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THE DEMAND FOR MONEY DURING HYPERINFLATION UNDER RATIONAL EXPECTATIONS: II*

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This paper estimates the parameters of Phillip Cagan's model of portfolio balance for hyperinflations under the assumption that the public's expectations about inflation are rational in the sense of John F. Muth [1961]. This paper is a sequel to a paper by Sargent [1977] which estimated and tested Cagan's model for the special circumstance in which the adaptive expectations scheme that Cagan assumed turns out to be rational. In that setting, econometric identification of Cagan's α — the slope of the log of real money balances with respect to the expected inflation rate — is very tenuous and requires prior information about the covariance of shocks to demand and supply schedules for money. In this paper, we show how knowledge of that covariance can be dispensed with provided that rational expectations turn out not to be adaptive, for then α appears in restrictions that Cagan's model imposes on the systematic part of the vector autoregression for the money creation, inflation process. Our strategy is to estimate α from those restrictions and to test the model by seeing how much those restrictions cost in terms of goodness of fit vis-à-vis unconstrained vector autoregression.

1. THE DEMAND FOR MONEY IN HYPERINFLATION WITH RATIONALLY FORMED EXPECTATIONS

We suppose that the portfolio balance schedule has the form that Cagan assumed,

$$m_t - p_t = \beta + \alpha\pi_t + \varepsilon_t \tag{1}$$

where m_t is the natural logarithm of the stock of money, p_t is the natural logarithm of the price level, π_t is the public's expectation formed at time t of the rate of inflation to occur between t and $t+1$, and ε_t is a random variable the properties of which are discussed shortly. Letting $X_t \equiv p_t - p_{t-1}$ and $\mu_t \equiv m_t - m_{t-1}$, it is further assumed that the public's expectations of inflation are rational, which amounts to imposing

$$\pi_t = E_t(X_{t+1}). \tag{2}$$

Here $E_t(y)$ denotes the linear least squares forecast of the random variable y based on information available at t which is assumed to consist of $(X_t, X_{t-1}, \ldots,$

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$\mu_t, \mu_{t-1}, \ldots)$.[1]

Let $\eta_t = \varepsilon_t - \varepsilon_{t-1}$. By first differencing (1) and substituting (2), one obtains

$$\mu_t - X_t = \alpha(E_t(X_{t+1}) - E_{t-1}(X_t)) + \eta_t. \tag{3}$$

It is assumed that η_t is a serially uncorrelated random variable with zero mean and that

$$E_{t-1}(\eta_t) = E(\eta_t \mid X_{t-1}, X_{t-2}, \ldots, \mu_{t-1}, \mu_{t-2}, \ldots) = 0. \tag{4}$$

Assumption (4) permits η_t to be correlated with current X and μ. Indeed, the sense of the model is that movements in η_t cause responses in current and future values of the price level and perhaps the money supply as well if the monetary authority is following a rule by which it relates current growth in m to current and past growth in p. Assumption (4) is in the nature of an arbitrary restriction on the disturbance process η_t with no economic reasoning behind it. Some such arbitrary restrictions on disturbance processes are commonly used to achieve identification (see Hatanaka [1975]). It should be emphasized that the estimates of α below are made under the joint hypothesis composed of Cagan's economic hypothesis, (1), the arbitrary restriction (4) on η_t, and the hypothesis that the public's expectations are rational. Similarly, the likelihood ratio tests proposed below test this joint hypothesis.

Using assumption (4) and projecting both sides of equation (3) on information known at time $(t-1)$ yields

$$E_{t-1}(\mu_t) = \alpha E_{t-1}(X_{t+1}) + (1 - \alpha)E_{t-1}(X_t). \tag{5}$$

Equation (5) is a non-linear restriction which the model (1) and assumption of rational expectations place on the systematic part of the vector autoregressive representation of the (X, μ) process. In general, estimating the vector autoregression of (X, μ) subject to restriction (5) will permit the identification of α, the important structural parameter of Cagan's model. However, it is important to observe that in one special case α is *not* identified by the restriction. If X_t has the representation

$$X_t = \frac{1 - \lambda}{1 - \lambda L} X_{t-1} + \phi_t \tag{6}$$

where $E_{t-1}(\phi_t) = 0$, and where L is the lag operator defined by $Ly_t = y_{t-1}$, then the adaptive expectations mechanism hypothesized by Cagan yields expectations which are rational in the sense of equation (2). Moreover, as Muth [1960] pointed out, equation (6) implies that

[1] Thus $E_t(y) \equiv E(y \mid \mu_t, \mu_{t-1}, \ldots, X_t, X_{t-1}, \ldots)$ where E is the linear least squares projection operator. Actually, all of our results go through so long as the conditioning information set includes at least $(\mu_t, \mu_{t-1}, \ldots, X_t, X_{t-1}, \ldots)$. That this is so can be shown by carrying along a larger information set and then applying the law of iterated projections to deduce implication for projections on the smaller set. (The law of iterated projections states $E(E(y \mid X, Z) \mid Z) = E(y \mid Z)$.)

$$E_{t-1}(X_{t+j}) = E_{t-1}(X_t) = \frac{1-\lambda}{1-\lambda L}X_{t-1} \qquad j = 1, 2, \ldots \tag{7}$$

Substituting (7) into (5) gives

$$E_{t-1}(\mu_t) = E_{t-1}(X_t), \tag{8}$$

in which α does not appear. So in this case, restriction (5) does not identify α. This is consistent with a result in Sargent [1977] which shows that in a setting in which Cagan's adaptive expectations scheme is rational α is *not* identified even from knowledge of the entire vector autoregressive representation of the (X, μ) process — not only the systematic portion but also the covariance matrix of the innovations. Hence our result in (8) is not surprising since ignoring information on the covariance structure of the innovations in the (X, μ) process can exacerbate but never ameliorate a problem of identification.

