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A Classical Macroeconometric Model for the United States

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A statistical definition of the natural unemployment rate hypothesis is advanced and tested. A particular illustrative structural macroeconomic model satisfying the definition is set forth and estimated. The model has "classical" policy implications, implying a number of neutrality propositions asserting the invariance of the conditional means of real variables with respect to the feedback rule for the money supply. The aim is to test how emphatically the data reject a model incorporating rather severe classical hypotheses.

This paper estimates a small, linear, classical macroeconometric model for the postwar United States. One reason for estimating the model is to produce a simple device capable of generating unconditional forecasts of key economic aggregates such as the unemployment rate, the price level, and the interest rate. But a more important reason is that as part of the estimation process the hypotheses underlying the model are subjected to empirical testing. Since these hypotheses are very "classical" and sharply at variance with Keynesian macroeconomics, it would be useful to know at what confidence levels the data reject them.

The present model is considerably more monetarist than is the St. Louis model.[1] Indeed, as interpreted and manipulated by its builders, the St. Louis model is incapable of rationalizing prominent monetarist positions. In particular, it implies that simple x percent growth rules for money can generally be improved upon by adopting rules with feedback from past endogenous variables to current money.[2] By way of contrast,

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[1] See Andersen and Carlson 1970.

[2] Cooper and Fischer (1974) have made this point.

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the present model is one in which an x percent growth rule for the money supply seems not to be dominated by any rule with feedback.[3]

The deterministic (nonrandom) classical model, the static analysis of which is enshrined in macroeconomics textbooks, has never been taken seriously, because its predictions seem so terribly at variance with the data. In particular, it is hard to explain the observed persistent movements in employment and unemployment with the textbook classical model. How meaningfully integrating random disturbances into the classical model would affect the analysis is a matter about which there is presently little agreement. On the one hand, in his American Economic Association presidential address, James Tobin (1972) seemed to assert that the presence of random disturbances in demand and supply schedules so alters the character of the general system that it sets up an exploitable trade-off between unemployment and inflation even in a system where all agents optimize. On the other hand, Robert Lucas (1972*b*) has analyzed a general equilibrium system in which agents cope optimally with the existence of uncertainty. While there exist "nonneutralities" in that system, there are no nonneutralities that the government can either exploit or offset by way of countercyclical policy.

This paper formulates, tests, and estimates a version of the classical model that has its origin in hypotheses that place severe restrictions on the random behavior of unemployment, output, and the interest rate. The model implies that those three "real" variables are econometrically exogenous with respect to variables measuring monetary and fiscal policies. As a consequence, government manipulations of monetary and fiscal policy variables have no predictable effects on unemployment, output, or the interest rate and hence are useless for pursuing countercyclical policy. Such implications are in the nature of neutrality results, albeit ones that require drawing some fairly fine econometric distinctions. The key elements of the model that provide the sources of the restrictions on the stochastic nature of output, unemployment, and interest are: (*a*) a drastic version of the natural unemployment rate hypothesis; (*b*) the expectations theory of the term structure of interest rates, and (*c*) the assumption that the public's expectations are "rational."

The chief novelty of this paper is its formulation of a drastic, statistical definition of the natural unemployment rate hypothesis. That definition is not dependent on any particular macroeconomic structural model, being compatible with a variety of structures one could imagine. The particular structural model presented in this paper is intended only as an illustrative example that satisfies this definition of the natural-rate hypothesis. This particular structure does, however, illustrate some of the

[3] Models with this property have previously been analyzed by Sargent (1973) and Sargent and Wallace (1975).

strong classical properties that will be possessed by models that satisfy the definition of the natural-rate hypothesis advanced here. A major aim of the paper is to indicate how this definition of the natural-rate hypothesis can be tested and to present some test results.

This paper is organized as follows. In section I a prototype of the model is described. However, no attempt is made here to rationalize in a deep way the equations comprising the model. Section I is designed to display the system briefly and to establish its classical nature. Section II then provides a definition of the natural-rate hypothesis that is the cornerstone of the model. Statistical tests of the hypothesis are described. Section II also describes how the rational-expectations theory of the term structure of interest rates is implemented in the model and how its central implications can be tested. Section III implements the econometric tests described in Section II. Finally, Section IV contains estimates of the complete model. The casual reader not interested in econometrics can read only Sections I and IV and find there estimates of the model and a description of how it works.

I. Overview of the Model

I begin by describing a simple prototype of the model. It differs from the model finally estimated in some minor ways but illustrates well the mechanics of the model.

The prototype consists of the following five equations:

$$Un_t = \gamma(p_t - E_{t-1}p_t) + \sum_{i=1}^{n_1} \lambda_i Un_{t-i} + u_{1t}, \tag{1.1}$$

where $\gamma < 0$ (a Phillips curve);

$$n_{ft} = \beta(p_t - E_{t-1}p_t) + dUn_t + \sum_{i=1}^{n_2} w_i n_{ft-i} + u_{2t}, \tag{1.2}$$

where $\beta > 0$, and $d < 0$ (a labor force participation equation);

$$y_t = \alpha_0 t + \alpha_1(n_{ft} - Un_t + pop_t) + u_{3t}, \tag{1.3}$$

where $\alpha_1 \simeq 1$ (a production function);

$$R_t = R_{t-1} + \xi(Z_t - E_{t-1}Z_t) + u_{4t} \tag{1.4}$$

(a martingale equation for the long-term interest rate); and

$$m_t - p_t = b_1 R_t + b_2 y_t + b_3(m_{t-1} - p_{t-1}) + u_{5t}, \tag{1.5}$$

where $b_1 < 0$ and $b_2, b_3 > 0$ (a portfolio-balance schedule). The variables are defined as: Un_t = unemployment rate; p_t = log of GNP deflator;

n_{ft} = log of labor force participation rate; y_t = log of real GNP; pop_t = log of population; R_t = long-term interest rate; m_t = log of money supply; Z_t = a vector of exogenous variables in the "IS" curve, including tax rates and government purchases; u_{jt} = mutually and serially independent random terms with zero means, so that $E_{t-1}u_{jt} = 0$, $j = 1, \ldots, 5$; and $E_{t-1}X_t$ = the mathematical expectation of X_t conditioned on information available at time $t - 1$. The variables Z_t, pop_t, and m_t are taken as exogenous.

Equation (1.1) is a Phillips curve that posits an inverse supply-side relationship between unemployment and the unexpected part of the current price level. The public's psychological expectation about the price level is supposed to be "rational," meaning that it equals $E_{t-1}p_t$. The equation embodies the natural unemployment rate hypothesis, since it asserts that unemployment does not depend on the anticipated part of the rate of inflation. Equation (1.1) is essentially Lucas's (1973) formulation of the Phillips curve.

Equation (1.2) is a labor force participation equation positing that the participation rate depends directly on the unexpected part of the price level and inversely on the unemployment rate (the "discouraged worker effect"). The presence of unemployment and the unexpected part of prices in equations describing labor force participation is not unusual (e.g., see Wachter [1972] and the work cited by him).

Upon noting that the log of employment approximately equals $(n_{ft} - Un_t + pop_t)$, equation (1.3) is seen to be a Cobb-Douglas production function that excludes capital. The regressions reported by Bodkin and Klein (1967) and Lucas (1970) suggest that little violence is done to the data by omitting capital from equation (1.3). That is, time-series regressions of the log of output against the logs of capital and employment typically display constant or increasing returns to employment and zero or slightly negative returns to capital. For my purposes, excluding capital from equation (1.3) permits the construction of a model in which there is no need to account for capital accumulation.

Equation (1.4) posits that the long-term interest rate is a martingale. Fiscal policy and other aggregate-demand variables influence the long rate in two ways. First, the unexpected components of Z_t influence the "innovation" in the long rate, that is, the part of the long rate that cannot be predicted from the past. Second, the foreseen or expected part of Z_t is already reflected in R_{t-1} and affects R_t in precisely the same way it affects R_{t-1}.

Equation (1.5) is a standard portfolio-balance schedule.

The model is five equations in the five endogenous variables Un_t, n_{ft}, Y_t, R_t, and p_t. The exogenous variables are Z_t, pop_t, and m_t.

To complete the model, the stochastic processes governing the exogenous variables m_t, Z_t, and pop_t must be specified. I will assume the autoregressive schemes

$$m_t = \sum_{i=1}^{n_3} \xi_i m_{t-i} + \varepsilon_{1t}, \tag{1.6a}$$

$$Z_t = \sum_{i=1}^{n_4} \psi_i Z_{t-i} + \varepsilon_{2t}, \tag{1.6b}$$

and

$$pop_t = \sum_{i=1}^{n_5} \omega_i pop_{t-i} + \varepsilon_{3t}, \tag{1.6c}$$

where the ξ_i's, ψ_i's, and ω_i's are parameters, and the ε_t's are serially independent random variables with means of zero; they are assumed to be distributed independently of the u's in the structural equations (1.1)–(1.5). To solve the model and to forecast with it, expected values of the exogenous variables, for example, $E_{t-1}m_t$ and $E_t m_{t+1}$, are required. These expected values are calculated using the autoregressions above for the exogenous variables. This is partly by way of imposing rationality, since the expected price $E_{t-1}p_t$ turns out to depend on $E_{t-1}m_t$, $E_{t-1}Z_t$, and $E_{t-1}pop_t$. Rationality amounts to requiring that the public's expectations of the exogenous variables m_t, Z_t, and pop_t equal the mathematical expectations computed from the appropriate objective probability distributions, that is, the autoregressions above.

The model has a standard aggregate-demand and -supply representation in the p, y plane. Substituting equations (1.1) and (1.2) into (1.3) gives the aggregate-supply schedule

$$\begin{aligned} y_t = {} & \alpha_0 t + \alpha_1 pop_t + [\alpha_1 \beta + (\alpha_1 d - \alpha_1)\gamma](p_t - E_{t-1}p_t) \\ & + (\alpha_1 d - \alpha_1) \sum_{i=1}^{n_1} \lambda_i Un_{t-i} + \alpha_1 \sum_{i=1}^{n_2} w_i n_{f\,t-i} \\ & + (\alpha_1 d - \alpha_1) u_{1t} + \alpha_1 u_{2t} + u_{3t}. \end{aligned}$$

Since $\alpha_1 \beta_1 + (\alpha_1 d - \alpha_1)\gamma > 0$, the aggregate-supply schedule is upward sloping in the p–y plane.

Substituting equation (1.4) into (1.5) gives the aggregate-demand schedule $p_t = m_t - b_1 R_{t-1} - b_1 \xi (Z_t - E_{t-1}Z_t) - b_2 y_t - b_3(m_{t-1} - p_{t-1}) - b_1 u_{4t} - u_{5t}$, which slopes downward in the p, y plane. Increases in m_t and in the aggregate-demand innovations $\xi(Z_t - E_{t-1}Z_t)$ cause the demand schedule to shift outward. The equilibrium p, y combination is determined at the intersection of the demand and supply curves.

