, the above result may give a hint on how this

decision might look: As demonstrated in Proposition 2, compared to an accrual–based tax system, a realization–based one discriminates consumer 2 while it favors consumer 1, i.e., it favors agents with relatively large accrued capital gains. It should be easy to find empirical evidence for the claim that such gains occur more often among the "wealthy" than the "poor" and more often among the "elderly" than the "young." Given these presumptions, a realization–based tax system in which the lock–in effect is present has to be refused if the political aim is to "close the gap" and "reduce the burden of future generations" respectively.

EXTENSIONS

The analysis can be extended in various directions, some of which are discussed in this section.

Concerns of Optimal Taxation

From the viewpoint of optimal taxation, the analysis presented so far investigates the question whether taxation upon accrual or realization is preferable for a given tax rate τ. Under Assumption 3(b), this is, in light of Lemma 1, equivalent to the assumption of an exogenous revenue requirement. As discussed above, the answer depends upon the welfare criterion used by the planner.

Alternatively, one may ask the following question: What is the optimal tax rate τ_M given a certain method of taxation M? And more specifically: Do the optimal tax rates under accrual and realization taxation coincide or differ (systematically)?

The problem can be studied within a slight extension of the above framework,

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[15] In light of the large literature on how to avoid the lock–in effect of capital gains taxation upon realization surveyed in Warren (2004) or Sahm (2007), the above result is somewhat surprising. However, there are some authors, like Kovenock and Rothschild (1987), who doubt that there is a strong negative welfare effect arising from a realization tax. The second–best analysis presented here may be seen as a further justification for this point of view.

in which the planner uses the tax revenues to provide a public good at the end of each period. Although the revenue for a given tax rate τ is identical under both taxation methods, as shown in Lemma 1, one easily verifies the following result, which is illustrated in the example provided in Appendix B: In general, the optimal tax rates under accrual and realization taxation differ, but not systematically (e.g., always $\tau^*_{acc} \leq \tau^*_{real}$). This finding is due to the fact that the method of taxation alters the distribution of utilities in the economy as stated in Proposition 2. According to his welfare criterion, the planner may want to correct for this change by adjusting the amount of public goods provided and, hence, may set different tax rates. Consequently, from the viewpoint of optimal taxation, a change in the method of taxation usually requires an adjustment of the corresponding tax rate. This point should not be overlooked in the ongoing discussions about reforms of capital gains taxation.

Liquidity Shocks and Ex–ante Welfare

The analysis has shown that neither method of taxation Pareto dominates the other. However, Pareto superiority may be too stringent a criterion in this context. One might wonder whether the result still holds from an ex–ante perspective if there is some uncertainty about the consumers' trading strategies.

Suppose, for example, that an individual experiences a liquidity shock influencing his saving decision. Within the formal framework, such a liquidity shock can easily be modeled regarding the period–1 income of consumer i, W^i, as a random variable drawn from some distribution Φ^i.[16] This setting allows us to analyze the welfare implications of the taxation methods from the following ex–ante criterion: Method M is said to be welfare superior to method M' from an ex–ante perspective, i.e., before the realization of W^i has taken place, if it gives at least one agent a higher expected utility without reducing the expected utility of the other agent.

Having extended the model and relaxed the welfare criterion in this way, it is evident that the challenged result still holds, i.e., neither method of taxation welfare dominates the other from an ex–ante perspective. To see this, note that, by Proposition 2, consumer 1 (2) is better (worse) off under realization than accrual taxation for *any* admissible income pair (W^1, W^2). Hence, the same has to be true in expectations.

Uncertain Projects and Risk Taking

The relation between uncertainty, risk taking, and the taxation of capital gains is mostly studied under an accrual tax system and the results depend upon how tax revenues are spent in the corresponding model.[17] However, the main argument made in this literature, that taxation might reduce the risk of individual investments and, hence, eventually lead to excessive risk taking, carries forward to a realization system as well. Thus, incorporating uncertainty within a comparison of the two methods of taxation, the central question is whether there is a difference in how—if at all—the method of taxation increases individual risk taking, and if so, how this influences the corresponding equilibria. However, a rigorous analysis of this question would require a much richer model.[18]

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[16] Obviously, under both methods of taxation, the normal commodity assumption implies $\partial S^i / \partial W^i > 0$.