That a structural parameter α is in general identified by restriction (5) on the systematic part of the vector autoregression is a special circumstance not encountered in usual macroeconomic applications. Usually identification of structural parameters requires restrictions on the covariance matrix of innovations and on the matrix of contemporaneous structural coefficients. There are two special features of the present model which permit only the systematic part of the vector autoregression to identify the structural parameter α. The first is that the demand for real balances varies with the public's expectation of the *one period forward* rate of inflation. The second is that the public's expectations are assumed to be rational. Models characterized by these features of timing and rationality will often place identifying restrictions on the systematic portions of vector autoregressions.

2. MAXIMUM LIKELIHOOD ESTIMATION OF α

Let the n-th order vector autoregressive representation for (μ_t, X_t) be

$$\begin{aligned} \mu_t &= \sum_{i=1}^{n} a_i\mu_{t-i} + \sum_{i=1}^{n} b_i X_{t-i} + \boldsymbol{a}_{\mu_t} \\ X_t &= \sum_{i=1}^{n} c_i\mu_{t-i} + \sum_{i=1}^{n} d_i X_{t-i} + \boldsymbol{a}_{X_t} \end{aligned} \tag{9}$$

where the innovations $\boldsymbol{a}_{\mu_t}$ and $\boldsymbol{a}_{X_t}$ obey the least squares orthogonality conditions $E\boldsymbol{a}_{\mu_t}X_{t-j} = E\boldsymbol{a}_{X_t}X_{t-j} = E\boldsymbol{a}_{\mu_t}\mu_{t-j} = E\boldsymbol{a}_{X_t}\mu_{t-j} = 0$ for $j = 1, \ldots, n$. The random processes $\boldsymbol{a}_{X_t}$ and $\boldsymbol{a}_{\mu_t}$ are the errors in forecasting X_t and μ_t, respectively, from n past values of X and μ. We have deleted constants and trends from (9), but will shortly return to a discussion of alternative ways of modeling trends.

As a device for compactly writing the restriction which the model places on the n-th order vector autoregression of X_t and μ_t it is convenient to write the n-th order vector autoregression in the following form

$$Z_t = AZ_{t-1} + \boldsymbol{a}_t \tag{9'}$$

where $Z_t = (\mu_t\ \mu_{t-1} \cdots \mu_{t-n+1}\ X_t\ X_{t-1} \cdots X_{t-n+1})'$ and $\boldsymbol{a}_t = (\boldsymbol{a}_{\mu_t}\ 0 \cdots 0\ \boldsymbol{a}_{X_t}\ 0 \cdots 0)'$ are $2n \times 1$ vectors and A is the $2n \times 2n$ matrix

$$A = \begin{bmatrix} a_1 & a_2 \cdots a_{n-1} & a_n & b_1 & b_2 \cdots b_{n-1} & b_n \\ 1 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 & 0 \\ 0 & 1 \cdots 0 & 0 & 0 & 0 \cdots 0 & 0 \\ \vdots & \vdots \quad \vdots & \vdots & \vdots & \vdots \quad \vdots & \vdots \\ 0 & 0 \cdots 1 & 0 & 0 & 0 \cdots 0 & 0 \\ c_1 & c_2 \cdots c_{n-1} & c_n & d_1 & d_2 \cdots d_{n-1} & d_n \\ 0 & 0 \cdots 0 & 0 & 1 & 0 \cdots 0 & 0 \\ \vdots & \vdots \quad \vdots & \vdots & \vdots & \vdots \quad \vdots & \vdots \\ 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 1 & 0 \end{bmatrix}.$$

The n-th order system has thus conveniently been expressed as a first order system in which $\boldsymbol{a}_{\mu_t}$ and $\boldsymbol{a}_{X_t}$ are the one step ahead prediction errors for the rates of money creation and inflation respectively.

Let γ be the $1 \times 2n$ row vector with one in the first column and zeros elsewhere and let δ be the $1 \times 2n$ row vector with one in the $(n+1)$-st column and zeros elsewhere. Then $\mu_t = \gamma \cdot Z_t$ and $X_t = \delta \cdot Z_t$. So restriction (5) can be expressed as

$$\gamma E_{t-1}(Z_t) = \alpha \delta E_{t-1}(Z_{t+1}) + (1-\alpha)\delta E_{t-1}(Z_t). \tag{10}$$

Now equation (9′) implies that

$$\begin{aligned} Z_{t+1} &= AZ_t + \boldsymbol{a}_{t+1} \\ &= A^2 Z_{t-1} + A\boldsymbol{a}_t + \boldsymbol{a}_{t+1} \end{aligned}$$

and since the orthogonality condition of linear projection requires that $E_{t-1}(\boldsymbol{a}_t) = 0$, we have that

$$E_{t-1}(Z_{t+1}) = A^2 Z_{t-1}.$$

Thus equation (10) implies

$$\gamma A = \delta[\alpha A^2 + (1-\alpha)A] \tag{11}$$

which is a set of $2n$ non-linear equations across α and the $4n$ parameters in A.

The reader may verify that equation (11) is equivalent to

$$a_i = \begin{cases} \dfrac{(1+\alpha d_1)c_i + \alpha(c_{i+1} - c_i)}{(1-\alpha c_1)} & i = 1, 2, \ldots, n-1 \\ \dfrac{(1+\alpha d_1)c_i - \alpha c_i}{(1-\alpha c_1)} & i = n. \end{cases} \tag{12}$$

$$b_i = \begin{cases} \dfrac{(1+\alpha d_1)d_i + \alpha(d_{i+1} - d_i)}{(1-\alpha c_1)} & i = 1, 2, \ldots, n-1 \\[2ex] \dfrac{(1+\alpha d_1)d_i - \alpha d_i}{(1-\alpha c_1)} & i = n \end{cases}$$

provided that $(1-\alpha c_1)$ is not equal to zero. In the estimation below, the constraints were imposed directly by taking $\{c_i, d_i\}$ and α to be free parameters and using (12) to compute the values for $\{a_i, b_i\}$ that are implied by the restrictions imposed by the model.