While the model is clearly simultaneous in determining the current values of the five endogenous variables, for generating forecasts it is recursive. The one-period-ahead forecast of Un_t is determined by taking expectations in equation (1.1) conditional on data known at $t - 1$:

$$E_{t-1}Un_t = \sum_{i=1}^{n_1} \lambda_i Un_{t-i}, \tag{1.1'}$$

which follows since $E_{t-1}(p_t - E_{t-1}p_t) = E_{t-1}p_t - E_{t-1}p_t = 0$. The forecast of n_{ft} is then given from equation (1.2) as

$$E_{t-1}n_{ft} = dE_{t-1}Un_t + \sum_{i=1}^{n_2} w_i n_{ft-i}.$$

Then from equation (1.3) we have the forecast of the log of GNP as $E_{t-1}y_t = \alpha_0 t + \alpha_1(E_{t-1}n_{ft} - E_{t-1}Un_t + E_{t-1}pop_t)$. From equation (1.4) the forecast of the long-term interest rate is simply $E_{t-1}R_t = R_{t-1}$, which follows since $E_{t-1}(Z_t - E_{t-1}Z_t) = 0$. Finally, from the portfolio-balance schedule, the forecast of p_t is $E_{t-1}p_t = E_{t-1}m_t - b_1E_{t-1}R_t - b_2E_{t-1}y_t - b_3(m_{t-1} - p_{t-1})$. To compute the forecasts of the endogenous variables, the forecasts $E_{t-1}m_t$ and $E_{t-1}pop_t$ of the exogenous variables are required.

The predictions of the model are obviously classical in spirit. The predictions of the "real" variables are all independent of the prediction of the money supply, which only influences the predicted price level. For predicting the long-term interest rate, predictions of the fiscal and other aggregate-demand variables add no information to that in the current long rate since they are already properly embedded in the current long-term rate. Finally, the model implies that the monetary authority does not have the option of pegging the nominal interest rate R_t via some feedback rule by letting the money supply be whatever it must to guarantee portfolio balance at that interest rate.[4] For suppose that the authority were to attempt to peg the interest rate via the feedback rule

$$R_t = F\theta_{t-1}, \tag{1.7}$$

where θ_{t-1} is a vector of observations on endogenous and exogenous variables dated $t - 1$ and earlier, and F is a vector of parameters conformable with θ_{t-1}. The predictions of R_t from equations (1.4) and (1.7) are clearly in general inconsistent, so that the interest rate is overdetermined. Thus, this model is characterized by Wicksell's classical overdeterminacy of the interest rate (and indeterminacy of the price level) under a pegged interest rate.

It bears emphasizing that, while for prediction the model has a very classical recursive structure, it is a simultaneous model when it comes to determining current variables. Thus, money is not a "veil" in the model,

[4] This is one of the options analyzed for a stochastic Keynesian model by William Poole (1970).

since (random) increases in money can be shown to stimulate both GNP and the price level. So will (random) increases in the aggregate-demand Z's. But it turns out that in this model it is best to predict as if money were a veil. The fact that variables are determined jointly simply cannot be exploited in prediction; neither can it be exploited for control.

I have indicated that to generate forecasts of the endogenous variables the exogenous variables should be set equal to the forecasts $E_t m_{t+1}$, $E_t Z_{t+1}$, and $E_t pop_{t+1}$, which are to be computed from the autoregressions (eqq. [1.6a]–[1.6c]) that actually govern those exogenous variables. It seems that something more is possible in the way of forecasting, but it turns out not to be useful to the policymaker. In particular it is possible to use the model to "predict" values of the endogenous variables in $t + 1$, conditional on alternative assumed values for the exogenous variables m_{t+1}, Z_{t+1}, and pop_{t+1}, *given* values of $E_t m_{t+1}$, $E_t Z_{t+1}$, and $E_t pop_{t+1}$. For example, for a given $E_t m_{t+1}$, different values of m_{t+1} will be associated with different values of the *real* variables output and unemployment. The larger $m_{t+1} - E_t m_{t+1}$ is, the larger will be the "predicted" value of output and the lower the "predicted" value of unemployment. But such "conditional" forecasts are of no use in forming policy. For example, it will not work to use the model to "forecast" unemployment for alternative values of m_{t+1}, given $E_t m_{t+1}$, and then to set m_{t+1} in order to achieve the unemployment rate desired by the monetary authority. Expecting that to work amounts to assuming that the public would continue to form its expectations about m_{t+1} by using equation (1.6a) even if the authority adopted the new and different rule for setting m implicit in the procedure above. That violates the assumption that expectations are rational. What affects unemployment and output is the gap between m_{t+1} and $E_t m_{t+1}$, and there is no way that the authority can expect to set this gap at some desired nonzero level.

This completes the overview of the model. I now turn to the task of setting forth more precisely the nature of the key hypotheses underlying the model. In the process, statistical tests of those hypotheses will be described and implemented.

II. The Stochastic Model of Unemployment and Interest Rates

This section sets forth and describes tests of a naive but powerful formulation of the hypothesis that there is a natural rate of unemployment. The hypothesis formulated here is much stricter than the usual statement of the natural-rate hypothesis, which posits that the government can persistently depress the unemployment rate below the "natural rate" only at the cost of accepting an accelerating inflation. In contrast, the present formulation implies that there is *no* way that the government can operate so that it can expect to depress the unemployment rate below the natural rate, even in the short run. Among other things, that implies that policy-makers face no "cruel choice" between inflation and unemployment over *any* relevant time frame.

The tests of the natural-rate hypothesis implemented here differ substantially from the usual one, which involves testing the hypothesis that certain sums of distributed-lag weights are unity or zero. This usual test has been harshly criticized on theoretical grounds[5] and furthermore is subject to the purely econometric objection that economic time-series data usually yield very little information about "long-run" magnitudes such as the sum of distributed-lag weights.[6] The tests implemented here do not seem to depend on estimating any such long-run properties of lag distributions.

The present statement of the natural-rate hypothesis is compatible with, but somewhat stronger than, the one presented and tested by Lucas. The strategy that I use to test the hypothesis is more naive and purely "statistical" than was Lucas's (1973) procedure, which involved actually estimating a concrete structural model.

The Natural-Rate Hypothesis

I begin with the univariate Wold representation of the unemployment rate, Un_t. Wold showed that if a variable, for example, Un_t, is an indeterministic, covariance-stationary process, it can be represented as a one-sided moving average of "white noise":

$$Un_t = \sum_{j=0}^{\infty} a_j u_{t-j}, \qquad \sum_{j=0}^{\infty} a_j^2 < \infty, \tag{2.1}$$

where the u's are serially uncorrelated with mean zero and finite variance σ^2. The model in equation (2.1) is obviously intended to apply to deviations of unemployment from its mean and any deterministic components. To make things simpler without really altering the essentials, I shall assume that the u's and the other white noises to be introduced below are serially independent.[7] I also assume that the roots of

$$\sum_{j=0}^{\infty} a_j \lambda^j = 0$$

[5] See Sargent 1971 and Lucas 1972*a*.

[6] See Sims 1972*a*. A lag distribution that embodies a wrong prior restraint on the sum of the lag weights but is sufficiently flexible can usually achieve a fit arbitrarily close to what could be achieved if the erroneous constraint on the sum of the lag weights were removed. (This assumes that the spectral density of the independent variable has no spike at zero frequency.)

[7] Dropping the assumption that the u's and other white noises are serially independent but only serially uncorrelated would necessitate replacing conditional mathematical expectations with linear least-squares forecasts in the subsequent argument. With that replacement the argument would go through. The statistical tests reported in the next section only utilize the assumption that the various white noises are serially uncorrelated.

lie outside the unit circle, so that Un_t possesses the autoregressive representation

$$Un_t = \sum_{j=1}^{\infty} g_i Un_{t-j} + u_t. \tag{2.2}$$

Even with these restrictions, equation (2.1) is a very general representation of a covariance-stationary, indeterministic process, the a_j's being chosen to enable the covariogram of $\sum a_j u_{t-j}$ to match that of Un_j. So far, then, I have not restricted the process for the unemployment rate very much.

Let the vector θ_t be the set of observations on all variables observed as of time t or earlier; θ_t includes observations on current and past GNP, interest rates, prices, and any other things, including unemployment itself, thought potentially to contribute to predicting unemployment. The following statement of the natural-rate hypothesis can now be advanced: the unemployment rate Un_t is said to obey the natural-rate hypothesis if in its (univariate) Wold representation (eq. [2.1]), the innovation u_t obeys

$$E(u_t | \theta_{t-1}) = 0, \tag{2.3}$$

so that the innovation in the unemployment rate is statistically independent of each component of θ_{t-1} and so cannot be predicted on the basis of the information in θ_{t-1}. This means that taking into account components of θ_{t-1} other than lagged Un_t's does not, on the least-squares criterion, improve the forecast of Un_t that can be made on the basis of lagged Un's alone. The least-squares forecast of Un_t on the basis of Un_{t-1}, Un_{t-2}, . . . , call it $\hat{U}n_t$, is given by

$$\hat{U}n_t = \sum_{i=1}^{\infty} g_i Un_{t-i} = \sum_{j=1}^{\infty} a_j u_{t-j}.$$

On our assumption that the u's are serially independent, $\hat{U}n_t = E(Un_t | Un_{t-1}, Un_{t-2}, \ldots) = E(Un_t | u_{t-1}, u_{t-2} \ldots)$.

The statement that the best forecast of u_t conditional on all past data is simply its unconditional mean of zero amounts to a very strict version of the natural-rate hypothesis. For θ_{t-1} includes past values of monetary and fiscal policy variables. Such variables are asserted to offer no aid in predicting the unemployment rate, once lagged unemployment rates are taken into account. Furthermore, equation (2.3) implies that the current value of any control variable determined via a deterministic feedback rule on θ_{t-1} is also of no use in predicting the unemployment rate. For example, suppose that the logarithm of the money supply at t, m_t, is determined according to the deterministic, very general feedback rule

$$m_t = f(\theta_{t-1}), \tag{2.4}$$

where f is some (perhaps very complicated) function that determines the monetary authority's feedback rule. Then the above version of the natural-rate hypothesis implies that once lagged Un's are taken into account, current m_t is of no use in predicting Un_t, so that $E(Un_t \mid m_t, Un_{t-1}, Un_{t-2}, \ldots) = E(Un_t \mid Un_{t-1}, Un_{t-2}, \ldots)$. This holds regardless of the nature of the function f or the particular parameter values characterizing f. Now feedback rules of the form of equation (2.4) form the class of rules for government-policy variables that control theory indicates to be optimal ones for macroeconometric models (fixed-coefficient stochastic-difference equations). The statement above of the natural-rate hypothesis implies that the choice of f has no effect on the mean of the unemployment rate, conditional on past data. This is a very strong implication about the conditional mean of the unemployment rate, one that denies, for example, that policymakers have any scope to trade a lower expected unemployment rate for a higher expected rate of inflation.[8] By way of contrast, the existing macroeconometric models, as usually manipulated, all imply that the parameters of f and the feedback rules for other government-policy variables *do* help determine the conditional mean of the unemployment rate, and that policymakers must face up to a hard choice between the unemployment rate and the inflation rate they can expect to achieve.

