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THE DEVELOPMENT OF UTILITY THEORY. II

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C. THE BERNOULLI HYPOTHESIS

The precise shape of the utility function received little attention in the main tradition of utility theory. Occasionally it was stated that the marginal utility of a necessity falls rapidly as its quantity increases and the like; and there were some mystical references to the infinite utility of subsistence. These were *ad hoc* remarks, however, and were not explicitly developed parts of the formal theory. Only one hypothesis about the marginal utility function ever achieved prominence: it was the Bernoulli hypothesis, which ultimately merged with the Weber-Fechner law, and to this literature we now turn.

In 1713 Nicholas Bernoulli proposed to a French mathematician, Montmort, five problems in probability theory,[113] one of which was equivalent to the following:

> Peter tosses a coin in the air repeatedly until it falls heads up. If this occurs on the first throw, he pays Paul \$1.00; if this occurs first on the second throw, he pays \$2.00; on the third throw, \$4.00; on the fourth throw, \$8.00; and on the nth throw, \$2.00^{n-1}. What is the maximum amount Paul should pay for this game?

Montmort replied, perhaps too easily, "Les deux derniers de vos cinq Problêmes n'ont aucune difficulté,"[114] for this was to become known as the St. Petersburg paradox.

Twenty-five years later Daniel Bernoulli introduced the paradox to fame.[115] Its paradoxical nature is easily explained: The probability of a head on the first throw is $\frac{1}{2}$, so the expected winning from the first throw is $\frac{1}{2}$ times \$1.00, or \$0.50. The probability of a first head on the second throw is $\frac{1}{4}$ ($\frac{1}{2}$ of tails on the first throw times $\frac{1}{2}$ of heads on the second), so the expected winning is $\frac{1}{4}$ times \$2.00, or \$0.50. The probability of a first head on the nth throw is $(\frac{1}{2})^n$, so the expected winnings are $(\frac{1}{2})^n$ times \$2.00^{n-1}, or \$0.50. Since these probabilities are exclusive, we add them to obtain the expected win-

[113] P. R. de Montmort, *Essay d'analyse sur les jeux de hazard* (2d ed.; Paris: Quillau, 1713), p. 402.

[114] *Ibid.*, p. 407.

[115] In *Specimen theoriae novae de mensura sortis;* references are to the German translation, *Versuch einer neuen Theorie der Wertbestimmung von Glücksfällen* (Leipzig: Duncker & Humblot, 1896).

nings from the game, which are \$0.50 times the infinite possible number of throws. Thus the expected winnings of Paul are infinity—an excessive price for Paul to pay for the game, as even the mathematicians saw.

Bernoulli's solution was to take into account the diminishing marginal utility of money. In the later words of Laplace, he distinguished the mathematical from the moral expectation of a chance event upon which a sum of money depended: the moral expectation was defined as the sum of the products of the various advantages accruing from various sums of money times their respective probabilities.[116] To Bernoulli, "it appears in the highest degree probable" that each equal increment of gain yields an advantage which is inversely proportional to the individual's wealth,[117] i.e.,

$$dU = k\,\frac{dx}{x},$$

where dU is the increment of utility resulting from an increment dx of wealth and k is a constant. It follows that total utility is a logarithmic function of wealth,

$$U = k \log \frac{x}{c},$$

where c is the amount of wealth necessary for existence.[118]

Bernoulli applied this formula to gambling, obtaining the now traditional result that mathematically fair bets are disadvantageous to both parties because the utility of the sum that may be gained is less than the utility of the sum that may be lost.[119] By a converse application, he calculated the maximum amount one should pay for insurance of specified risks.[120] Finally, he solved the paradox: a person with \$1,000 should pay \$6; etc.[121]

We should notice one further point in this beautiful memoir:

> If [the initial wealth] appears to be infinitely large relative to the greatest possible gain, the arc [of the total utility curve from initial wealth to initial wealth plus the gain] may be considered an infinitely short straight line, and in this case the usual rule [for calculating mathematical expectations] is again applicable. This case is closely approximated in all games in which relatively small sums are at stake.[122]

Thus Bernoulli suggested the assumption of a constant marginal utility of wealth for small variations of wealth.

We cannot follow the immense literature of the paradox in mathematics, but a few views may be noticed.[123] Some

[116] *Ibid.*, p. 27.

[117] *Ibid.*, pp. 27–28. Marshall properly remarked on the difficulties raised by the use of wealth instead of income (*Principles* [8th ed.], p. 842).

[118] On integrating the differential expression, we obtain

$$U = k \log x + \text{constant},$$

and the constant is determined by the condition that, when wealth is at the subsistence level c, $U = 0$.