Let $p = \{a_1, \ldots, a_n, b_1, \ldots, b_n, \ldots, c_1, \ldots, c_n, d_1, \ldots, d_n\}$ and let $\hat{\boldsymbol{a}}_{\mu_t}(p, \alpha)$ and $\hat{\boldsymbol{a}}_{X_t}(p, \alpha)$ be respectively the estimates of $\boldsymbol{a}_{\mu_t}$ and $\boldsymbol{a}_{X_t}$ implied by (9) and (12) with a given set of parameters and a given data record of length T. Let

$$V(p, \alpha) = \begin{bmatrix} \dfrac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{a}}^2_{\mu_t}(p, \alpha) & \dfrac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{a}}_{\mu_t}(p, \alpha)\hat{\boldsymbol{a}}_{X_t}(p, \alpha) \\[2ex] \dfrac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{a}}_{\mu_t}(p, \alpha)\hat{\boldsymbol{a}}_{X_t}(p, \alpha) & \dfrac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{a}}^2_{X_t}(p, \alpha) \end{bmatrix}. \tag{13}$$

Then if $(\boldsymbol{a}_{\mu_t}, \boldsymbol{a}_{X_t})$ is a jointly normal stochastic process, maximum likelihood estimates of p and α under the restrictions (12) imposed by the model are obtained by maximizing

$$-T \log 2\pi - (1/2)T(\log |V(p, \alpha)| + 2) \tag{14}$$

subject to (12) and with respect to p and α. As shown by Wilson [1973], equation (14) gives the value of the log likelihood function at the optimum values for p and α.

We proceeded as follows with estimating and testing the model. First, unconstrained maximum likelihood estimates of the vector autoregression parameters in (9) were obtained. Next, p and α were estimated subject to the restrictions (12) that are imposed by the model.[2] We then calculated a statistic for testing the validity of the restrictions (12), namely the likelihood ratio

$$L(u, r) = T(\log |V_r| - \log |V_u|) \tag{15}$$

where V_u and V_r are given by (13) in the unconstrained and constrained cases respectively. Under the null hypothesis that the restrictions (12) are correct, the likelihood ratio is asymptotically distributed as chi-square with q degrees of freedom, where q is the number of restrictions imposed. Next, the vector autoregressions were estimated under the restrictions (12) imposed by the model and under the additional restriction that the rate of inflation is exogeneous in the sense of Sims [1972] or "failed to be caused" by money creation, in Granger's [1969] sense. In the context of equation (9) this amounts to imposing in addition to (12),

[2] The estimation was performed by using the Maxlik subroutine package for constrained maximum likelihood estimation described in Kaplan and Elston [1972].

$$(16) \qquad c_i = 0 \qquad i = 1, 2, \ldots, n.$$

In a "model-free" setting Sargent and Wallace [1973] and Salemi [1976] found evidence for the exogeneity of the inflation process during the European hyperinflations.[3] It is interesting to explore whether this result holds up under the less free parameterization involved in testing for exogeneity while maintaining the model (1) and (4) underlying the restrictions (12). However, one caveat is in order about testing the exogeneity of inflation by imposing (16). Because we have short time series, we shall set n equal to two or four. For standard omitted-variables reasons, premature truncation of a vector autoregression can lead to erroneous rejection of an hypothesis of exogeneity. Therefore, the exogeneity tests, especially with $n=2$, but also with $n=4$, should be interpreted very cautiously. Ideally, one would like to perform these tests with larger values for n.

On the other hand, the theory predicts that premature truncation will leave restriction (12) intact. Equation (12) is a valid restriction on the n-th order autoregression even if a larger-than-n-th order autoregression would fit better.[4] Thus, restrictions (12) and (16) are predicted to display differing degrees of robustness with respect to setting n too low.

The estimates were in most cases obtained by an algorithm which searched the likelihood surface directly. In several cases, however, a variation of the Newton Raphson technique converged to a likelihood maximum when the direct search routine was hampered by an extremely flat portion of the likelihood surface. The data employed are basically those found in Cagan [1956] with two exceptions. First, in the case of Germany, Cagan created price and money stock data by using a complicated procedure designed to take advantage of the finer observations available in 1923. Our study reports the results using both Cagan's data and the X_t and μ_t series derived directly from the monthly average series. Second, we have corrected an apparent error in Cagan's price series for Austria.[5]

[3] They also found evidence for rather substantial feedback from inflation to money creation.

[4] Application of the law of iterated projections to (5) establishes this.

[5] For his study of the German hyperinflation, Cagan used monthly series on money stocks and wholesale prices published in a special issue of *Wirtschaft and Statistik, Zahlen zur Geldenwertung in Deutchland,* 1914 *bis* 1923, (*ZzG*). Although this study begins with the same raw material as did Cagan's, the data for the inflation and money creation series are different from his.

The rate of inflation, X_t, is the first difference of the natural log of the index of wholesale prices, p_t, observed as a monthly average of daily rates throughout the hyperinflation (*ZzG*, p. 16). Observations on p_t are available at 10 day intervals during the first half of 1923. Cagan took the monthly average p_t as a proxy for the midmonth price level before 1923 and the point in time observation near the 15th of the month as a proxy for the midmonth price level in 1923. As a rationale, Cagan argued that monthly averages would grossly overstate the midmonth price level in the latter months of hyperinflation. Since point in time data are not available throughout the hyperinflation and monthly average data are available, this study also uses the monthly average data throughout in the view that the mixing of the average and point in time data could potentially distort estimates of distributed lags more than the consistent use of the monthly average series (Sims [1971]).

(*Continued on next page*)

We tried three different models of "trends", which led to different estimation procedures and important differences in results. Model 1 takes the (μ_t, X_t)'s that appear in (9) to be deviations from a constant and a trend. Our procedure was first to regress money creation and inflation each against a constant and linear trend, and then to use the residuals from these regressions as the data modeled in (9). Notice that this procedure ignores the restrictions that the model would impose across the trends if (5) is interpreted as applying to the raw (i.e., not detrended) data.