[17] See Sandmo (1985) for a survey of this topic and, for example, Konrad (1991) or Christiansen (1995) for some later neutrality results in general equilibrium models.

[18] Particularly, in order to study an endogenous decision on risk taking, such a model has to include several (real) investment opportunities with different risk characteristics (see the subsection on endogenous production below).

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The purpose of this subsection is to show that, within the simple framework of this paper, uncertainty does not generally change the results derived in Propositions 1 and 2. To see this, consider the following modifications of the model: There is uncertainty with respect to the period–2 payout of the asset; with probability π, the payout is P_2, and with probability $1 - \pi$, it is 0. In analogy to Assumption 2, assume a monotone (expected) price path, i.e.,

$$[17] \quad P_0 \leq P_1 \leq \pi P_2.$$

In the good state of the world, consumer 1's budget constraints still are given by equations [8] and [14] respectively. In the bad state of the world, these constraints become

$$[18] \quad c_1^1 = (1 - S^1)P_1 - \tau(P_1 - P_0)$$

$$c_2^1 = S^1 \tau P_1$$

under accrual taxation and

$$[19] \quad c_1^1 = (1 - S^1)[P_1 - \tau(P_1 - P_0)]$$

$$c_2^1 = S^1 \tau P_0$$

under realization taxation. As has been shown by the analysis so far, in the good state of the world, the budget constraint for consumer 1 is weaker under realization than accrual taxation. As one can see from a comparison of constraints [18] and [19], in the bad state of the world, the opposite is true; i.e., the realization tax system leaves more risk at the individual level of consumer 1 than the accrual system. Consequently, on the one hand, the chance of ending up in the good state of the world provides an incentive for consumer 1 to save more under realization than accrual taxation. But, on the other hand, in the presence of risk, saving more means tak-

ing more risk. Therefore, if consumer 1 is risk–averse, the chance of ending up in the bad state of the world might have the opposite effect and provide an incentive to save less under realization than accrual taxation. However, with

$$E(W^{acc}) = \pi \frac{P_1 - \tau(P_1 - P_0)}{P_1}[P_2 - \tau(P_2 - P_1)]$$
$$+ (1 - \pi)\tau[P_1 - \tau(P_1 - P_0)],$$

$$E(W_H^{real}) = \pi[P_2 - \tau(P_2 - P_0)] + (1 - \pi)\tau P_0,$$

the exact analog to relation [1] holds in expectations; i.e., under the assumption of inequality [17], $E(W_H^{real}) \geq E(W^{acc})$. Put differently, for any given level of period–1 consumption c_1^1, the expected level of period–2 consumption $E(c_2^1)$ is higher under realization than accrual taxation.

This finding may be interpreted as if the realization tax would come along with some *risk premium* as compared to the less–risky accrual tax. As a consequence, consumer 1 still has an incentive to save more under realization than accrual taxation if the risk premium is sufficient to compensate for the associated higher risk. Put differently, if risk aversion is not too pronounced, total demand for the asset and, hence, its price are higher under realization than accrual taxation even under uncertainty. Hence, for sufficiently low levels of risk aversion, Proposition 1 still holds.

Following similar strategies as in the welfare analysis above, one concludes that the same is true for Lemma 1[19] and Proposition 2. Moreover, note that a more–risk–averse consumer 1 can use the sell–and–repurchase strategy—thereby enforcing accrual–like taxation—to insure against the higher volatility of a realization tax. Hence, independently from the consumers' risk aversions, consumer 1 will never be worse off under realization

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[19] Note that without discounting, the expected present value of tax revenues is equal to $\tau(\pi P_2 - P_0)$ under both methods of taxation.

than accrual taxation even with uncertain projects.