It is important to note that the definition above of the natural-rate hypothesis does not rule out the possibility that there are correlations between the unemployment rate and other variables such as prices or wages or the money supply. It does imply, however, that any such correlations that exist cannot be exploited in predicting the unemployment rate. To take an example, Lucas's model of the Phillips curve is

$$Un_t = \sum_{i=1}^{\infty} g_i Un_{t-1} + g_0(X_t - EX_t|\theta_{t-1}) + u_t',$$

where X_t is the price level at time t, $EX_t|\theta_{t-1}$ is the mathematical expectation of the price level at t, conditional on information available at time $t-1$, and u_t' is a well-behaved disturbance term, one that satisfies $Eu_t'|\theta_{t-1} = 0$.[9] The equation above posits a correlation between the innovations of Un_t and X_t; but notice that

$$E(Un_t|\theta_{t-1}) = \sum_{i=1}^{\infty} g_i Un_{t-1},$$

[8] It does not necessarily follow that the distribution of the innovation in unemployment is independent of the feedback rule for policy—only that its conditional mean is. The empirical tests reported in this paper are of neutrality-in-conditional-means propositions. Stronger neutrality propositions, asserting invariance of the entire probability distribution of some real economic variables with respect to the policy rule, obtain in some macroeconomic models (see, e.g., Sargent and Wallace 1975).

[9] The notations $E_{t-1}X_t$ and $EX_t \mid \theta_{t-1}$ are alternatives denoting the same concept, so that $E_{t-1} X_t \equiv EX_t \mid \theta_{t-1}$.

so that such a correlation does not help in predicting the unemployment rate. Obviously, the same sort of result would obtain were X_t interpreted as a vector of exogenous and endogenous variables.

As another example of correlation between unemployment and another variable that does not aid in forecasting unemployment, consider the system

$$Un_t = \sum_{i=1}^{m} g_i Un_{t-i} + u_t$$

$$X_t = \sum_{i=1}^{n} \lambda_i X_{t-i} + \sum_{i=1}^{q} \gamma_i Un_{t-i} + \varepsilon_t,$$

where the g's, λ's, and γ_i's are parameters, and u_t and ε_t are mutually uncorrelated and serially independent random variables with finite variances. In this system, unemployment helps predict X, even taking lagged X's into account; but once lagged Un's are taken into account, lagged values of X are of no aid in predicting unemployment.

Testing the Hypothesis

Granger (1969) and Sims (1972*b*) have described the statistical theory that can be used to construct tests of the natural-rate hypothesis as formulated above. According to Granger, ". . . We say that Y_t is *causing* X_t if we are better able to predict X_t using all available [past] information than if the information apart from [past] Y_t had been used" (p. 428). The formulation above of the natural-rate hypothesis thus posits that the unemployment rate is *caused*, in Granger's sense, by *no* other variables. From Granger's paper, a direct statistical test of that hypothesis is available. Consider the unemployment rate Un_t and some other variable Y_t. Using the method of least squares, estimate the linear regression of Un_t on lagged Un's and lagged Y's as

$$\hat{Un}_t = \sum_{j=1}^{m} \hat{\alpha}_j Un_{t-j} + \sum_{j=1}^{n} \hat{\beta}_j Y_{t-j}, \tag{2.5}$$

where the $\hat{\alpha}_j$'s and $\hat{\beta}_j$'s are least-squares estimates. On the null hypothesis that Y does *not* cause Un, the parent parameters $\beta_j, j = 1, \ldots, n$, equal zero. The natural-rate hypothesis can then be tested by testing the null hypothesis $\beta_j = 0$ (when $j = 1, \ldots, n$), for various choices of Y. Alternatively, lagged values of several variables can be added to the right side of equation (2.5). On the natural-rate hypothesis, all such variables bear zero coefficients.

An alternative way of testing the natural-rate hypothesis as posed here is to employ the test for Granger causality proposed by Sims (1972*b*). Assume that Un and some other series Y are jointly covariance stationary and that they are purely indeterministic. Then the generalization of Wold's representation theorem to n dimensions implies that Un_t and Y_t have the moving-average representation

$$Un_t = \sum_{i=0}^{\infty} a_i \varepsilon_{t-i} + \sum_{i=0}^{\infty} b_i \eta_{t-i} \tag{2.6a}$$

$$Y_t = \sum_{i=0}^{\infty} c_i \varepsilon_{t-i} + \sum_{i=0}^{\infty} d_i \eta_{t-i}, \tag{2.6b}$$

where ε and η are serially uncorrelated and mutually uncorrelated with finite variances; equations (2.6a) and (2.6b) are very general representations of the two processes Un_t and Y_t, the a's, b's, c's, and d's being chosen to make the cross-covariogram between the moving sums on the left-hand sides of the two equations match that between Un and Y. Sims showed that Y does not cause Un in Granger's sense if and only if either all of the a_i's or all of the b_i's in those equations are zero.[10] On the basis of this result, Sims showed that Y_t could be expressed as a one-sided distributed lag of Un_t with a disturbance uncorrelated with past, future, and current Un's if and only if Y fails to "cause" Un. Sims's test for exogeneity of Un is to regress Y on past, present, and future Un's and then to test the null hypothesis that coefficients on future Un's are zero. That is, by least-squares estimate,

$$Y_t = \sum_{i=-n}^{n} \gamma_i Un_{t-i} + e_t,$$

where e_t is a residual. On the null hypothesis that Y does not cause Un, $\gamma_i = 0$ for $i < 0$.

The Interest Rate

The equation for the long-term interest rate is motivated by the rational-expectations version of the expectations theory of the term structure.[11] Let R_{nt} be the yield to maturity on an n-period bond at time t, where n is large in relation to unit increments in t. I approximate the rational-expectations theory of the term structure as asserting

$$R_{nt} = \frac{1}{n}(R_{1t} + E_t R_{1t+1} + \ldots + E_t R_{1t+n-1}), \tag{2.7}$$

[10] This is a very important result, since it establishes the coincidence between Granger causality and the econometrician's definition of statistical exogeneity. (It is assumed that the process $[Un_t/Y_t]$ possesses an autoregressive representation.)

[11] For expositions of the rational-expectations theory of the term structure and evidence that it performs acceptably well, see Shiller (1972) and Modigliani and Shiller (1973).

so that the n-period rate is an average of the current short rate R_{1t} and expected future short rates $E_t R_{1t+j}, j = 1, \ldots, n-1$. Expectations about future short rates are assumed to be rational. Subtracting R_{nt} from R_{nt+1} gives $R_{nt+1} - R_{nt} = \eta_{nt+1} + (1/n)(E_t R_{1t+n} - R_{1t})$, where $\eta_{nt+1} = (1/n)[(R_{1t+1} - E_t R_{1t+1}) + (E_{t+1}R_{1t+2} - E_t R_{1t+2}) + \ldots + (E_{t+1}R_{1t+n-1} - E_t R_{1t+n-1})]$. The term η_{nt+1} is of the nature of an "innovation" and as an implication of rationality obeys $E_t \eta_{nt+1} = 0$. Furthermore, for large n and well-behaved (i.e., flat enough) yield curves, $(1/n)(E_t R_{1t+n} - R_{1t}) \simeq 0$. Consequently, for large n, there obtains the approximation

$$E_t R_{nt+1} = R_{nt}, \tag{2.8}$$

which says that the n-period rate is a martingale process.

Suppose that the reduced form for the short-term interest rate is $R_{1t} = \beta Z_t$, where β is conformable to Z_t and where Z_t is a vector of exogenous variables including government expenditures, tax rates, the money supply, and other determinants of the real rate of interest and the expected rate of inflation. That gives us $E_t R_{1t+j} = \beta E_t Z_{t+j}$. Then equation (2.7) becomes $R_{nt} = (1/n)\beta(Z_t + E_t Z_{t+1} + \ldots + E_t Z_{t+n-1})$. So we have

$$\begin{aligned} R_{nt+1} - R_{nt} = (1/n)\beta[(Z_{t+1} - E_t Z_{t+1}) + (E_{t+1}Z_{t+2} - E_t Z_{t+2}) + \ldots \\ + (E_{t+1}Z_{t+n-1} - E_t Z_{t+n-1})] + (1/n)\beta(E_t Z_{t+n} - Z_t). \end{aligned} \tag{2.9}$$

Supposing that Z_t is a vector autoregressive process, it is easy to show that[12]

$$(E_{t+1}Z_{t+j} - E_t Z_{t+j}) = \Gamma_{j-1}(Z_{t+1} - E_t Z_{t+1}), \tag{2.10}$$

where Γ_{j-1} is a square matrix conformable with Z, one whose elements are functions of the parameters of the autoregression for Z. Substituting equation (2.10) into (2.9), we obtain $R_{nt+1} - R_{nt} = (1/n)\beta(I + \Gamma_1 + \Gamma_2 + \ldots + \Gamma_{n-2})(Z_{t+1} - E_t Z_{t+1}) + (1/n)\beta(E_t Z_{t+n} - Z_t)$. Upon imposing our flat-yield-curve approximation $(1/n)(E_t Z_{t+n} - Z_t) = 0$, the equation above becomes

$$R_{nt+1} - R_{nt} = \xi(Z_{t+1} - E_t Z_{t+1}), \tag{2.11}$$

where $\xi = (1/n)\beta(I + \Gamma_1 + \ldots + \Gamma_{n-2})$. This is a version of equation (1.5). As before, we have the implication of equation (2.8), $E_t R_{nt+1} = R_{nt}$.

According to equation (2.8), a regression of $R_{nt+1} - R_{nt}$ against any

[12] This is an implication of Wold's chain rule of forecasting (see, e.g., Shiller 1972 and Modigliani and Shiller 1973).

variables dated t or earlier ought to have coefficients of zero. For example, a regression of $R_{nt+1} - R_{nt}$ against prices or rates of inflation dated t or earlier ought to have zero regression coefficients. The reason is that R_{nt} already has built into it expectations of inflation over almost all of the horizon for R_{nt+1} and that any revisions in those expectations between t and $t + 1$ cannot be predicted on the basis of information available at time t, by virtue of the rationality of those expectations.

Another way to test equation (2.7) is to note that it implies that R_{nt} is not caused by any variable. That can be tested by fitting two-sided distributed lags of causal candidates against R_{nt} and testing the null hypothesis that the coefficients on future R_n's are zero.

For my purposes, the important implication of the theory is that R_n cannot be predicted better by taking into account other variables, once lagged values of R_n have been taken into account. So it would be perfectly acceptable to modify equation (2.8) to read

$$E_t R_{nt+1} = \sum_{i=0}^{n} w_i R_{nt-i}, \tag{2.8'}$$

which carries the crucial implication that R_{nt} is caused by no other variables. Equation (2.8′) should perhaps be preferred over equation (2.8) according to certain theories about the liquidity premiums that allegedly infest the term structure.[13]

The assertion that other variables such as monetary aggregates and fiscal-policy variables contain no information (over and above that contained in lagged values of the long rate) that can be used to predict the long rate is one that contradicts the implications of all existing macroeconometric models, as they are usually manipulated.[14] Stochastic simulations of these models will in general generate data for which a variety of monetary, fiscal, and other variables "cause" the long rate and thereby aid in its prediction.

[13] A more adequate approximation than eq. (2.8) is available, one that does not ignore the term $1/n(E_t R_{1t+n} - R_{1t})$. Notice that the term-structure eq. (2.7) implies that

$$E_t n R_{nt+1} = (n + 1)R_{n+1,t} - R_{1t}. \tag{2.8''}$$

Equation (2.8″) implies that R_n is caused by (i.e., not exogenous with respect to) R_{n+1} and R_1 but is *not* caused by (i.e., is exogenous with respect to) any other variables once R_{n+1} and R_1 are taken into account. Equation (2.8″) shares the classical character of the less adequate approximation equation (2.8). Essentially, (2.8″) asserts that as a block the term structure of interest rates is statistically exogenous or not caused by other variables. This is enough to preserve the classical nature of the model but is weaker than requiring the interest rate on bonds of a given maturity to be statistically exogenous with respect to all other variables.