Model two is

$$\mu_t = \sum_{i=1}^{n} a_i \mu_{t-i} + \sum_{i=1}^{n} b_i X_{t-i} + \text{con}_\mu + \text{trend}_\mu \cdot t + \boldsymbol{a}_{\mu_t}$$

$$X_t = \sum_{i=1}^{n} c_i \mu_{t-i} + \sum_{i=1}^{n} d_i X_{t-i} + \text{con}_X + \text{trend}_X \cdot t + \boldsymbol{a}_{X_t}$$

where $\boldsymbol{a}_{\mu_t}$ and $\boldsymbol{a}_{X_t}$ are again least squares residuals and con_μ, trend_μ, con_X, and trend_X are constants. The restriction (5) can easily be shown to imply the following restrictions across these constants:

$$\text{con}_\mu = \frac{(1 + \alpha d_1)\,\text{con}_X + \alpha\,\text{trend}_X}{(1 - \alpha c_1)} \tag{17}$$

$$\text{trend}_\mu = \frac{(1 + \alpha d_1)\,\text{trend}_X}{(1 - \alpha c_1)}. \tag{18}$$

These restrictions hold in addition to restrictions (12). In model two, we imposed (17) and (18), so that model two takes into account the restrictions that (5) imposes across trend terms.

Model three is simply

$$\mu_t = \sum_{i=1}^{n} a_i \mu_{t-i} + \sum_{i=1}^{n} b_i X_{t-i} + \text{con}_\mu + \boldsymbol{a}_{\mu_t}$$

(*Continued*)

The rate of money creation, μ_t, is taken to be the first difference of the natural log of the money stock, M_t, which equals the sum of Reichbank notes, Loan Bank certificates called *Darlehnskassenscheine,* currency called *Reichskassenscheine,* private bank notes, and coins. The series is available (*ZzG*, p. 45) as an end of month nominal quantity throughout the hyperinflation. In addition, during 1923 the gold value of the money stock is available weekly. Cagan generated a midmonth money stock series by using Mark-Dollar exchange data to convert the 1923 series to nominal terms and by interpolating to obtain midmonth estimates. For months before 1923, Cagan interpolated linearly; for the first half of 1923 he interpolated log linearly; and for the second half of 1923 he interpolated double log linearly. In this study a log linear interpolation of the series is employed.

The data on money and prices for the Austrian, Hungarian, and Polish hyperinflations are essentially the same as employed by Cagan and are cited in Young [1925, VII], the *International Abstract of Economic Statistics, 1919–1930*, and Walre de Bordes [1924, pp. 48–50, pp. 88–89]. In the case of Austria, however, Cagan failed to observe that while the money stock data are end of month quotations, the price index quoted for each month is obtained at the end of the previous month. (See Walre de Bordes, [1925, p. 86]).

$$X_t = \sum_{i=1}^{n} c_i \mu_{t-i} + \sum_{i=1}^{n} d_i X_{t-i} + \text{con}_X + \boldsymbol{a}_{X_t}$$

where $\boldsymbol{a}_{\mu_t}$ and $\boldsymbol{a}_{X_t}$ are least squares residuals. Here any trend-like elements are forced to manifest themselves through the elements of A in (9′). Eigen values of A exceeding one would lead to "explosive" or trend-like sample paths for the (X, μ) process. The constraint implied by (5) was imposed across the constant terms. In effect, method three shares with method two the feature that it imposes the constraints implied by (5) across any trend components of (X_t, μ_t).

By way of summary, Table 1 reports our estimates of α for $n=2$ and $n=4$ under restrictions (12) and (12) and (16) jointly. Table 1 also reports Cagan's [1956] and Sargent's [1977] estimates for purposes of comparison. Table 1 reveals that the Method two and three estimates are generally more reasonable economically, and are larger in absolute value relative to their estimated asymptotic standard errors. This could be taken as evidence that the trend components of the (X, μ) process contain valuable information about α which method one ignores. (It could alternatively be interpreted as evidence that the two step procedure of method one — which involves detrending first, then computing the autoregression — is inappropriate and distorts the estimated autoregression.) Overall, the method two and three estimates are not as impressive as were Cagan's in terms of their estimated "t-ratios", but they are certainly encouraging relative to the method one estimates.

We now turn to a more detailed discussion of the estimation results.[6] Table 2 records the marginal significance values for the likelihood ratios calculated to test the two sets of restrictions (12) and (12) jointly with (16). If there are q such restrictions, the marginal significance is the probability that a random variable distributed $X^2(q)$ would realize a value greater than or equal to the computed likelihood ratio. Thus small marginal significance values offer stronger evidence to reject the hypothesis that the restrictions hold than do larger values. The estimates of the asymptotic standard errors were obtained by a three step procedure. First, W, the matrix of numerical second partial derivatives of the likelihood function with respect to the unconstrained parameters, was computed. Second, $-W$ was inverted via Gauss-Jordan elimination and an accuracy test performed — $(-W)^{-1}$ is the estimate of the asymptotic variance covariance matrix of the unconstrained parameters. Third, the variances and covariances of the constrained parameters were computed as appropriately weighted sums of the elements of $(-W)^{-1}$. Table 4 reports the method one results for Germany.

There are several characteristics of the method one results which merit description. In fourteen of seventeen of the constrained estimations the estimate of α is negative as economic theory would predict. However, the data apparently

[6] An appendix of tables of estimation results for all three methods is available from the authors on request. Tables 4–6 report the detailed results for Germany when the monthly average data set described in footnote 5 was employed.