Costs of Asset Ownership

The problems of liquidity and valuation coming along with accrual taxation may be reinterpreted as higher costs of asset ownership compared to a realization system. The simple model discussed in the previous section abstracts from differences in such costs. This seems reasonable from a theoretic point of view. As has been mentioned in the review of the literature, an accrual system can be imitated by retrospective taxation on a realization basis, thereby avoiding the problems of liquidity and valuation. Accordingly, a "fair" comparison between accrual and realization taxation should neglect differences in ownership costs.

However, assuming higher costs under an accrual than a realization tax reinforces the main results of the model. Suppose, reasonably enough, that ownership costs in each holding period increase in the asset share owned. Such higher costs will then, ceteris paribus, decrease asset demand and, thus, prices, thereby fostering the price differential between accrual and realization taxation as well as its incidence for individual welfare levels. Moreover, a difference in ownership costs obviously biases the welfare analysis towards the method that creates fewer ones.

Multiple Assets and Endogenous Production

So far, only the distortion of the liquidation decision and not of the portfolio choice, is incorporated in the model. To take this secondary lock–in effect into account, one may consider a situation with alternative saving opportunities. Remember that the advantage of tax defer-

ral that comes with a realization system is more pronounced the higher the accrued capital gains are. Consequently, in such an extended framework, one would expect that the realization requirement biases a net seller's (consumer–1–type) portfolio towards assets with higher accrued capital gains as in Auerbach (1991). However, besides influencing the composition of consumers' portfolios, the deferral advantage under realization taxation would—in the presence of accrued capital gains—still increase overall savings, i.e., the overall demand for assets. Hence, the qualitative results of Propositions 1 and 2 are likely to hold in the case of several saving opportunities, too. In line with this reasoning, Viard (2000) applies a model with multiple assets where both taxation methods coexist, finding that the current realization tax increases asset prices, thereby shifting the burden of taxation (partly) to the buyer of an asset.

One of the most challenging extensions and a further step towards reality certainly would be to switch from the framework of a pure exchange economy to a model with endogenous production, i.e., endogenous asset supply. As long as production is exogenously given, the lock–in effect is at most able to distort the decisions on liquidation and portfolio selection but not on real investment. Real investment distortions may, however, have a significant impact on asset prices and, in particular, welfare.

Incorporating endogenous investment decisions requires the consideration of multiple assets and periods, which considerably complicates the analysis.[20] Auerbach (1992) uses a simulation model featuring three periods, one risky and one safe asset with exogenously given periodical pre–tax returns, and homogenous households in order to compute the efficiency gains from accrual taxa-

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[20] For the framework presented here, this would require explicitly modeling the real investment decision in period 0, i.e., "endogenizing" the "price" P_0, as well as allowing for additional "new" investments in period 1.

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tion compared to a realization tax. In the course of his analysis, he finds that—for very similar reasons as in the model presented above—the overall asset demand is higher under realization than accrual taxation. In the model presented here, where asset supply is fixed, this higher asset demand increases the asset price, whereas in the Auerbach (1992) model it is absorbed by a perfectly elastic asset supply raising savings, i.e., investments. Combining these two observations, one may expect that in a setting with endogenous production *and* endogenous investment returns, i.e., asset prices, both effects would occur, though they would be less pronounced:[21] savings *and* asset prices should be higher under a realization system than an accrual tax. Such considerations admit the conjecture that the qualitative result of Proposition 1 holds even in the very general setting of a production economy.

Since Auerbach (1992) assumes homogenous agents, distributional aspects cannot be studied within his model. However, the additional distortion of the real investment decision that comes with realization taxation in a production economy leaves more room for efficiency gains from a switch to accrual taxation compared to an exchange economy. Although this observation suggests that, in a production economy with heterogeneous agents, it is probably harder to identify situations in which some individuals remain better off under realization taxation, such situations may still exist. The welfare analysis presented in this paper possibly helps narrow the search for respective interest groups down to the party of net sellers. However, further research is required to understand better the welfare implications of the realization requirement in a production economy.