[14] The St. Louis model is no exception.

Observations on the Tests

The restrictions imposed by the statistical models for unemployment and the interest rate outlined here are stricter than what is really necessary to deliver the classical policy implications of the model. Thus, suppose that X_t is a vector of "real" economic aggregates at time t including variables such as real GNP, unemployment, layoffs, interest rates, and so on; X_t excludes variables measuring the composition of output, such as aggregate consumption and investment and outputs of particular commodities. Let g_t be a list of monetary and fiscal-policy variables at time t. Then a model in general will have classical policy implications if it satisfies

$$E(X_t|X_{t-1}, X_{t-2}, \ldots ; g_{t-1}, g_{t-2}, \ldots) = E(X_t|X_{t-1}, X_{t-2}, \ldots), \quad (2.12)$$

so that as a block the aggregate real variables X are statistically exogenous with respect to (not caused by, in Granger's sense) the variables in g. For a system satisfying equation (2.12), movements in the components of g do not have predictable effects on subsequent values of the real variables in X. So equation (2.12) exhibits the same sort of neutrality of certain real variables with respect to monetary and fiscal policy as does the model in Section I.

While the model of Section I is an example of a system satisfying equation (2.12), (2.12) is more general. There are systems satisfying equation (2.12) that violate the hypothesis for the unemployment rate and the interest rate described here in Section II which are key hypotheses underlying the model of Section I. Thus, equation (2.12) does *not* imply $E(Un_t|Un_{t-1}, Un_{t-2}, \ldots ; g_{t-1}, g_{t-2}, \ldots) = E(Un_t|Un_{t-1}, Un_{t-2}, \ldots)$, even though Un_t is a component of X_t. A simple example that illustrates this is a system satisfying equation (2.12) in which, say, layoffs help cause unemployment. Suppose that some components of g_t are set via a feedback rule on layoffs. Then even though g does not cause (help predict) Un when lagged unemployment *and* lagged layoffs are taken into account, components of g will help predict unemployment when only lagged unemployment is taken into account. This is because g contains some information about lagged layoffs. This is a "spurious" type of causality from g to Un in which an omitted variable (layoffs) is causing both g and Un (see Granger 1969); when layoffs are omitted, g only appears to cause Un because it is standing in for the omitted lagged-layoff rates.

The possibility of such spurious apparent causality running from components of g to Un is noteworthy, since the statement above of the natural-rate hypothesis is so very strict. In particular, it rules out even the possibility that other real variables (the components of X in eq. [2.12]) cause unemployment. This seems too drastic, since it is easy to imagine structures in which there is extensive causality from, say, GNP and layoffs to unemployment that satisfy equation (2.12) and so are basically classical in nature. For such a system, our tests might well reject the very strict version of the natural-rate hypothesis adopted above.

While failure of monetary and fiscal-policy variables to cause unemployment and other real variables is sufficient to deliver classical policy implications, it is not really necessary. One can imagine structures in which policy variables cause (help predict) unemployment and other real variables, but in which switching from one deterministic rule for setting the policy variable to another leaves the stochastic behavior of unemployment unchanged. As an example, consider the structural system

$$Un_t = \sum_{i=1}^{n_1} \lambda_i Un_{t-i} + \beta_0(m_t - E_{t-1}m_t) + \beta_1(m_{t-1} - E_{t-2}m_{t-1}) + u_t \tag{2.13}$$

and

$$m_t = \sum_{i=1}^{n_2} \delta_i m_{t-i} + \varepsilon_t, \tag{2.14}$$

where ε_t and u_t are random variables, and $E_{t-1}\varepsilon_t = E_{t-1}u_t = 0$. For the structure above, it is easy to calculate

$$E(Un_t | Un_{t-1}, \ldots, m_{t-1}, m_{t-2}, \ldots) = \sum_{i=1}^{n_1} \lambda_i Un_{t-i} + \beta_1\left(m_{t-1} - \sum_{i=1}^{n_2} \delta_i m_{t-i-1}\right).$$

It follows that m helps predict (causes) Un_t. But notice that according to equation (2.13) switching from one deterministic rule for m (i.e., a rule for which $m_t = E_{t-1}m_t$) to any other deterministic rule will leave the stochastic behavior of unemployment unaltered. Even though m causes unemployment in this system, it is true that one deterministic rule is as good as any other, so that there is no scope for countercyclical policy by way of "leaning against the wind."

The preceding observations suggest reasons for believing that this paper tests versions of classical hypotheses that are really stronger than what is necessary to deliver classical policy conclusions, so that the tests seem biased against the natural-rate hypothesis and other classical hypotheses. However, it is important to note that the tests are not uniformly biased against classical hypotheses, since it is possible to concoct nonclassical systems that will mimic the classical characteristics that my tests look for. Thus, the tests might be fooled into failing to reject the natural-rate hypothesis in a system for which that hypothesis is false. Suppose that the true reduced form for Un_t is

$$Un_t = \sum_{i=1}^{2} \lambda_i Un_{t-1} + \alpha_0 m_t + \varepsilon_t, \tag{2.15}$$

where $E(\varepsilon_t | Un_{t-1}, \ldots, m_t, m_{t-1}) = 0$ and where the λ's and α's are fixed parameters. Suppose that the authority sets m_t according to the deterministic feedback rule

$$m_t = \sum_{i=1}^{2} \delta_i Un_{t-i}.$$

Then clearly

$$E(Un_t | Un_{t-1}, Un_{t-2}, \ldots ; m_t, m_{t-1}) = \sum_{i=1}^{2} (\lambda_i + \alpha_0 \delta_i)\, Un_{t-i},$$

Here Un is not caused by m, in Granger's sense, because the authority, by making m_t an exact function of past Un's, eliminates any value from the m series for predicting Un.

While the tests might be fooled by such a structure, that structure itself seems unlikely to me. In particular, if the reduced form were equation (2.15) and the authority were to set m_t by a feedback only on lagged Un's, and not also on other variables, presumably the authority would want to minimize the variance of Un_t, which it could accomplish by eliminating any serial correlation in Un_t. That is, in our example, it could minimize the variance in Un by setting $\lambda_1 + \alpha_0\delta_1 = 0$, $\lambda_2 + \alpha_0\delta_2 = 0$. Then the variance of unemployment would equal the variance of ε_t. But in reality, variables like unemployment and the deviation of GNP from trend are highly serially correlated. That makes it hard to believe that any failure of, say, m to cause Un is due to the authority's manipulating m in response to past movements in Un, since that requires imputing to the authority a perverse objective, that is, one tolerating much serial correlation and variance in Un.

III. Empirical Results

Tables 1–6 report the results of performing tests along the lines proposed by Granger and Sims for quarterly data on the dependent variables spanning the period 1952 II–1972 III. The unemployment rate for all civilian workers is used for Un, while Moody's Baa corporate bond index is taken for the long-term interest rate R. The variables used as candidates for the "causal" variables Y are the logarithm of the money supply, currency plus demand deposits, (m); the federal, state, and local government surplus on the national-income-accounts basis in 1958 dollars (*surp*); the logarithms of the GNP deflator (p); a straight-time wage index in manufacturing (w); federal, state, and local purchases of goods and services in 1958 dollars (g); and federal, state, and local purchases in current dollars ($g\$$).[15]

Each of the series has been seasonally adjusted by taking the Fourier transform of the series, setting its real and imaginary parts to zero in a band of width $\pi/12$ about the seasonal frequencies, and then taking the inverse Fourier transform to obtain a seasonally adjusted series.[16] This method has the virtue of applying a seasonal-adjustment filter with the same frequency-response function to each series, thereby avoiding the distortions in estimating distributed lags between variables that can be caused where the series have been adjusted asymmetrically (see Sims 1974 and Wallis 1974). Furthermore, the method reduces the spectral power of the series to zero at the seasonal frequencies, which Sims (1974) has argued helps eliminate bias in the form of seasonal patterns showing up in estimated distributed-lag coefficients.

Table 1 reports the results of implementing Granger's test for causality between *Un* and each of the Y candidates listed above. For each Y, the test is run in both directions: first *Un* is regressed on lagged *Un*'s and lagged Y's to permit testing the null hypothesis that Y does not cause *Un* (i.e., that the coefficients on lagged Y's are zero), then Y is regressed on lagged Y's and lagged *Un*'s to permit testing the null hypothesis that *Un* does not cause Y (i.e., that the coefficients on lagged *Un*'s are zero). Regressions in both directions include a constant and a linear trend. The regressions include four lagged values of the dependent variable and six lagged values of the other variable. The F-statistic pertinent for

[15] The wage is an index of the straight-time manufacturing wage (w), which is seasonally adjusted and reported on a monthly basis in *Employment and Earnings* (Department of Labor, Bureau of Labor Statistics). The civilian unemployment rate (*Un*) seasonally unadjusted, on a monthly basis, was taken from *Employment and Earnings*. For population I used the civilian noninstitutional population aged 16 and over, constructed by subtracting armed forces numbers from the total population aged 16 and over. The noninstitutional population aged 16 and over was interpolated from annual figures compiled by the Current Population Survey and reported in table 1, Bureau of Labor Statistics *Handbook of Labor Statistics*, 1973. Armed forces numbers were obtained by averaging monthly numbers reported in *Employment and Earnings*. The civilian labor force aged 16 years and older was taken from *Employment and Earnings* and divided by pop_t to obtain the labor force participation rate. The money supply (m) is $M1$, currency plus adjusted demand deposits taken from the *Federal Reserve Bulletin*. The Baa rate (R) was obtained from Moody's Investor's Service. For R, w, and *Un*, the monthly figures were averaged to obtain quarterly figures. The GNP deflator (p); federal, state, and local purchases in current dollars ($g\$$); and federal, state, and local purchases in 1958 dollars (g) were all taken from the National Income Accounts; the federal, state, and local government surplus in current dollars was also taken from the National Income Accounts and then divided by the GNP deflator to obtain the surplus in 1958 dollars (*surp*).

[16] A deterministic trend was extracted before taking the Fourier transform and then added back in after taking the inverse transform. The degrees of freedom for the F-statistics have been adjusted for the loss of degrees of freedom due to setting the seasonal bands to zero. The appropriate correction is described by Sims (1974).