TABLE 1

ESTIMATES OF α

Restrictions Imposed†	Method 1				Method 2			
	R $n=2$	R, E $n=2$	R $n=4$	R, E $n=4$	R $n=2$	R, E $n=2$	R $n=4$	R, E $n=4$
Germany (Cagan data) Feb. 1921–June 1923*	.651 (.16)	—	−335.77 (101.06)	−.128 (.57)	−133.62 (1456.4)	−4.28 (1.60)	−10.53 (7.64)	−3.64 (.71)
Germany (m.a. data) Sept. 1920–June 1923	−106.07 (966.03)	−4.776 (9.15)	−41.79 (180.11)	−21.322 (59.22)	−3.92 (1.25)	−3.95 (1.59)	−4.76 (1.26)	−4.01 (1.57)
Poland Aug. 1921–Nov. 1923	−2.039 (2.07)	−1.103 (1.26)	−2.258 (2.09)	−1.966 (2.01)	−1.27 (.77)	−1.61 (.50)	−1.69 (.53)	−1.34 (.49)
Hungary Dec. 1921–Feb. 1924	−7.745 (9.95)	−3.336 (2.75)	−25.565 (92.34)	−9.52 (11.82)	−5.39 (1.96)	−4.79 (1.66)	−4.87 (2.13)	−4.92 (1.38)
Austria April 1921–Aug. 1922	.577 (.30)	.596 (.761)			2.62 (2.31)	3.88 (7.70)		

Restrictions Imposed†	Method 3					
	R $n=2$	R, E $n=2$	R $n=4$	R, E $n=4$	Cagan's**	Sargent's
Germany (Cagan data) Feb. 1921–June 1923*	−6.03 (2.18)	−.35 (.70)	−4.90 (1.79)	−.018 (.39)	−5.46	−5.97 (4.62)
Germany (m.a. data) Sept. 1920–June 1923	−4.60 (2.09)	−.50 (.61)	−5.01 (1.73)	−6.67 (6.84)		
Poland Aug. 1921–Nov. 1923	−.74 (.42)	−1.13 (.82)	−.53 (.64)	−.87 (.56)	−2.30	−2.53 (.86)
Hungary Dec. 1921–Feb. 1924	1.68 (1.38)	−.03 (.49)	3.02 (3.56)	1.91 (1.65)	−8.70	−1.84 (.40)
Austria April 1921–Aug. 1922	9.20 (35.44)	−9.38 (55.61)			−8.55	−.311 (1.57)

* Periods are for estimates with $n=4$ (except for Austria, where period is for $n=2$). Periods for estimates with $n=2$ start two months earlier and end in same month listed here.

† R denotes only restriction (12) imposed, i.e., rationality.
R, E denotes restrictions (12) and (16) imposed jointly, i.e., rationality and exogeneity of inflation.

** Periods for Cagan's and Sargent's estimates do not precisely match those for ours.
Estimated asymptotic standard errors are in parentheses beneath each estimate.

contain very imprecise information as to the size of α. In most cases the absolute value of α is very small relative to the estimated asymptotic standard error of α. For example, with Cagan's German data and lag length equal to four a local likelihood maximum was discovered at which $\hat{\alpha}=-335.77$ with an estimated standard error of 101.06 and a value for likelihood of 108.619. But a second local maximum was discovered which gives $\hat{\alpha}=.445$ with standard error of .16 and a likelihood value of 106.025 — less than 2.5% smaller than that associated with the first maximum. Extreme flatness of the likelihood surface along the α axis was particularly characteristic of the German case and held whether Cagan's data or the monthly average data were employed. In fact, with the Cagan data and lag length two neither the direct search nor the Newton Raphson technique would converge.

By and large the data do *not* contain strong evidence to reject the validity of the constraints imposed on the autoregression by the model of money demand. In Table 2 are presented the marginal significance values for the test of the hypothesis that the equation (12) restrictions hold. Only for Hungary are the restrictions rejected by a 95% test for lag length two and four. For all other countries, a 95% test would *not* lead to rejection in the four lag case although a 95% test would lead to rejection in the two lag case when Cagan's data are used for Germany. Far stronger, however, is the evidence to reject the restrictions placed on the autoregression parameters by the model and the additional assumption that the inflation process is exogenous. Only for the cases of Austria and Poland would a 95% test *not* lead to rejection of the restrictions in the two lag case. In the four lag case, the restrictions would be rejected at the .95 level in four of the five cases tested.

TABLE 2

MARGINAL SIGNIFICANCES OF THE LIKELIHOOD RATIO STATISTICS COMPUTED TO TEST THE RESTRICTIONS IMPOSED BY THE MODEL

Method	1				2				3			
Lag Length	$n=2$		$n=4$		$n=2$		$n=4$		$n=2$		$n=4$	
†Restrictions – TYPE	R	R, E	R	R, E	R	R, E	R	R, E	R	R, E	R	R, E
†Restrictions – NUMBER	4	6	8	12	6	8	10	14	5	7	9	13
Germany (Cagan Data)	.035	—	.052	.057	.002	.000	.011	.000	.008	.000	.009	.000
Germany (m.a. Data)	.099	.000	.164	.000	.037	.000	.055	.000	.465	.000	.179	.000
Poland	.910	.057	.656	.006	.505	.017	.443	.139	.812	.009	.471	.188
Hungary	.005	.001	.010	.018	.021	.009	.008	.023	.013	.003	.190	.332
Austria	.307	.091	—	—	.122	.044	—	—	.718	.241	—	—

If q is the number of restrictions, each entry reports the probability that $X^2(q)$ is greater than or equal to the observed likelihood ratio.

† R denotes only restriction (12) imposed, i.e., rationality.

R, E denotes restrictions (12) and (16) imposed jointly, i.e., rationality and exogeneity of inflation.