CONCLUSIONS

This paper has investigated the effects of accrual- and realization–based taxation of capital gains in a simple general equilibrium model of an exchange economy with heterogeneous agents. It has been shown that in the presence of accrued capital gains, equilibrium asset prices are higher under a realization tax than under an accrual tax. However, though taxation upon realization is never Pareto efficient in the presence of accrued capital gains, the impact of the taxation method on welfare is ambiguous due to distributional effects.

Such ambiguity gives rise to clientele considerations that may be regarded as an important reason for the political reluctance to switch from realization to accrual taxation. Hence, for the political debate on the methods of capital gains taxation, it is essential to identify how the respective interest groups form up. The welfare analysis presented in this paper may be regarded as a first step towards a better understanding of these issues.

Acknowledgments

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[21] Viard (2000, p. 483) argues that the impact the realization requirement has on current investment 'depends upon the tax treatment of new production relative to secondary trading'. In particular, for the case of reinvested corporate earnings, which increase the value of the existing shares but shareholders are not allowed a basis deduction for, he expects lower current investment under realization than accrual taxation. Given this, the asset price differential between an accrual and a realization tax would be even more pronounced in a production economy.

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APPENDIX A: COMPARATIVE STATICS

The subsequent analysis makes intensive use of the fact that, due to the normality assumption, for an interior solution to consumer i's problem of utility maximization the following inequalities hold:

[20]
$$\frac{\partial^2 U^i}{\partial c_1^{i^2}} - \left(\frac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i}\right)\frac{\partial^2 U^i}{\partial c_2^i \partial c_1^i} < 0,$$

$$\frac{\partial^2 U^i}{\partial c_1^i \partial c_2^i} - \left(\frac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i}\right)\frac{\partial^2 U^i}{\partial c_2^{i^2}} > 0.$$

The Case of No Taxation

Applying the implicit function theorem on the first order conditions [5], yields

[21]
$$\frac{dS^i}{dP_1} = -\frac{\dfrac{P_2}{P_1^2} + \dfrac{\partial\left(\dfrac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i}\right)}{\partial P_1}}{\dfrac{\partial\left(\dfrac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i}\right)}{\partial S^i}}$$

for $i \in \{1, 2\}$. Using inequalities [20] and the budget constraints [3] and [4] respectively, one can determine the sign of the following terms:

[22]
$$\frac{\partial\left(\dfrac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i}\right)}{\partial S^i} = \frac{\dfrac{dc_1^i}{dS^i}\left[\dfrac{\partial^2 U^i}{\partial c_1^{i^2}} - \left(\dfrac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i}\right)\dfrac{\partial^2 U^i}{\partial c_2^i \partial c_1^i}\right] + \dfrac{dc_2^i}{dS^i}\left[\dfrac{\partial^2 U^i}{\partial c_1^i \partial c_2^i} - \left(\dfrac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i}\right)\dfrac{\partial^2 U^i}{\partial c_2^{i^2}}\right]}{\partial U^i / \partial c_2^i},$$

[23]
$$\frac{\partial\left(\dfrac{\partial U^1 / \partial c_1^1}{\partial U^1 / \partial c_2^1}\right)}{\partial P_1} = \frac{\dfrac{dc_1^1}{dP_1}\left[\dfrac{\partial^2 U^1}{\partial c_1^{1^2}} - \left(\dfrac{\partial U^1 / \partial c_1^1}{\partial U^1 / \partial c_2^1}\right)\dfrac{\partial^2 U^1}{\partial c_2^1 \partial c_1^1}\right] + \dfrac{dc_2^1}{dP_1}\left[\dfrac{\partial^2 U^1}{\partial c_1^1 \partial c_2^1} - \left(\dfrac{\partial U^1 / \partial c_1^1}{\partial U^1 / \partial c_2^1}\right)\dfrac{\partial^2 U^1}{\partial c_2^{1^2}}\right]}{\partial U^1 / \partial c_2^1},$$