TABLE 1

REGRESSION OF $Y(t) = \sum_{s=1}^{4} \alpha(s)Y(t-s) + \sum_{l=1}^{6} \beta(l)X(t-l) + \delta_1 t + \delta_0$

1952II–1972III

Y	X	$\alpha(1)$	$\alpha(2)$	$\alpha(3)$	$\alpha(4)$	$\beta(1)$	$\beta(2)$	$\beta(3)$	$\beta(4)$	$\beta(5)$	$\beta(6)$	δ_1	δ_0	$\bar{R}^2$	SE of Est. Adj.	D-W	F-Statistic on All β $F(6, 60)$
Un	*m*....	1.317	−0.541	−0.029	0.113	−14.244	7.787	7.578	−24.943	26.849	−0.862	−0.010	−9.526	.918	0.346	1.973	2.627*
		(10.25)	(−2.64)	(−0.14)	(0.94)	(−1.89)	(0.57)	(0.53)	(−1.75)	(1.85)	(−0.10)	(−1.32)	(−1.86)				
m	*Un* ..	1.279	−0.367	0.022	0.063	−0.002	0.003	0.0005	0.00002	−0.002	−0.0009	0.0002	0.022	.999	0.006	1.991	1.506
		(9.75)	(−1.60)	(0.09)	(0.44)	(−0.88)	(0.82)	(0.13)	(0.01)	(−0.56)	(−0.44)	(1.33)	(0.26)				
Un	*surp* ..	1.435	−0.540	−0.121	0.134	−0.020	0.035	−0.035	0.029	−0.002	−0.007	−0.0004	0.439	.906	0.372	1.995	0.954
		(10.84)	(−2.33)	(−0.52)	(1.02)	(−1.32)	(1.58)	(−1.56)	(1.33)	(−0.07)	(−0.57)	(−0.21)	(2.36)				
surp	*Un* ..	1.143	−0.399	0.149	−0.248	−0.319	0.268	−0.848	2.875	−3.330	1.979	−0.017	−3.200	.814	3.323	1.992	1.034
		(8.60)	(−2.02)	(0.75)	(−1.90)	(−0.26)	(0.13)	(−0.40)	(1.36)	(−1.73)	(1.77)	(−1.04)	(−1.84)				
Un	*g*	1.498	−0.641	−0.027	0.057	0.814	−1.085	−0.037	1.152	−1.336	0.143	0.002	2.036	.902	0.380	1.917	0.489
		(11.99)	(−2.87)	(−0.12)	(0.40)	(0.75)	(−0.82)	(−0.03)	(0.88)	(−1.14)	(0.20)	(0.28)	(0.44)				
g	*Un* ..	0.685	−0.042	0.069	−0.110	−0.016	−0.004	0.024	−0.065	0.034	0.004	0.003	1.840	.960	0.043	1.979	4.564**
		(5.20)	(−0.27)	(0.46)	(−0.96)	(−1.10)	(−0.15)	(0.88)	(−2.42)	(1.31)	(0.22)	(3.28)	(3.30)				
Un	*g$* ...	1.506	−0.614	−0.134	0.147	0.204	−4.415	7.494	0.211	−6.820	3.455	−0.002	−0.052	.908	0.368	2.056	1.145
		(12.28)	(−2.76)	(−0.57)	(1.09)	(0.07)	(−0.95)	(1.73)	(0.05)	(−1.61)	(1.56)	(−0.19)	(−0.02)				
g$	*Un* ..	1.135	−0.066	0.099	−0.292	−0.014	0.021	−0.006	−0.008	0.004	−0.003	0.002	0.530	.999	0.014	1.806	2.504*
		(9.84)	(−0.37)	(0.57)	(−2.99)	(−2.87)	(2.45)	(−0.63)	(−0.86)	(0.43)	−(0.55)	(3.99)	(3.84)				
Un	*p*	1.469	−0.610	−0.120	0.132	−15.845	22.334	−0.608	−12.832	14.059	−3.717	−0.018	−14.386	.906	0.372	2.063	0.936
		(11.55)	(−2.73)	(−0.54)	(1.04)	(−1.34)	(1.23)	(−0.03)	(−0.71)	(0.85)	(−0.34)	(−1.48)	(−1.43)				
p	*Un* ..	1.090	0.003	0.034	−0.120	0.0004	−0.002	0.002	−0.002	0.001	−0.0004	0.00002	−0.026	.999	0.004	2.019	1.222
		(8.42)	(0.02)	(0.17)	(−0.87)	(0.31)	(−0.94)	(0.78)	(−0.88)	(0.54)	(−0.31)	(0.20)	(−0.30)				
Un	*w*	1.238	−0.437	−0.204	0.122	−18.346	25.905	−20.746	20.274	1.146	1.050	−0.084	−2.469	.917	0.350	2.027	2.371*
		(8.02)	(−1.94)	(−0.92)	(0.95)	(−1.76)	(1.81)	(−1.44)	(1.46)	(0.08)	(0.10)	(−2.44)	(−2.36)				
w	*Un* ..	1.066	−0.347	0.431	−0.074	−0.003	0.00007	0.00001	0.004	−0.006	0.002	−0.0006	−0.010	.9996	0.005	1.969	1.638
		(7.05)	(−1.66)	(2.03)	(−0.43)	(−1.50)	(0.02)	(0.00)	(1.08)	(−1.88)	(1.15)	(−1.72)	(−0.79)				

NOTE.—t-statistics for coefficients appear in parentheses below relevant coefficients.
* Significant at 5%.
** Significant at 1%.

testing the null hypothesis that the dependent variable is not caused by the other variable is reported in the last column.

The F-statistic for m as the causal variable influencing Un is significant at the 95 percent confidence level, though not at the 99 percent level. Similarly, the F-statistic for w as a causal variable for Un is significant at the 95 percent confidence level. None of the other causal candidates obtains an F that would require rejecting the null hypothesis that they do not cause unemployment. In particular, notice that the GNP deflator does not appear to cause unemployment.

In the other direction, the F-statistics reveal that the hypothesis that Un does not cause g or $g\$$ can be rejected at the 95 percent confidence level. The hypothesis that Un does not cause the other four variables cannot be rejected.

Tables 2 and 3 report summary statistics for the regressions implementing Sims's test for unemployment.[17] Two-sided distributed lags were calculated in each direction, one with Un as the dependent variable and the causal candidate Y as the "independent" variable, the other with Un and Y reversed. The data were quasi-differenced by applying the filter $(1-.75L)^2$. Each regression included a constant and a trend, with four lead variables and 12 lagged variables. The regressions were first estimated by the method of least squares. Then the Fourier transform of the distributed-lag coefficients was calculated. The amplitude of the Fourier transform was inspected to see if peaks occurred at the seasonal frequencies. In those cases where a peak occurred, indicating a seasonal pattern in the coefficients, the regressions were recomputed using Theil's mixed estimator to incorporate weak, stochastic prior information stating that there is no seasonal pattern in the distributed lag. In particular, suppose the regression estimated is

$$Un_t = \sum_{i=-4}^{12} \hat{b}_i Y_{t-i} + \text{residual}_t,$$

and that a seasonal pattern characterizes the b_i's. The regression was then recalculated by adding observations on the three constraints

$$\begin{aligned} b_{-4} + b_0 + b_4 + b_8 &= b_{-3} + b_1 + b_5 + b_9 + U_1, \\ b_{-4} + b_0 + b_4 + b_8 &= b_{-2} + b_2 + b_6 + b_{10} + U_2, \\ b_{-4} + b_0 + b_4 + b_8 &= b_{-1} + b_3 + b_7 + b_{11} + U_3, \end{aligned}$$

where the U's are random variables obeying $EU_1 = EU_2 = EU_3 = 0$. Theil's mixed estimator requires estimates of the standard error of the

[17] To save space, the graphed lag distributions and various summary statistics of the two-sided tests have been relegated to a mimeographed appendix that is available from the author on request. The graphs and various statistics from the Hannan efficient regressions discussed below are also in this appendix.

TABLE 2

F-STATISTIC—TWO-SIDED TESTS

VARIABLE NAME (Y)	INDEPENDENT VARIABLE Un* (1)	Y† (2)
m	0.401‡	0.951§
$surp$	1.09‡	0.396§
g	0.344‖	2.854‖
$g\$$	0.374‡	1.255‡
p	0.647‖	0.479§
w	1.472‖	1.238§

NOTE.—All F's are $F(4, 50)$; significance levels are 2.56 for .95% confidence, 3.72 for .99% confidence:

Col. 1 regressions: $Y_t = \sum_{i=-4}^{12} w_i Un_{t-i}$; col. 2 regressions: $Un_t = \sum_{i=-4}^{12} w'_i Y_{t-i}$.

* F-statistic is pertinent for testing null hypothesis $w_{-4} = w_{-3} = w_{-2} = w_{-1} = 0$.
† F-statistic is pertinent for testing null hypothesis $w'_{-4} = w'_{-3} = w'_{-2} = w'_{-1} = 0$.
‡ Theil constraint used with $\sigma_u = \max w_i - \min w_i$.
§ No Theil constraint used.
‖ Theil constraint used with $\sigma_u = (\max w_i - \min w_i)/2$.

disturbances in the regression and the standard errors of U_1, U_2, and U_3. The former was taken as equal to the standard error of the residuals in the original least-squares regression. The latter standard errors were taken as equal to one another at σ_u, which was set at either $(\max_i \hat{b}_i - \min_i \hat{b}_i)$ or $(\max_i \hat{b}_i - \min_i \hat{b}_i)/2$, where the $\hat{b}_i$'s are from the original least-squares regression. The covariance of each U with all other random variables was assumed to be zero. Estimation incorporating this prior information in most cases sufficed to eliminate the seasonal in the distributed-lag coefficients.

Table 2 summarizes the F-statistics pertinent for testing the null hypothesis that the coefficients on future values of the right-side variable are zero, that is, the null hypothesis that the left-side variable does not cause the right-side variable. For no causal candidate Y does the F-statistic indicate rejecting that Y does not cause Un at the 95 percent confidence level. In particular, notice that in contrast to the results from applying the direct Granger test, it is not possible to reject the hypothesis that m or w does not cause Un. In the other direction, the F-statistic reveals that the hypothesis that g is not caused by Un must be rejected at the 95 percent confidence level. The next highest F is for $g\$$, though it is not significant at the 95 percent confidence level. Qualitatively, the overall pattern of the results is similar to that obtained by applying Granger's test, with the important exceptions of the different results rendered for whether m causes Un and for whether w causes Un.

Table 3 reports F-statistics pertinent for testing whether the coefficients on current and lagged right-hand side variables are zero in the one-sided regressions corresponding to those in table 2. Only the F-statistics

TABLE 3

F-STATISTICS FOR COEFFICIENTS ON CURRENT AND LAGGED VARIABLES:

$$\text{I. } Y_t = \sum_{\alpha=0}^{12} \alpha_i Un_{t-i} + \beta_0 + B_1 t, \text{ II. } Un_t = \sum_{\alpha=0}^{12} \gamma_i Y_{t-i} + \delta_0 + \delta_1 t$$

VARIABLE NAME (Y_t)	INDEPENDENT VARIABLE	
	Un	Y_t
m	0.883[a]	0.911[b]
surp	1.685[a]	1.999*[b]
g	1.454[c]	0.520[c]
g\$	1.785[a]	1.441[a]
p	0.571[c]	1.347[b]
w	1.003[c]	2.355*[b]

NOTE.—All *F*'s are $F(13, 54)$; all data are filtered: $(1 - .75L)^2$.
[a] Theil constraint used with σ_u = max w_i − min w_i.
[b] No Theil constraint used.
[c] Theil constraint used with σ_u = (max w_i = min w_i)/2.
* Significant at 5%.

for the regression of *Un* on *surp* and *Un* on *w* are significant at the 95 percent confidence level.