A key feature of the estimates is altered by the method two estimation: in general, the reestimates of α are not only of the correct sign but also much sharper relative to estimated standard errors. Some of the method 2 point estimates are similar to those of Cagan. For Germany, Cagan's estimate for α is -5.46 with a .90 confidence interval of $(-6.13, -5.05)$. With Cagan's data in the four lag case, our estimate for α is -10.53 with a standard error equaling 7.64. For the monthly average data we estimate that α equals -4.76 with a standard error of 1.26. Cagan estimated that for Poland α equaled -2.30 with a .90 confidence interval of $(-3.94, -1.74)$. Again our results are similar and for the four lag case we estimate that α equals -1.69 with a standard error of .53. Cagan found that money demand was most sensitive to anticipated inflation in the Hungarian case. For Hungary he estimated that α equaled -8.70 but found that the estimate was imprecise relative to those for other countries. However, our Hungarian estimates are similar to those for Germany: We estimate that α equals -4.87 with a standard error of 2.13. Finally, in the case of Austria Cagan's estimate of α is -8.55 with a .90 confidence interval of $(-31.0, -4.43)$. In the two lag case, our estimate of α is 2.62 with a standard error of 2.21. However, in one sense it is fair to say that the Austrian data do *not* contain much information about α since setting $\alpha = -1000$, and permitting the algorithm to choose the autoregression parameters led only to a 1.40% decrease in likelihood. The method two estimation results for Germany are reported in Table 5.

The results thus differ somewhat from those reported in Sargent [1977]. Sargent demonstrated that if Cagan's adaptively formed expectations are rational, α is *not* econometrically identifiable. However, conditional *not only* on the assumption that adaptive expectations are rational *but also* on the assumption that shocks to the money demand and supply schedules are contemporaneously uncorrelated, Sargent showed that α could be consistently estimated. Sargent's estimates are less sharp than those reported here for Germany and Poland and sharper than those for Hungary. It is worthwhile to point out again that it is possible to identify α in our model without the zero correlation assumption required by Sargent precisely because we assume that expectations are rational but *not* adaptive.

The likelihood ratio statistics for method two are reported in Table 2. In two of four cases with lag length four one would reject at .95 significance level the hypothesis that restrictions placed by the model of money demand across the autoregression parameters are true. The evidence is significantly stronger to reject the joint hypothesis that two sets of restrictions hold: those placed by the money demand model under rational expectations, and those placed by the assumption that the rate of inflation is exogenous. The joint hypothesis would be rejected at the .95 level for each country in the two lag case and for all countries except Poland in the four lag case. Nevertheless, even when exogeneity is imposed the method two estimation returns correctly signed estimates of α that for all countries except Austria are sharp relative to those returned by method one.

The method three results have two distinctive features. First, likelihood ratio statistics in Table 2 show that the evidence is *not* strong to reject the restrictions imposed by the Cagan model under rational expectations (but is much stronger to reject the joint hypothesis that the portfolio balance model (1) and (4) is true, that expectations are rational, and that the rate of inflation is exogenous). Second, the estimates of α returned by this method are *not* as clearly in accord with economic theory as those returned by method two. For Germany (with either data set), the estimate of α is appropriately signed and fairly sharp: for Cagan's data with $n=4$, $\hat{\alpha}=-4.90$ with a standard error of 1.79; for the moving average data with $n=4$, $\hat{\alpha}=-5.01$ with a standard error of 1.73. But for Poland, Hungary, and Austria the estimates are always small relative to estimated standard errors and most often of the wrong sign. The method three estimation results for Germany are reported in Table 6.

3. STABILITY AND HYPERINFLATION

Cagan estimated a "stability parameter" which indicated that the hyperinflations were *not* self-generating but rather sustained by the rapid money creation undertaken by the central banks of the hyperinflation countries. A different but related matter concerning stability may be easily addressed in the context of our treatment of the Cagan model. In estimating equation (9) as constrained by the model of money demand under rational expectations, we estimated a feedback rule for the money supply — an estimate of the decision rule followed by the hyperinflation country monetary authority linking the current rate of money creation to past rates of money creation and inflation. For example, for Germany the estimated money creation rule was estimated by method two to be:

$$\begin{aligned}\mu_t = {} & \underset{(.21)}{1.03}\,\mu_{t-1} - \underset{(.28)}{.75}\,\mu_{t-2} + \underset{(.30)}{1.06}\,\mu_{t-3} - \underset{(.20)}{.56}\,\mu_{t-4} \\ & + \underset{(.30)}{.15}\,X_{t-1} + \underset{(.05)}{.30}\,X_{t-2} - \underset{(.05)}{.07}\,X_{t-3} + \underset{(.05)}{.05}\,X_{t-4} - \underset{(.010)}{.009} + \underset{(.001)}{.001}\,t.\end{aligned}$$

In addition, we estimated subject to the constraints imposed by the model the projection of the current inflation rate on past observations of the rates of inflation and money creation. It is this projection which the rational agent would use to predict the one period forward rate of inflation. For Germany the estimated projection is:

$$\begin{aligned}X_t = {} & \underset{(.25)}{.57}\,\mu_{t-1} - \underset{(.15)}{.18}\,\mu_{t-2} + \underset{(.18)}{.39}\,\mu_{t-3} - \underset{(.22)}{.40}\,\mu_{t-4} \\ & + \underset{(.07)}{.12}\,X_{t-1} + \underset{(.07)}{.01}\,X_{t-2} - \underset{(.05)}{.01}\,X_{t-3} + \underset{(.04)}{.04}\,X_{t-4} - \underset{(.068)}{.003} + \underset{(.004)}{.007}\,t.\end{aligned}$$

Because it is hypothesized that expectations are formed rationally, the self-generating inflation which Cagan considered is *not* possible in this model. It is an implication of the model under rational expectations that the expected rate

TABLE 3

ESTIMATION OF THE DOMINANT EIGEN VALUE OF THE UNCONSTRAINED AUTOREGRESSION PARAMETER MATRIX FOR EUROPEAN HYPERINFLATIONS

Country	Lag Length	Method Two Estimates		Method Three Estimates (with TREND suppressed)	
		Dominant Eigen Value Modulus	A.S.E.	Dominant Eigen Value Modulus	A.S.E.
Germany	2	.660	(.250)	.959	(.066)
(Cagan data)	4	1.007	(.095)	.999	(.100)
Germany	2	.698	(.232)	.982	(.072)
(Monthly average data)	4	1.059	(.129)	1.065	(.189)
Poland	2	1.087	(.159)	1.208	(.092)
	4	1.140	(.124)	1.254	(.082)
Hungary	2	.764	(.052)	.624	(.112)
	4	.915	(.093)	.804	(.098)
Austria	2	.808	(.135)	.730	(.160)