[24]
$$\frac{\partial\left(\dfrac{\partial U^2 / \partial c_1^2}{\partial U^2 / \partial c_2^2}\right)}{\partial P_1} = \frac{\dfrac{dc_1^2}{dP_1}\left[\dfrac{\partial^2 U^2}{\partial c_1^{2^2}} - \left(\dfrac{\partial U^2 / \partial c_1^2}{\partial U^2 / \partial c_2^2}\right)\dfrac{\partial^2 U^2}{\partial c_2^2 \partial c_1^2}\right] + \dfrac{dc_2^2}{dP_1}\left[\dfrac{\partial^2 U^2}{\partial c_1^2 \partial c_2^2} - \left(\dfrac{\partial U^2 / \partial c_1^2}{\partial U^2 / \partial c_2^2}\right)\dfrac{\partial^2 U^2}{\partial c_2^{2^2}}\right]}{\partial U^2 / \partial c_2^2}.$$

While the terms in [22] and [24] are positive, the term in [23] is negative. Hence, the sign of the expressions in [21] are determined as stated in [6].

The Case of Accrual Taxation

Applying the implicit function theorem on the first order conditions [10],

[25]
$$\frac{dS^i}{dP_1} = -\frac{\dfrac{P_2(1 - \tau)}{P_1^2} + \dfrac{\partial\left(\dfrac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i}\right)}{\partial P_1}}{\dfrac{\partial\left(\dfrac{\partial U^i / \partial c_1^i}{\partial U^i / \partial c_2^i}\right)}{\partial S^i}},$$

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[26] $\quad \dfrac{dS^i}{d\tau} = -\dfrac{\dfrac{P_2 - P_1}{P_1} + \dfrac{\partial\left(\dfrac{\partial U^i/\partial c_1^i}{\partial U^i/\partial c_2^i}\right)}{\partial\tau}}{\dfrac{\partial\left(\dfrac{\partial U^i/\partial c_1^i}{\partial U^i/\partial c_2^i}\right)}{\partial S^i}},$

[27] $\quad \dfrac{dS^1}{dP_0} = -\dfrac{\dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial P_0}}{\dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial S^1}}$

hold for $i \in \{1, 2\}$. Using inequalities [20] and the budget constraints [8] and [9] respectively, one can determine the sign of the terms in [22], [23], and [24] as well as of following terms:

[28] $\quad \dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial\tau} = \dfrac{\dfrac{dc_1^1}{d\tau}\left[\dfrac{\partial^2 U^1}{\partial c_1^{1^2}} - \left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)\dfrac{\partial^2 U^1}{\partial c_2^1 \partial c_1^1}\right] + \dfrac{dc_2^1}{d\tau}\left[\dfrac{\partial^2 U^1}{\partial c_1^1 \partial c_2^1} - \left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)\dfrac{\partial^2 U^1}{\partial c_2^{1^2}}\right]}{\partial U^1/\partial c_2^1},$

[29] $\quad \dfrac{\partial\left(\dfrac{\partial U^2/\partial c_1^2}{\partial U^2/\partial c_2^2}\right)}{\partial\tau} = \dfrac{\dfrac{dc_1^2}{d\tau}\left[\dfrac{\partial^2 U^2}{\partial c_1^{2^2}} - \left(\dfrac{\partial U^2/\partial c_1^2}{\partial U^2/\partial c_2^2}\right)\dfrac{\partial^2 U^2}{\partial c_2^2 \partial c_1^2}\right] + \dfrac{dc_2^2}{d\tau}\left[\dfrac{\partial^2 U^2}{\partial c_1^2 \partial c_2^2} - \left(\dfrac{\partial U^2/\partial c_1^2}{\partial U^2/\partial c_2^2}\right)\dfrac{\partial^2 U^2}{\partial c_2^{2^2}}\right]}{\partial U^2/\partial c_2^2},$