Tables 4, 5, and 6 report the results of applying Granger's and Sims's tests to determine whether the long-term interest rate, as measured by the Baa yield index, is statistically exogenous as implied by our theory. Table 4 records the results of applying the direct Granger test. The *F*-statistic is the one pertinent for testing that the coefficients on lagged values of the causal candidate *Y* are all zero, so that *Y* does not cause or help predict the dependent variable. Where *R*Baa is the dependent variable, *w* is the only causal candidate that obtains an *F*-statistic that is significant at the 95 percent confidence level. At that confidence level, the results are thus consistent with the implications of the theory, with the exception of the results for *w*, which indicate that *w* causes *R*Baa. In the reverse direction, the hypothesis that *R*Baa does not cause the money supply must be rejected at the 95 percent confidence level.

For Sims's test, table 5 summarizes the *F*-statistics pertinent for testing the null hypothesis of no causality for the interest rate. The results are compatible with those obtained from applying Granger's test. The hypothesis that *R*Baa is not caused by the causal candidate can be rejected at the 95 percent confidence level only for *w*. In the reverse direction, the hypothesis that *R*Baa fails to cause *m* must be rejected at the 95 percent confidence level.

Table 6 reports the *F*-statistics pertinent for testing the null hypothesis that coefficients on current and lagged values of the causal candidates are zero in the one-sided regressions corresponding to those in table 5. The *F*'s for *w* on *R*Baa and *R*Baa on *w* are the only ones significant at the 95 percent confidence level, though a couple of others are marginal

TABLE 4

REGRESSION OF $Y(t) = \sum_{s=1}^{4} \alpha(s)Y(t-s) + \sum_{l=1}^{6} \beta(l)X(t-l) + \delta_1 t + \delta_0$

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Y	X	α(1)	α(2)	α(3)	α(4)	β(1)	β(2)	β(3)	β(4)	β(5)	β(6)	δ_1	δ_0	$\bar{R}^2$	SE of Est. Adj.	D-W	F-Statistic on All β F(6, 60)
RBaa	m	1.447	−0.734	0.553	−0.378	0.635	3.311	−2.633	−4.584	5.248	−1.016	−0.0004	−4.251	.992	0.161	1.867	0.805
		(11.54)	(−3.26)	(2.41)	(−2.87)	(0.17)	(0.50)	(−0.39)	(−0.68)	(0.79)	(−0.27)	(−0.11)	(−1.29)				
m	RBaa....	1.337	−0.187	−0.317	0.194	−0.012	0.005	0.017	−0.011	−0.002	0.001	0.00001	−0.129	.999	0.006	1.925	2.730*
		(10.39)	(−0.80)	(−1.27)	(1.38)	(−2.55)	(0.62)	(1.89)	(−1.19)	(−0.21)	(0.21)	(0.14)	(−1.30)				
RBaa	surp	1.471	−0.715	0.491	−0.290	0.009	−0.011	0.010	−0.007	0.002	0.001	0.003	0.141	.992	0.163	1.867	0.593
		(11.45)	(−3.16)	(2.14)	(−2.12)	(1.48)	(−1.15)	(1.06)	(−0.74)	(0.19)	(0.19)	(1.61)	(1.58)				
surp	RBaa ...	1.107	−0.331	0.156	−0.265	0.889	−7.705	9.335	−7.956	5.698	−1.244	0.053	2.155	.828	3.194	1.960	1.948
		(8.83)	(−1.77)	(0.85)	(−2.18)	(0.34)	(−1.66)	(1.89)	(−1.58)	(1.17)	(−0.44)	(1.41)	(1.19)				
RBaa	g	1.485	−0.720	0.423	−0.254	−0.044	0.110	0.421	−0.426	−0.030	0.276	0.002	−1.139	.992	0.164	1.894	0.462
		(11.82)	(−3.20)	(1.88)	(−2.00)	(−0.10)	(0.20)	(0.75)	(−0.76)	(−0.06)	(0.90)	(0.72)	(−0.97)				
g	RBaa ...	0.918	−0.080	0.036	−0.096	0.060	−0.085	0.067	−0.070	0.047	−0.025	0.002	0.990	.946	0.050	1.971	0.711
		(7.15)	(−0.46)	(0.21)	(−0.76)	(1.49)	(−1.15)	(0.86)	(−0.88)	(0.63)	(−0.59)	(2.51)	(2.92)				
RBaa	g$	1.485	−0.758	0.470	−0.282	1.130	−1.303	1.216	−1.598	1.161	−0.145	−0.002	−1.640	.992	0.164	1.888	0.502
		(11.75)	(−3.40)	(2.11)	(−2.25)	(0.90)	(−0.67)	(0.64)	(−0.84)	(0.62)	(−0.15)	(−0.50)	(−1.27)				
g$	RBaa ...	1.193	−0.151	0.218	−0.352	−0.003	0.026	−0.039	0.024	−0.012	0.007	0.001	0.369	.999	0.015	1.650	0.656
		(10.38)	(−0.82)	(1.22)	(−3.61)	(−0.21)	(1.19)	(−1.63)	(0.98)	(−0.53)	(0.52)	(3.60)	(3.31)				
RBaa	p	1.388	−0.687	0.375	−0.187	5.516	−3.821	3.812	6.530	−11.143	−1.321	0.007	2.144	.992	0.156	1.962	1.589
		(10.86)	(−3.20)	(1.74)	(−1.30)	(1.11)	(−0.50)	(0.50)	(0.88)	(−1.62)	(−0.29)	(0.92)	(0.24)				
p	RBaa ...	1.083	0.003	0.021	−0.106	0.003	−0.004	0.010	−0.008	0.0003	−0.0002	0.00002	−0.004	.999	0.004	2.016	1.482
		(8.24)	(0.02)	(0.11)	(−0.78)	(0.99)	(−0.75)	(1.57)	(−1.31)	(0.04)	(−0.06)	(0.09)	(−0.02)				
RBaa	w	1.531	−0.770	0.267	−0.117	7.898	−15.196	15.141	0.587	−12.076	3.635	0.005	0.154	.993	0.146	2.017	3.199**
		(11.87)	(−3.45)	(1.21)	(−0.92)	(2.19)	(−2.66)	(2.48)	(0.09)	(−2.00)	(0.93)	(0.60)	(0.36)				
w	RBaa ...	1.148	−0.333	0.361	−0.282	0.004	0.002	−0.004	−0.004	0.005	0.002	0.001	0.038	.9996	0.005	2.065	1.704
		(8.95)	(−1.65)	(1.70)	(−2.04)	(0.94)	(0.24)	(−0.45)	(−0.52)	(0.63)	(0.45)	(2.20)	(2.45)				

NOTE.—t-statistics for coefficients appear in parentheses below relevant coefficients.
* Significant at 5%.
** Significant at 1%.

TABLE 5

F-STATISTICS—TWO-SIDED TESTS

VARIABLE NAME (Y)	INDEPENDENT VARIABLE: RBaa* (1)	INDEPENDENT VARIABLE: Y† (2)
m	0.886‡	2.808‡
$surp$	0.708‡	1.454‡
g	0.285§	0.373‡
$g\$$	0.853§	0.661‡
p	1.339§	0.450§
w	3.251§	1.932§

NOTE.—All F's are $F(4, 50)$; significance levels are 2.56 for .95% confidence, 3.72 for .99% confidence:

Col. 1 regressions: $Y_t = \sum_{i=-4}^{12} w_i R\mathrm{Baa}_{t-i}$; col. 2 regressions: $R\mathrm{Baa}_t = \sum_{i=-4}^{12} w'_i Y_{t-i}$.

* F-statistic is pertinent for testing null hypothesis $w_{-4} = w_{-3} = w_{-2} = w_{-1} = 0$.
† F-statistic is pertinent for testing null hypothesis $w'_{-4} = w'_{-3} = w'_{-2} = w'_{-1} = 0$.
‡ No Theil constraint used.
§ Theil constraint used with $\sigma_u = (\max w_i - \min w_i)/2$.

TABLE 6

F-STATISTICS FOR COEFFICIENTS ON CURRENT AND LAGGED VARIABLES:

I. $Y_t = \sum_{i=0}^{12} \alpha_i R\mathrm{Baa}_{t-i} + \beta_0 + \beta_1 t$ (SEASONAL DUMMIES INCLUDED),

II. $R\mathrm{Baa}_t = \sum_{i=0}^{12} \gamma_i Y_{t-i} + \delta_0 + \delta_1 t$

VARIABLE NAME (Y_t)	INDEPENDENT VARIABLE: RBaa (1)	INDEPENDENT VARIABLE: Y_t (2)
m	1.854[a]	0.511[a]
$surp$	1.771[a]	1.648[a]
g	0.344[b]	0.446[a]
$g\$$	0.798[b]	0.408[a]
p	1.751[b]	1.025[b]
w	2.554**[b]	2.004*[b]

NOTE.—All F's are $F(13, 54)$; all data are filtered with $(1 - .75L)^2$.
[a] No Theil constraint used.
[b] Theil constraint used with $\sigma_u = (\max w_i - \min w_i)/2$.
* Significant at 5%.
** Significant at 1%.

and may be understated because possibly too many lagged variables have been included.

Table 7 reports F-statistics pertinent for testing whether the labor force participation rate nf is exogenous with respect to various causal candidates. The model implies that n_{ft} is exogenous with respect to all variables in the model, with the possible exception of the unemployment rate. The unemployment rate can cause the labor force participation rate, say through the "discouraged-worker effect," while not destroying the

TABLE 7

F-STATISTICS—TWO-SIDED TESTS

VARIABLE NAME (*Y*)	INDEPENDENT VARIABLE *nf** (1)	*Y*† (2)
Un	3.060‡	0.945§
m	1.591§	0.390‡
surp	1.320‡	0.514§
g	1.471§‖	1.594‡
g$	0.499‡	0.819§‖
p	0.901‡	1.487‡
w	0.586‡	1.498§‖

NOTE.—All *F*'s are $F(4, 50)$; significance levels are 2.56 for .95% confidence, 3.72 for .99% confidence:

$$\text{Col. 1 regressions: } Y_t = \sum_{i=-4}^{12} w_i nf_{t-i}; \text{ col. 2 regressions: } nf_t = \sum_{i=-4}^{12} w'_i Y_{t-i}.$$

* *F*-statistic is pertinent for testing null hypothesis $w_{-4} = w_{-3} = w_{-2} = w_{-1} = 0$.

† *F*-statistic is pertinent for testing null hypothesis $w'_{-4} = w'_{-3} = w'_{-2} = w'_{-1} = 0$.

‡ No Theil constraint used.

§ Theil constraint used with $\sigma_u = (\max w_i - \min w_i)/2$.

‖ Some seasonal remains in the distributed lag weights despite the imposition of Theil smoothness prior.

"recursive" structure of the model which prevents monetary and fiscal policy variables from causing the real variables *Un*, *nf*, and *y*.

The *F*-statistics in table 7 emerge from implementing Sims's test. The only *F*-statistic that is significant at the 95 percent confidence level is the one pertinent for testing the null hypothesis that *Un* fails to cause *nf*. At that significance level the null hypothesis must be rejected, which is compatible with the presence of a discouraged-worker effect that is useful for predicting labor force participation. While none of the other *F*-statistics is significant, the regression of m_t against nf_t did obtain several large and statistically significant coefficients on leading values of *nf*. This indicates that one ought perhaps to be cautious about the null hypothesis that *m* does not cause *nf*, despite the insignificant *F*-statistic. With this possible exception, the regressions summarized in table 7 are consistent with the causal structure imposed by the model upon *nf*.