ESTIMATION OF THE DOMINANT EIGEN VALUE OF THE CONSTRAINED AUTOREGRESSION PARAMETER MATRIX FOR EUROPEAN HYPERINFLATION

Country	Lag Length	Method Two Estimates		Method Three Estimates (with TREND suppressed)	
		Dominant Eigen Value Modulus	A.S.E.	Dominant Eigen Value Modulus	A.S.E.
Germany	2	.995	(.057)	.959	(.050)
(Cagan data)	4	.928	(.047)	.984	(.056)
Germany	2	.864	(.098)	.967	(.050)
(Monthly average data)	4	.944	(.073)	.960	(.057)
Poland	2	1.034	(.228)	1.183	(.092)
	4	.872	(.969)	1.125	(1.149)
Hungary	2	.614	(.144)	.997	(.047)
	4	.773	(.224)	.994	(.035)
Austria	2	.851	(.117)	.964	(.134)

of inflation is a weighted sum of future expected rate of money creation.[7] Thus if the monetary authority were known to have ceased increasing the supply of money, our model implies that the hyperinflation would end. In Cagan's model, a self-generating inflation was possible precisely because expectations of inflation could persist even after the government ceased printing money.

It is possible, however, to address a related stability question: Do our estimates of the monetary feedback rule and the elasticity of demand for money with respect to expected inflation suggest that the hyperinflation was bound to explode? Put another way, did the monetary authority choose a rule which, given the money holding behavior of agents, implied an ever accelerating inflation

[7] See, for example, Sargent and Wallace [1973].

rate? This stability question may be answered by estimating the eigen values of the parameter matrix A of the vector autoregression of Z_t given by equation (9′). The system will be stable if and only if the largest eigen value of A has modulus less than 1.0. The eigen values were estimated both for the constrained and

TABLE 4

METHOD ONE ESTIMATION OF THE AUTOREGRESSIVE REPRESENTATION OF Z_t

Germany — Monthly Average Data

$n=2$

July, 1920–June, 1923

Variable	Uncon-strained Estimates	Asymptotic Standard Error	Estimates Constrained By (12)	Asymptotic Standard Error	Estimates Constrained By (12) and (16)	Asymptotic Standard Error
a_1	.81	(.35)	.55	(.33)	—	
a_2	−.05	(.28)	.16	(.26)	—	
b_1	.23	(.04)	.21	(.05)	.234	(.07)
b_2	−.05	(.08)	.04	(.08)	.228	(.07)
c_1	2.17	(1.41)	.02	(.23)	—	
c_2	−1.78	(1.16)	.005	(.05)	—	
d_1	.20	(.19)	.008	(.07)	.081	(.13)
d_2	−.76	(.34)	.001	(.01)	.042	(.07)
α	—		−106.07	(966.03)	−4.776	(9.15)

$n=4$

September, 1920–June, 1923

Variable	Uncon-strained Estimates	Asymptotic Standard Error	Estimates Constrained By (12)	Asymptotic Standard Error	Estimates Constrained By (12) and (16)	Asymptotic Standard Error
a_1	1.20	(.30)	.98	(.20)	—	
a_2	−1.19	(.50)	−.71	(.28)	—	
a_3	1.62	(.41)	1.11	(.31)	—	
a_4	−.86	(.34)	−.62	(.21)	—	
b_1	.166	(.04)	.12	(.03)	.125	(.05)
b_2	−.059	(.09)	.03	(.05)	.306	(.05)
b_3	−.007	(.13)	−.08	(.05)	.016	(.05)
b_4	−.003	(.13)	.06	(.05)	.381	(.06)
c_1	1.77	(1.74)	.08	(.33)	—	
c_2	−3.74	(3.19)	−.02	(.10)	—	
c_3	4.04	(2.24)	.05	(.21)	—	
c_4	−2.00	(2.17)	−.06	(.27)	—	
d_1	.330	(.22)	.01	(.06)	.038	(.11)
d_2	−.708	(.58)	.002	(.01)	.032	(.09)
d_3	.522	(.91)	−.001	(.01)	.018	(.05)
d_4	−.494	(.95)	.006	(.03)	.018	(.05)
α	—		−41.79	(180.11)	−21.322	(59.72)

unconstrained cases. Also the eigen values were estimated both for the method two estimates (when a trend coefficient was estimated jointly with the A matrix

TABLE 5

METHOD TWO ESTIMATION OF AUTOREGRESSIVE REPRESENTATION OF Z_t

Germany — Monthly Average Data

$n=2$

July, 1920–June, 1923

	Unconstrained		Constrained by (12)		Constrained by (12) & (16)	
	Estimates	A.S.E.	Estimates	A.S.E.	Estimates	A.S.E.
a_1	.65	(.34)	.66	(.32)	—	—
a_2	.07	(.28)	.04	(.26)	—	—
b_1	.25	(.04)	.22	(.04)	.24	(.07)
b_2	−.02	(.08)	.02	(.08)	.23	(.07)
c_1	2.33	(1.47)	.43	(.19)	—	—
c_2	−1.91	(1.21)	.02	(.15)	—	—
d_1	.19	(.19)	.15	(.05)	.10	(.04)
d_2	−.80	(.35)	.01	(.05)	.05	(.02)
con_μ	−.011	(.017)	−.013	(.01)	−.06	(.03)
trend_μ	.001	(.001)	.001	(.001)	.007	(.002)
con_X	−.149	(.072)	−.017	(.06)	−.03	(.04)
trend_X	.021	(.006)	.007	(.004)	.011	(.002)
α			−3.92	(1.25)	−3.95	(1.59)