[30] $\quad \dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial P_0} = \dfrac{\dfrac{dc_1^1}{dP_0}\left[\dfrac{\partial^2 U^1}{\partial c_1^{1^2}} - \left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)\dfrac{\partial^2 U^1}{\partial c_2^1 \partial c_1^1}\right] + \dfrac{dc_2^1}{dP_0}\left[\dfrac{\partial^2 U^2}{\partial c_1^2 \partial c_2^2} - \left(\dfrac{\partial U^2/\partial c_1^2}{\partial U^2/\partial c_2^2}\right)\dfrac{\partial^2 U^2}{\partial c_2^{2^2}}\right]}{\partial U^1/\partial c_2^1}.$

The terms in [22] and [24] are positive, the ones in [29] and [30] are negative, and the sign of the terms in [23] and [28] is indeterminate. Hence, the sign of the expressions in [25], [26], and [27] are determined as stated in [6], [11], and [12] respectively.

The Case of Realization Taxation

For $i = 2$ the respective equations [25] and [26] remain valid since the first order condition [10] for consumer 2 stays unchanged. Applying the implicit function theorem on the first order condition [15] for consumer 1 yields

[31] $\quad \dfrac{dS^1}{dP_1} = -\dfrac{(1-\tau)\dfrac{P_2 - \tau(P_2 - P_0)}{(P_1 - \tau(P_1 - P_0))^2} + \dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial P_1}}{\dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial S^1}},$

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[32] $\dfrac{dS^1}{d\tau} = -\dfrac{\dfrac{(P_2-P_0)[P_1-\tau(P_1-P_0)]-(P_1-P_0)[P_2-\tau(P_2-P_0)]}{[P_1-\tau(P_1-P_0)]^2} + \dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial\tau}}{\dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial S^1}},$

[33] $\dfrac{dS^1}{dP_0} = -\dfrac{\dfrac{\tau(1-\tau)(P_2-P_1)}{[(1-\tau)P_1+\tau P_0]^2} + \dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial P_0}}{\dfrac{\partial\left(\dfrac{\partial U^1/\partial c_1^1}{\partial U^1/\partial c_2^1}\right)}{\partial S^1}}.$

Using inequalities [20] and the budget constraints [14] and [9] respectively, one verifies that the terms in [22] and [24] are positive, the ones in [23] and [29] are negative, and the sign of the terms in [28] and [30] is indeterminate. Hence, the sign of the expressions in [31], [25], [32], [26], and [33] are determined as stated in [6], [11], and [16] respectively.

APPENDIX B: AN EXAMPLE WITH QUASI–LINEAR LOGARITHMIC PREFERENCES

The considerations concerning optimal taxation briefly discussed in the body of the paper are illustrated by an example using quasi–linear logarithmic preferences. To this end, consider a slight extension of the model presented above: A benevolent social planner spends the tax revenues of period i to provide a certain amount g_i of a public good at the end of period i, which enters the utility function of the consumers in a quasi–linear way:

$$U^i(c_1^i, c_2^i, g_1, g_2) = \ln(c_1^i) + \ln(c_2^i) + \theta(g_1+g_2).$$

The taste parameter θ expresses the valuation of the public goods. By Lemma 1, $g_1 + g_2 = \tau(P_2 - P_0)$ under both methods of taxation. For illustrative purposes, assume $W^1 = 0$ and $W^2 = W$.

Individual Utility Maximization

The consumers take the tax rate τ and, hence, the amount $g_1 + g_2$ as given and maximize their utilities $U^i(c_1^i, c_2^i, g_1, g_2)$ deciding on their savings S^i in period 1. The relevant budget constraints for consumer 1 are described by [8] under accrual taxation and by [14] under realization taxation, whereas for consumer 2 they look the same under both methods of taxation and are given by [9]. From the first order conditions for the consumers' utility maxima one can compute their optimal savings $S^i(P_1)$ for a given asset price in period 1. Under an accrual tax, consumer 1's asset demand equals $S^1_{acc} = \dfrac{1}{2}\dfrac{P_1-\tau(P_1-P_0)}{P_1}$; under a realization tax, it is given by $S^1_{real} = 1/2$.