Table 8 reports the results of applying Granger's test to *nf* and various causal candidates. At the 95 percent confidence level, *nf* appears to cause *w*, *p*, *g*, and *Un*, while only *Un* appears to cause *nf*.[18]

[18] While the Durbin-Watson statistics from most of the two-sided regressions are close to two, there is a possibility that the presence of higher than first-order serial correlation is making inappropriate the *F*-statistics in the text. For this reason, the two-sided regressions were recomputed using a version of Hannan's efficient estimator, which is asymptotically equivalent to generalized least squares allowing for high-order serial correlation in the disturbances. The results are reported in the mimeographed appendix to this paper (n. 17 above). The general pattern agrees with the results in the text, though there are differences in details. For example, in the Hannan efficient results, *w* does not seem to cause the Baa rate or the unemployment rate. If anything, then, the Hannan efficient regressions seem more favorable to the exogeneity hypotheses imposed by the classical model than are the two-sided regressions reported in the text.

TABLE 8

REGRESSION OF $Y(t) = \sum_{s=1}^{8} \alpha(s)Y(t-s) + \sum_{l=1}^{6} \beta(l)X(t-l) + \delta_1 t + \delta_0$

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Y	*X*	$\alpha(1)$	$\alpha(2)$	$\alpha(3)$	$\alpha(4)$	$\alpha(5)$	$\alpha(6)$	$\alpha(7)$	$\alpha(8)$	$\beta(1)$	$\beta(2)$
nf	*Un* . .	1.027	−0.151	−0.063	0.273	−0.227	0.101	−0.147	0.115	−0.0007	−0.0005
		(8.14)	(−0.83)	(−0.34)	(1.50)	(−1.24)	(0.60)	(−0.89)	(0.95)	(−0.52)	(−0.20)
Un	*nf* . . .	1.540	−0.757	0.159	−0.390	0.430	0.017	−0.162	0.069	−21.500	22.961
		(12.02)	(−3.22)	(0.64)	(−1.57)	(1.68)	(0.07)	(−0.75)	(0.57)	(−1.75)	(1.29)
nf	*m* . . .	0.919	−0.176	0.090	−0.086	0.190	−0.116	−0.101	0.028	−0.057	0.002
		(6.76)	(−0.99)	(0.50)	(−0.49)	(1.08)	(−0.67)	(−0.60)	(0.22)	(−0.64)	(0.01)
m	*nf* . . .	1.441	−0.525	−0.117	0.465	−0.474	0.352	−0.228	0.121	−0.036	−0.073
		(10.29)	(−2.01)	(−0.41)	(1.62)	(−1.60)	(1.10)	(−0.71)	(0.70)	(−0.17)	(−0.25)
nf	*surp* . .	0.927	−0.086	0.028	0.114	0.022	0.013	−0.131	0.027	0.00009	0.00003
		(6.92)	(−0.48)	(0.16)	(0.64)	(0.12)	(0.08)	(−0.74)	(0.20)	(0.55)	(0.11)
surp	*nf* . . .	1.089	−0.374	0.163	−0.398	0.326	−0.210	−0.102	−0.024	−22.533	−144.945
		(8.34)	(−1.93)	(0.81)	(−2.02)	(1.74)	(−1.13)	(−0.58)	(−0.20)	(−0.22)	(−1.06)
nf	*g* . . .	0.943	−0.071	−0.031	0.143	0.081	−0.119	−0.155	0.105	0.013	−0.026
		(7.42)	(−0.41)	(−0.18)	(0.84)	(0.46)	(−0.64)	(−0.85)	(0.78)	(1.19)	(−1.72)
g	*nf* . . .	0.931	−0.087	0.052	−0.129	0.140	−0.090	0.081	−0.147	1.917	1.033
		(7.19)	(−0.51)	(0.31)	(−0.78)	(0.92)	(−0.69)	(0.62)	(−1.57)	(1.25)	(0.50)
nf	*g$* . .	0.969	−0.128	0.014	0.037	0.022	0.046	−0.144	0.052	0.046	−0.047
		(7.27)	(−0.73)	(0.08)	(0.21)	(0.13)	(0.27)	(−0.84)	(0.41)	(1.45)	(−0.94)
g$	*nf* . . .	1.082	−0.100	0.242	−0.438	0.182	−0.052	0.052	−0.098	0.440	0.553
		(8.45)	(−0.52)	(1.26)	(−2.35)	(0.93)	(−0.27)	(0.29)	(−1.04)	(0.89)	(0.80)
nf	*p* . . .	0.938	−0.185	−0.011	0.039	0.050	−0.085	−0.179	0.098	0.181	−0.078
		(6.73)	(−1.02)	(−0.06)	(0.22)	(0.28)	(−0.47)	(−0.99)	(0.66)	(1.18)	(−0.37)
p	*nf* . . .	0.853	0.038	0.138	0.196	−0.185	0.086	−0.145	−0.048	0.360	−0.107
		(6.61)	(0.22)	(0.79)	(1.15)	(−1.14)	(0.51)	(−0.84)	(−0.45)	(3.10)	(−0.68)
nf	*w* . . .	0.920	−0.175	−0.020	0.021	0.075	−0.091	−0.173	0.081	0.207	−0.073
		(6.93)	(−0.95)	(−0.11)	(0.11)	(0.40)	(−0.50)	(−0.96)	(0.57)	(1.85)	(−0.42)
w	*nf* . . .	1.049	−0.302	0.297	−0.058	0.030	−0.065	0.104	−0.143	0.553	−0.235
		(8.08)	(−1.51)	(1.47)	(−0.30)	(0.16)	(−0.34)	(0.57)	(−1.18)	(3.62)	(−1.11)

Y	*X*	$\beta(3)$	$\beta(4)$	$\beta(5)$	$\beta(6)$	δ_1	δ_0	$\bar{R}^2$	SE of Est Adj.	D–W	*F*-Statistic on All $F(6, 56)$
nf	*Un* . .	0.003	−0.007	0.009	−0.004	0.00003	−0.036	.864	0.004	2.143	2.886*
		(0.97)	(−2.62)	(3.54)	(−2.81)	(1.29)	(−0.98)				
Un	*nf* . . .	−5.060	21.901	−32.547	24.041	−0.001	5.604	.916	0.351	2.097	2.223*
		(−0.29)	(1.31)	(−1.92)	(1.99)	(−0.63)	(1.76)				
nf	*m* . . .	0.262	−0.384	0.312	−0.103	−0.0002	−0.288	.853	0.004	2.031	1.983
		(1.47)	(−2.12)	(1.76)	(−1.02)	(−1.74)	(−2.31)				
m	*nf* . . .	0.041	0.075	−0.039	−0.137	−0.0001	−0.258	.999	0.006	2.000	0.309
		(0.13)	(0.25)	(−0.14)	(−0.69)	(−0.59)	(−1.25)				
nf	*surp* . .	−0.0001	0.0003	−0.0003	0.000001	0.00002	−0.045	.841	0.004	2.026	1.132
		(−0.51)	(1.06)	(−1.19)	(0.01)	(1.04)	(−1.18)				
surp	*nf* . . .	91.625	128.847	−69.150	−53.598	−0.012	−37.639	.826	3.190	1.967	0.980
		(0.67)	(0.94)	(−0.50)	(−0.53)	(−0.66)	(−1.39)				
nf	*g* . . .	−0.004	0.020	−0.016	0.016	−0.000006	−0.073	.852	0.004	2.054	1.938
		(−0.24)	(1.38)	(−1.23)	2.09	(−0.11)	(−1.58)				
g	*nf* . .	−0.555	−5.256	5.180	−2.354	0.002	1.066	.953	0.047	2.081	2.578*
		(−0.27)	(−2.62)	(2.44)	(−1.51)	(2.88)	(1.93)				
nf	*g$* . .	0.046	−0.105	0.073	−0.003	−0.0001	−0.113	.845	0.004	2.015	1.385
		(0.96)	(−2.16)	(1.46)	(−0.11)	(−1.23)	(−1.97)				
g$	*nf* . . .	0.028	−0.746	−0.055	0.192	0.002	0.750	.999	0.015	1.692	1.343
		(0.04)	(−1.14)	(−0.09)	(0.41)	(3.86)	(3.14)				
nf	*p* . . .	0.160	−0.372	0.192	−0.033	−0.0002	−0.397	.840	0.004	2.060	1.072
		(0.74)	(−1.75)	(0.97)	(−0.26)	(−1.17)	(−1.58)				
p	*nf* . . .	−0.089	0.131	0.123	−0.223	0.0004	0.410	.9995	0.003	2.104	2.935*
		(−0.56)	(0.83)	(0.82)	(−1.86)	(2.38)	(2.10)				
nf	*w* . . .	0.010	−0.167	0.132	−0.060	−0.0004	−0.217	.841	0.004	2.002	1.125
		(0.06)	(−0.95)	(0.78)	(−0.56)	(−1.15)	(−2.00)				
w	*nf* . . .	−0.125	0.219	0.017	−0.053	0.0008	0.242	.9996	0.005	1.972	3.137**
		(−0.59)	(1.04)	(0.08)	(−0.32)	(2.25)	(2.25)				

NOTE.—*t*-statistics for coefficients appear in parentheses below relevant coefficients.
*Significant at 5%.
**Significant at 1%.

All in all, the empirical results provide some evidence that the causal structure imposed on the data by the classical model of Section I is not obscenely at variance with the data. The evidence that m seems to be caused by RBaa means that the assumption that m is exogenous, embedded in the assumed autoregression of equation (1.6), must be abandoned. But this is not essential, since for the purpose for which the model is intended (unconditional forecasting), the regression in table 4 will do just as well. Findings that contradict the model are that w seems to cause both RBaa and Un, according to both Sims's and Granger's tests. Also, according to Granger's test, m seems to cause Un, but according to Sims's test, it does not. This last discrepancy requires reconciling, as does the apparently general tendency of Granger's test to reject exogeneity more readily than does Sims's test.[19]

I do not believe that these results render a verdict on the model of Section I sufficiently negative for me to stop now before presenting estimates of the model. The causal candidate that does the most damage to the hypotheses of the model is the money wage w, which does not appear itself as a variable in the model of Section I. Causal candidates drawn from the list of variables actually appearing in the model usually do not seem to violate the hypotheses of the model, which gives some encouragement to the project of estimating the model.

IV. Estimates of the Model[20]

To estimate the model, a proxy for $E_{t-1}p_t$ was required.[21] As in a procedure previously used (Sargent 1973), the proxy for $E_{t-1}p_t$ was formed by regressing p_t against a list of variables dated $t - 1$ and earlier.[22] In

[19] In implementing Granger's test, I specified a maximal number of lagged own terms, usually four, upon which a variable was permitted to depend. If the variable in question is exogenous but follows a mixed moving-average, autoregressive process so that its autoregression is of infinite order, this misspecification could lead to erroneous rejection of the hypothesis of exogeneity. With Sims's test, premature truncation of the lag distribution will lead to too frequent rejection of the hypothesis of exogeneity when it is true. (Christopher Sims points out to me that since the autoregressive part of the direct Granger regression whitens the residuals, thereby reseasonalizing them, it is not possible for the Granger test to "ignore" the seasonal bands, as the Sims test as applied here does. This could conceivably account for some of the differences in the results of the two tests.)