$n=4$

September, 1920–June, 1923

	Unconstrained		Constrained by (12)		Constrained by (12) & (16)	
	Estimates	A.S.E.	Estimates	A.S.E.	Estimates	A.S.E.
a_1	1.09	(.33)	1.03	(.21)	—	—
a_2	−1.20	(.44)	−.75	(.28)	—	—
a_3	1.42	(.50)	1.06	(.30)	—	—
a_4	−.61	(.37)	−.56	(.20)	—	—
b_1	.19	(.05)	.15	(.03)	.20	(.04)
b_2	−.02	(.08)	.03	(.05)	.36	(.05)
b_3	.03	(.08)	−.07	(.05)	.05	(.04)
b_4	.02	(.08)	.05	(.05)	.41	(.06)
c_1	1.73	(1.87)	.57	(.25)	—	—
c_2	−3.73	(2.51)	−.18	(.15)	—	—
c_3	4.04	(2.86)	.39	(.18)	—	—
c_4	−1.98	(2.09)	−.40	(.22)	—	—
d_1	.34	(.27)	.12	(.07)	.41	(.12)
d_2	−.70	(.43)	.01	(.07)	.30	(.04)
d_3	.53	(.45)	−.01	(.05)	.16	(.02)
d_4	−.49	(.45)	.04	(.04)	.12	(.03)
con_μ	−.010	(.02)	−.009	(.010)	.010	(.014)
trend_μ	.001	(.002)	.001	(.001)	−.002	(.001)
con_X	−.132	(.09)	−.003	(.068)	−.037	(.088)
trend_X	.021	(.009)	.007	(.004)	.003	(.006)
α	—	—	−4.76	(1.26)	−4.01	(1.57)

TABLE 6

METHOD THREE ESTIMATION OF AUTOREGRESSIVE REPRESENTATION OF Z_t
(Trend Suppressed)

Germany — Moving Average Data

$n=2$

July, 1920–June, 1923

	Unconstrained		Constrained by (12)		Constrained by (12) & (16)	
	Estimates	A.S.E.	Estimates	A.S.E.	Estimates	A.S.E.
a_1	.54	(.31)	.57	(.28)	—	—
a_2	.19	(.24)	.15	(.22)	—	—
b_1	.27	(.04)	.24	(.04)	.35	(.07)
b_2	.01	(.07)	.04	(.07)	.34	(.07)
c_1	.22	(1.57)	.52	(.21)	—	—
c_2	.39	(1.20)	.11	(.14)	—	—
d_1	.53	(.19)	.20	(.06)	.36	(.06)
d_2	−.25	(.37)	.03	(.04)	.26	(.09)
con_μ	.0002	(.009)	.001	(.009)	.026	(.019)
trend_μ	—	—	—	—	—	—
con_X	.069	(.046)	.076	(.047)	.032	(.026)
trend_X	—	—	—	—	—	—
α	—	—	−4.60	(2.09)	−.50	(.61)

$n=4$

September, 1920–June, 1923

	Unconstrained		Constrained by (12)		Constrained by (12) & (16)	
	Estimates	A.S.E.	Estimates	A.S.E.	Estimates	A.S.E.
a_1	1.04	(.27)	1.05	(.20)	—	—
a_2	−1.23	(.39)	−.80	(.28)	—	—
a_3	1.39	(.41)	.96	(.29)	—	—
a_4	−.51	(.29)	−.42	(.18)	—	—
b_1	.20	(.04)	.16	(.03)	.20	(.04)
b_2	−.002	(.06)	.03	(.05)	.32	(.05)
b_3	.06	(.06)	−.05	(.05)	.03	(.04)
b_4	.04	(.06)	.07	(.05)	.34	(.05)
c_1	.85	(1.72)	.70	(.29)	—	—
c_2	−4.42	(2.54)	−.23	(.19)	—	—
c_3	3.37	(2.54)	.48	(.21)	—	—
c_4	−.12	(1.85)	−.37	(.25)	—	—
d_1	.61	(.22)	.18	(.08)	.53	(.08)
d_2	−.30	(.37)	.04	(.07)	.30	(.04)
d_3	1.06	(.38)	.01	(.05)	.14	(.05)
d_4	−.12	(.41)	.06	(.04)	.08	(.06)
con_μ	−.0002	(.008)	.002	(.008)	−.007	(.010)
trend_μ	—	—	—	—	—	—
con_X	.050	(.051)	.080	(.054)	.003	(.006)
trend_X	—	—	—	—	—	—
α	—	—	−5.01	(1.73)	−6.67	(6.84)

parameters) and for the method three estimates (where the trend was suppressed). The estimates and their standard errors are reported in Table 3.[8]

Surprisingly, neither the method two nor the method three evidence strongly suggests that the system is unstable. For method three it was estimated that the modulus of the dominant eigen value is slightly less than one for each country except Poland. However, it is true that the estimated values lie less than one standard error below unity. As might be expected, the moduli of the dominant eigen values are in general smaller for round two where the trend term is *not* suppressed. In several round two cases (notably Hungary and Austria with lag length two), the estimated moduli are significantly less than one. A comparison of the two parts of Table 3 demonstrates that imposing the restrictions implied by the money demand model under rational expectations apparently has a slightly stabilizing effect since it is more often true that the dominant eigen values have a smaller modulus in the constrained case than in the corresponding unconstrained case.

4. CONCLUSION

The reader can undoubtedly draw conclusions from our estimates of α and the likelihood ratio statistics that we present as competently as we can. For what it is worth, our own conclusions are these. If one takes our method 2 and 3 results, Cagan's model does in some sense survive our attempt to estimate the model using procedures not subject to the simultaneity bias characterizing Cagan's own estimator. Most, though by no means all, of the estimates of α in Table 1 are of the correct sign. On the other hand, the standard errors around our estimates typically indicate much wider confidence bands than those reported by Cagan.

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[8] The standard errors of the moduli of the dominant eigen values were estimated by a procedure described in Theil and Boot [1962]. It should be pointed out that the Theil-Boot procedure is valid only under the hypothesis that the system is stable, that is, that the dominant eigen value has modulus less than 1.0.

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