Consumer 2's asset demand $S^2 = \dfrac{1}{2}\dfrac{W}{P_1}$ is the same under both methods of taxation. For Assumption 2 to hold, it is sufficient to assume that the parameters of the model fulfill $P_0 < W < P_2$.

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Making use of the market clearing condition [2], the equilibrium asset price is then computed to be, in fact, higher under realization than accrual taxation, as stated in Proposition 1:[22]

$$P^*_{1acc} = \frac{W + \tau P_0}{1 + \tau} \quad < \quad W = P^*_{1real} .$$

Substituting these prices into the corresponding asset demand functions and budget constraints, one derives the equilibrium values for savings and consumption. For consumer 1, this yields

$$S^{1^*}_{acc} = \frac{1}{2} \frac{W + \tau(2P_0 - W)}{W + \tau P_0}$$

$$\leq \frac{1}{2} \qquad\qquad = S^{1^*}_{real} ,$$

$$c^{1^*}_{1acc} = \frac{1}{2} W - \frac{\tau}{1+\tau}(W - P_0)$$

$$\leq \frac{1}{2} W - \frac{\tau}{2}(W - P_0) \qquad\qquad = c^{1^*}_{1real} ,$$

$$c^{1^*}_{2acc} = \frac{1}{2}(1-\tau)P_2 \frac{W + \tau P_0 - \tau(W - P_0)}{W + \tau P_0} + \frac{1}{2}\tau \frac{W + \tau(2P_0 - W)}{1+\tau} ,$$

$$\frac{1}{2}(1-\tau)P_2 + \frac{1}{2}\tau P_0 \qquad\qquad = c^{1^*}_{2real} ,$$

and for consumer 2,

$$S^{2^*}_{acc} = \frac{1}{2} \frac{W + \tau W}{W + \tau P_0}$$

$$\geq \frac{1}{2} \qquad = S^{2^*}_{real} ,$$

$$c^{2^*}_{1acc} = \frac{1}{2} W \qquad = c^{2^*}_{1real} ,$$

$$c^{2^*}_{2acc} = \frac{1}{2}(1-\tau)P_2 \frac{W + \tau W}{W + \tau P_0} + \frac{1}{2}\tau W$$

$$\geq \frac{1}{2}(1-\tau)P_2 + \frac{1}{2}\tau W \qquad = c^{2^*}_{2real} .$$

Using those results one observes that for the consumers' equilibrium utility levels

[34] $\quad U^{2^*}_{acc} \geq U^{2^*}_{real} \quad$ and $\quad U^{1^*}_{acc} \leq U^{1^*}_{real}$

hold as stated in Proposition 2. For consumer 2, this is obvious from his respective equilibrium consumption levels; for consumer 1, one can verify it by comparing the products $c^1_{1acc} \, c^1_{2acc}$ and $c^1_{1real} \, c^1_{2real}$ respectively; note that $U^i(c^i_1, c^i_2, g_1, g_2) = \ln(c^i_1 \, c^i_2) + \theta(g_1 + g_2)$.

Moreover, for $t \in \{1, 2\}$ and $M \in \{acc, real\}$ the consumers' equilibrium consumption levels satisfy $c^2_{tM} \geq c^1_{tM}$ in this example. Hence, in equilibrium the utility level of consumer 2 is at least as high as that of consumer 1 under either method of taxation:

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[35] $\quad U^{2^*}_M \geq U^{1^*}_M .$

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[22] Note that up to this point all results of the example hold as well assuming uncertainty (as modeled in the extension on Risk Considerations) and $P_0 < W < \pi P_2$.