[119] *Op. cit.*, pp. 39–40.

[120] *Ibid.*, pp. 42–44.

[121] The moral expectation of the individual with initial wealth a is

$$\begin{aligned} U &= \tfrac{1}{2}k \log \frac{a+1}{c} + \tfrac{1}{4}k \log \frac{a+2}{c} \\ &\qquad + \tfrac{1}{8}k \log \frac{a+4}{c} + \ldots \\ &= k \log \left(\frac{a+1}{c}\right)^{1/2} \left(\frac{a+2}{c}\right)^{1/4} \\ &\qquad \times \left(\frac{a+4}{c}\right)^{1/8} \ldots \\ &= k \log \frac{v}{c}, \end{aligned}$$

where v is the sum of money whose utility equals the moral expectation. Hence

$$v = (a+1)^{1/2}(a+2)^{1/4}(a+4)^{1/8}\ldots$$

and $(v - a)$ is the sum of money whose utility equals the expected gain of utility from playing the game.

[122] *Op. cit.*, p. 33.

[123] For the eighteenth century see I. Todhunter, *A History of the Mathematical Theory of Probability* (London: Macmillan, 1865).

mathematicians—the foremost was Laplace[124]—accepted Bernoulli's solution. Some, like Poisson, solved the problem by taking into account Peter's inability to pay if he had a sufficiently long run of tails, so Paul should pay an amount for the game determined by Peter's fortune.[125] Perhaps the most amusing solution was one by Buffon, which was based on the "lemma" that all probabilities smaller than .0001 are equal to zero (because this was the probability of dying during the day for a man of fifty-six, which was commonly treated as negligible).[126] Cournot, here as in demand theory, refused to look at utility and resorted to the market evaluation of the game.[127]

Perhaps the most surprising characteristic of this literature to the economist is the mathematicians' chief requisite of a solution: that a finite value be found for the value of the game. This is the only merit one can attach to the "limited-fortune" solution of Poisson and others, and even its spurious plausibility depends upon the particular formulation of the problem.[128] Bernoulli was right in seeking the explanation in utility (or alternatively, as Cournot did, in market appraisals), and he was wrong only in making a special assumption with respect to the shape of the utility curve for which there was no evidence and which he submitted to no tests.[129]

In 1860 this line of thought was joined by the independent series of researches that culminated in the Weber-Fechner law. E. H. Weber had proposed the hypothesis: the just noticeable increment to any stimulus is proportional to the stimulus (R — *Reiz*), or

$$\frac{dR}{R} = k .$$

Fechner made this constant of just noticeable differences the unit of sensation (S), to obtain

$$dS = C \frac{dR}{R},$$

or, integrating, $S = C \log R/R_0$, where R_0 is the threshold of sensation. Fechner performed a vast number of experiments on weight, temperature, tonal, and other types of discriminations which the formula fitted fairly well, and in the process he devised several methods of measurement (such as the constant method, in which Weber's k is determined by the

[124] *Théorie analytique des probabilités* (3d ed.; Paris: Gauthier-Villars, 1886), pp. xix–xx, chap. x.

[125] S. D. Poisson, *Recherches sur la probabilité des jugements* (Paris: Bachelier, 1837), pp. 74–76. Thus if $F = 2^k$ is Peter's fortune, Paul's expected winnings are

$$\tfrac{1}{2}\cdot 1 + \tfrac{1}{4}\cdot 2 + \ldots + \frac{1}{2^k}\cdot 2^{k-1} + 2^k \times\left(\frac{1}{2^{k+1}} + \frac{1}{2^{k+2}} + \ldots\right) = \frac{k}{2} + 1 .$$

[126] Todhunter, *op. cit.* At the present time the critical probability is .00005.

[127] *Exposition de la théorie des chances* (Paris: L. Hachette, 1843), pp. 108–9, 334. He reformulated the problem: the state (chosen to avoid Poisson's solution) issues tickets: No. 1 pays $1.00 if the first throw is heads; No. 2 pays $2.00 if the first heads comes on the second throw; etc. He argued that no one would buy the high-numbered tickets.

[128] J. Bertrand was surely right in this respect: "If one plays with centimes instead of francs, with grains of sand instead of centimes, with molecules of hydrogen instead of grains of sand, the fear of insolvency may be reduced without limit" (*Calcul des probabilités* [Paris: Gauthier-Villars, 1889], p. 64). Alternatively, one may alter the game, increasing the probability of longer runs and decreasing the rewards correspondingly.