[20] The estimates of the model use the data seasonally adjusted by setting their Fourier transforms equal to zero in the seasonal bands, the same data used in the tests in Section III. Estimates of the model using officially seasonally adjusted data were also made. The results, which are qualitatively similar to those summarized here, are in the mimeographed appendix (n. 17 above).

[21] For population (pop), I took the civilian population over 16 years old, while for the labor force I used the civilian labor force over 16. The labor force participation rate nf was measured as the ratio of the latter to the former. The total civilian unemployment rate was used. Notice that $nf_t + pop_t - Un_t$ approximately equals civilian employment, so my production function views GNP as a function only of civilian employment.

[22] To form the proxy for $E_{t-1}p_t$, p_t was regressed on a constant, trend, three seasonal dummies, and p, w, nf, and Un, each lagged one through four times.

each case, this list included all the predetermined variables that appear on the right side of the equation in which $E_{t-1}p_t$ appears.

Since the model is a simultaneous one, an instrumental-variables estimator was used to estimate the coefficients. Current endogenous variables that appear on the right side of an equation were replaced by the systematic part of a regression of that variable on the same variables that were used to form the proxy for $E_{t-1}p_t$ plus current values of the exogenous variables.[23]

The estimates are reported in table 9. The production function includes current and four lagged values of $n_t \equiv (n_{ft} + pop_t - Un_t)$. The estimates of the production function (item 3) are compatible with increasing returns to labor in the short run and slightly decreasing returns to labor in the long run.

The estimates reported in table 9 possess signs that agree with a priori expectations. Unexpected increases in the price level are estimated to increase the labor force participation rate and decrease the unemployment rate. Increases in the unemployment rate decrease the labor force participation rate, which is consistent with a discouraged-worker effect.

In the estimates reported in table 9, I have not included innovations in Z_t as determinants of R, so that the equation for R (the Baa rate) is simply an autoregression. Two pairs of equations for portfolio equilibrium are reported. The first pair regresses the reciprocal of the log of velocity $(m - p - y)$ against current- and lagged-interest rates, one member including and the other excluding trend. Including trend is seen to increase the coefficients on current- and lagged-interest rates and to make their sum positive. This is a common, though widely ignored, result: including a trend in postwar estimates of demand schedules for money for the United States tends to eliminate any inverse dependence of velocity on interest rates. The second pair of portfolio balance equations regresses $m - p$ on current and lagged y's and R's, again with and without trend. Including trend again has important effects on the coefficients. For my purposes, any of these four or any other reasonable demand schedule for money is suitable. Notice also that the model will work in the same "recursive" way if a demand schedule for money is dropped and replaced by a regression of $p_t + y_t$ on current and lagged m, the sort of equation estimated by Sims (1972*b*) and Andersen and Carlson (1970).

[23] The endogenous variables were replaced by the systematic part of a regression of themselves against pop_t, m_t, g_t, $surp_t$, the log of current government employment, and all of the variables reported in n. 22 above. The reader may wonder whether eqq. (1) and (2), which have lagged endogenous variables as regressors, can be consistently estimated by the technique employed. If the residuals are serially correlated, my estimates are not consistent. But it is straightforward to show that, e.g., the *Un* vs. p exogeneity tests of Section III can be viewed as tests for serial correlation of the disturbances in eq. (1), failure to reject exogeneity of unemployment (p's failing to cause *Un*) being consistent with no serial correlation. In effect, then, some testing for the null hypothesis of no serial correlation has been carried out, with results favorable to the null hypothesis.

TABLE 9

ESTIMATES OF THE MODEL (1951 I–1973 III)

Variable	Estimate
1. Un_t	$\underset{(2.0)}{-0.287}(\hat{p}_t - E_{t-1}p_t) + \underset{(2.5)}{0.0043} + \underset{(0.5)}{0.0000007t} + \underset{(12.8)}{1.47Un_{t-1}} - \underset{(2.9)}{0.59Un_{t-2}} - \underset{(0.1)}{0.03Un_{t-3}} + \underset{(0.3)}{0.04Un_{t-4}}$*
2. nf_t	$\underset{(0.9)}{0.149}(\hat{p}_t - E_{t-1}p_t) - \underset{(1.9)}{0.075\hat{Un}_t} - \underset{(1.3)}{0.038} + \underset{(2.1)}{0.00004t} + \underset{(8.2)}{0.94nf_{t-1}} - \underset{(0.7)}{0.11nf_{t-2}} - \underset{(0.2)}{0.02nf_{t-3}} + \underset{(1.0)}{0.12nf_{t-4}}$†
3. y_t	$\underset{(3.5)}{1.09\hat{n}_t} + \underset{(1.0)}{0.24n_{t-1}} - \underset{(1.0)}{0.24n_{t-2}} - \underset{(0.6)}{0.14n_{t-3}} - \underset{(0.1)}{0.02n_{t-4}} + \underset{(1.8)}{0.35} + \underset{(4.5)}{0.0009t}$‡
4. R_t	$\underset{(13.1)}{1.52R_{t-1}} - \underset{(3.7)}{0.77R_{t-2}} + \underset{(2.4)}{0.44R_{t-3}} - \underset{(2.1)}{0.24R_{t-4}} + \underset{(1.8)}{0.15} + \underset{(2.0)}{0.0034t}$§
5a. $m_t - p_t - y_t$	$\underset{(0.0)}{-0.0004\hat{R}_t} - \underset{(0.0)}{0.0004R_{t-1}} - \underset{(1.4)}{0.010R_{t-2}} + \underset{(2.4)}{0.021R_{t-3}} + \underset{(0.9)}{0.007R_{t-4}} + \underset{(2.1)}{0.015R_{t-5}} - \underset{(1.6)}{0.012R_{t-6}} + \underset{(1.0)}{0.007R_{t-7}} - \underset{(212.0)}{0.91} - \underset{(14.7)}{1.37 \times 10^{-3}} \times t$ ‖
5b. $m_t - p_t - y_t$	$\underset{(2.0)}{-0.032\hat{R}_t} - \underset{(0.7)}{0.006R_{t-1}} - \underset{(3.2)}{0.027R_{t-2}} + \underset{(1.6)}{0.014R_{t-3}} - \underset{(0.9)}{0.007R_{t-4}} - \underset{(0.0)}{0.0003R_{t-5}} - \underset{(2.0)}{0.018R_{t-6}} - \underset{(0.5)}{0.004R_{t-7}} - \underset{(59.2)}{0.22}$#
5c. $m_t - p_t$	$\underset{(0.5)}{-0.0060\hat{R}_t} - \underset{(1.2)}{0.0059R_{t-1}} - \underset{(1.7)}{0.0091R_{t-2}} + \underset{(2.6)}{0.0143R_{t-3}} + \underset{(1.6)}{0.0080R_{t-4}} + \underset{(2.0)}{0.0107R_{t-5}} - \underset{(0.4)}{0.0022R_{t-6}} + \underset{(0.8)}{0.0042R_{t-7}} + \underset{(3.3)}{0.45\hat{y}_t} + \underset{(1.9)}{0.16y_{t-1}} + \underset{(2.3)}{0.19y_{t-2}} + \underset{(1.1)}{0.09y_{t-3}} - \underset{(0.9)}{0.06y_{t-4}} + \underset{(0.3)}{0.02y_{t-5}} + \underset{(0.8)}{0.05y_{t-6}} + \underset{(1.1)}{0.06y_{t-7}} - \underset{(2.6)}{0.22} - \underset{(2.1)}{0.0003t}$**
5d. $m_t - p_t$	$\underset{(0.2)}{-0.0023\hat{R}_t} - \underset{(1.5)}{0.0075R_{t-1}} - \underset{(1.6)}{0.0089R_{t-2}} + \underset{(1.7)}{0.0088R_{t-3}} + \underset{(0.9)}{0.0045R_{t-4}} + \underset{(0.9)}{0.0043R_{t-5}} - \underset{(0.8)}{0.0044R_{t-6}} + \underset{(0.1)}{0.0003R_{t-7}} + \underset{(2.5)}{0.25\hat{y}_t} + \underset{(0.9)}{0.06y_{t-1}} + \underset{(1.3)}{0.09y_{t-2}} - \underset{(0.1)}{0.01y_{t-3}} - \underset{(2.2)}{0.14y_{t-4}} - \underset{(0.3)}{0.02y_{t-5}} + \underset{(0.2)}{0.01y_{t-6}} + \underset{(0.6)}{0.04y_{t-7}} - \underset{(1.7)}{0.052}$††

NOTE.—t-statistics are in parentheses beneath coefficients. Hatted variables (^) are systematic parts of regressions against instrumental variables.

* $\bar{R}^2 = .908$; SE $= .371$; D-W $= 2.07$.

† $\bar{R}^2 = .867$; SE $= .0040$; D-W $= 2.00$.

‡ $\bar{R}^2 = .95$; SE $= .00964$; D-W $= 2.03$; filter: $(1 - .6L)^2$; sum of weights on $n = +.93$; $\hat{n}_t \equiv (nf_t - \hat{Un}_t + pop_t)$; $n_t \equiv (nf_t - \hat{Un}_t + pop_t)$.

§ $\bar{R}^2 = .99$; SE $= .158$; D-W $= 1.89$; sum of weights $= .95$.

‖ $\bar{R}^2 = .93$; SE $= .00856$; D-W $= 1.97$; filter: $(1 - .6L)^2$; sum of coefficients on $R = +.024$.

\# $\bar{R}^2 = .28$; SE $= .01085$; D-W $= 1.91$; filter: $(1 - .8L)^2$; sum of coefficients on $R = -.080$.

** $\bar{R}^2 = .29$; SE $= .00561$; D-W $= 2.10$; filter: $(1 - .8L)^2$; sum of weights on $R = +.014$; sum of weights on $y = +.96$.

†† $\bar{R}^2 = .25$; SE $= .00576$; D-W $= 1.99$; filter: $(1 - .8L)^2$; sum of weights on $R = -.005$; sum of weights on $y = +.28$.

V. Conclusions

This paper has estimated and tested a macroeconometric model with "classical" or "monetarist" policy implications, even though it has "Keynesian" short-run properties. Some evidence for rejecting the model has been turned up, but it is far from being overwhelming and decisive. The evidence that seems most damaging to the model comes from the role that the money wage plays in apparently "causing" unemployment and the long-term interest rate. On the other hand, the tests have turned up little evidence requiring us to reject the key hypothesis of the model that government monetary and fiscal-policy variables do not cause unemployment or the interest rate. The fact that such evidence has been hard to turn up ought to be disconcerting to users of the existing macroeconometric models, since as usually manipulated those models all imply that monetary and fiscal policy *do* help cause unemployment and the interest rate.

Models of the kind presented in this paper imply that there is no scope for the government to engage in activist countercyclical policy, so that it might as well employ rules without feedback for fiscal and monetary policy, for example, Friedman's x percent growth rule for the money supply. In contradistinction, macroeconometric models as they are usually manipulated imply that it is optimal for the government to use rules with feedback, which may imply "leaning against the wind," contrary to Friedman's rule. If we are to have any reason to believe that rules with feedback are superior to rules without feedback, there should be empirical evidence in hand that some existing macroeconometric model can outperform models of the class studied in this paper. It is my impression that such evidence does not yet exist.

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