Socially Optimal Method of Taxation

Of course, consumer i's equilibrium utility level does depend not only on the method of taxation $M \in \{acc, real\}$ but also on the tax rate $\tau \in]0,1[$. His indirect utility is henceforth denoted by $U_M^{i*}(\tau)$. Now consider the problem of a social planner who maximizes social welfare by a choice in his two–dimensional policy space, i.e., deciding on the method of taxation and the tax rate. For the sake of concreteness, assume a weighted utilitarian type of (indirect) social welfare function V with

[36] $\quad V(M,\tau) = \alpha U_M^{1*}(\tau) + (1 - \alpha)U_M^{2*}(\tau),$

where $0 \le \alpha \le 1$. The socially optimal policy can be found by computing the optimal tax rates under either method of taxation τ_M^* and choosing the method of taxation that yields the highest level of welfare resulting from taxation at the corresponding optimal rate: $M^* \in argmax_M(V(M, \tau_M^*))$.

However, corresponding to the discussion in the subsection on Concerns of Optimal Taxation in the body of the paper, first look at a situation in which the planner faces an exogenous revenue requirement, i.e., he has to provide a fixed amount $g_1 + g_2$ of public goods and, therefore, sticks to the tax rate τ with $g_1 + g_2 = \tau(P_2 - P_0)$. From inequalities [34] it is obvious that a cut–off value $0 < \alpha_0 < 1$ exists such that the planner chooses accrual taxation for all $0 \le \alpha \le \alpha_0$ and a realization tax for all $\alpha_0 < \alpha \le 1$. Put differently, whenever he puts enough weight on the utility of consumer 1, he chooses taxation upon realization.

Socially Optimal Tax Rate

Now consider the planner's problem to maximize social welfare by choosing the optimal tax rate for a given method of taxation. The purpose of this subsection is to illustrate the fact discussed in the body of the paper, that in general the optimal tax rates differ, although, for a given tax rate, revenue is the same under both methods (see Lemma 1).

To this end, suppose $\alpha = 0$, i.e., the planner only cares about the well–being of consumer 2; by inequality [35], here this is equivalent to the assumption of a Maxmax social welfare function.[23] Using the optimal consumption levels of consumer 2, one derives the first order condition for the optimal tax rate under taxation upon realization

$$\frac{\partial V_{real}}{\partial \tau} = \frac{\partial U_{real}^{2*}}{\partial \tau} = \frac{W - P_2}{(1 - \tau)P_2 + \tau W} + \theta(P_2 - P_0) = 0.$$

The second order condition for a maximum is fulfilled ($\partial^2 V_{real}/\partial \tau^2 < 0$) and the optimal realization tax rate is given by $\tau^* real = \dfrac{P_2}{P_2 - W} - \dfrac{1}{\theta(P_2 - P_0)}$. If one assumes the taste parameter to be

$\theta = \dfrac{2(P_2 - W)}{(P_2 - P_0)(P_2 + W)}$, then the optimal realization tax rate equals $\tau_{real}^* = 1/2$.

Now it is shown that the optimal accrual tax rate τ_{acc}^* must differ from $1/2$. Again, using the optimal consumption levels of consumer 2, the first order condition for the optimal tax rate under taxation upon accrual equals

[37] $\quad \dfrac{\partial V_{acc}}{\partial \tau} = \dfrac{\partial U_{acc}^{2*}}{\partial \tau} = \dfrac{W - P_2 \left[\dfrac{W + \tau W}{W + \tau P_0} + \tau \dfrac{W - P_0}{(W + \tau W)^2} \right]}{(1 - \tau)P_2 \dfrac{W + \tau W}{W + \tau P_0} + \tau W} + \dfrac{2(P_2 - W)}{P_2 + W} = 0.$

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[23] Analogously, $\alpha = 1$ is equivalent to the assumption of a Maxmin (Rawlsian) social welfare function.

However, substituting $\tau = 1/2$ into [37] and rearranging terms, the expression on the left–hand side proves to be negative and, therefore, $\tau^*_{acc} < 1/2 = \tau^*_{real}$, since $\partial^2 V_{acc}/\partial\tau^2 < 0$. Put differently, shifting from accrual to realization taxation, the planner tries to compensate consumer 2 for his reduction in private utility by a higher level of public good provision, which can be achieved by choosing a higher tax rate.[24]

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[24] Of course, this is no general result but hinges on the special structure of the chosen example.

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