[129] The arbitrariness is illustrated by the fact that the Genevese mathematician, Cramer, had suggested that the utility of income be taken as proportional to the square root of income, in a letter to Nicholas Bernoulli, from which Daniel Bernoulli quotes an extract (*op. cit.*, pp. 55 ff.). It should be noted that, unless the utility of income has an upper bound, it is possible to devise some variant of the St. Petersburg paradox which will have an infinite moral expectation.

proportion of [e.g.] "greater" to total responses in weight comparisons).[130] This was construed—by Fechner also—as proof of Bernoulli's hypothesis, with stimulus identified with income, sensation with pleasure.[131]

We need not follow the detailed evolution of psychologists' treatment of the Fechner law. For decades it was a lively topic of discussion,[132] but for a generation or more it has been declining in importance. Many exceptions have been found to Fechner's formula.[133] The concept of sensation has been severely restricted in meaning, and the form of response of a subject was found to affect his sensitivity.[134] At present Fechner's *Elemente* is important chiefly for the basic methods of measurement he invented and improved.

Many economists in this later period noticed the Bernoulli or Weber-Fechner "laws." The majority simply referred to the hypothesis, favorably or otherwise, and made no real use of the theory. In this group we may list Edgeworth,[135] Pareto,[136] and Wicksell,[137] as well as many lesser figures.[138]

Marshall took the Bernoulli hypothesis much more seriously than did any other leading economist. In 1890 he was prepared to apply it directly to whole income classes:

> If however it should appear that the class affected [by a particular event] in the one case is on the average, say, ten times as rich as in the other, then we shall probably not be far wrong in supposing that the increment of happiness measured by a given sum of money in the one case is, so far at least as its direct results go, about one-tenth as great as in the other.[139]

Whatever the reason, this use of the hypothesis disappeared in the second edition, but lesser evidences of Marshall's affection for the Bernoulli theory persisted.[140]

A group of writers on tax justice, mostly Dutch, made considerable use of the theory in discussions of the ideal rate of income-tax progression.[141] The enthusiasm for the Bernoulli hypothesis di-

[130] *Elemente der Psychophysik* (reprint; 2 vols.; Leipzig: Breitkopf & Härtel, 1889). See also E. G. Boring, *A History of Experimental Psychology* (New York: Appleton-Century, 1929), chap. xiii.

[131] *Psychophysik*, I, 236 ff.

[132] For a summary see E. B. Titchener, *Experimental Psychology* (New York: Macmillan, 1905), II, xiii–clxx.

[133] J. P. Guilford, *Psychometric Methods* (New York: McGraw-Hill Book Co., 1936), chaps. iv and v.

[134] H. M. Johnson, "Did Fechner Measure 'Introspectional' Sensations?" *Psychological Review*, XXXVI (1929), 257–84. Johnson reports a subject whose sensitivity was 18 per cent greater when distinguishing weights by voice than when distinguishing them by pushing the heavier weight toward the experimenter. It would be interesting to know the effect on sensitivity of pushing money.

[135] *Mathematical Psychics*, pp. 7, 62; *Papers*, I, 210; II, 107 ff. Edgeworth flirted with the theory at first but later rejected it as arbitrary and accepted the equally arbitrary view that the marginal utility of income falls faster than the Bernoulli hypothesis suggests.

[136] "Considerazioni . . . ," *Giornale degli economisti*, Series 2, VI (1893), 1–8. Pareto also deemed it arbitrary and pointed out that strictly it pertained to consumption, not to possessions.

[137] "Zur Verteidigung der Grenznutzenlehre," *Zeitschrift für die gesamte Staatswissenschaft*, LVI (1900), 580. Wicksell thought the Weber-Fechner work might eventually permit interpersonal comparisons of utility.

[138] E.g., O. Effertz, *Les Antagonismes économiques* (Paris: Giard & Bière, 1906), pp. 30–32; he encountered the theory first at a beer party where a professor of physiology made a "humorous and detailed application to the consumption of beer" (F. A. Lange, *Die Arbeiterfrage* [5th ed.; Winterthur: Ziegler, 1894], pp. 113 ff., 143 ff.; F. A. Fetter, *Economic Principles* [New York: Century, 1915], pp. 40–41).

[139] *Principles* (1st ed., 1890), pp. 152–53; also p. 180.

[140] *Principles* (8th ed., 1920), pp. 135, 717, 842–43.

[141] For references and summaries see E. Sax, "Die Progressivsteuer," *Zeitschrift für Volkswirtschaft, Sozialpolitik und Verwaltung*, I (1892), 43 ff.

minished when it was discovered that it led to proportional taxation under the equal sacrifice doctrine (each taxpayer to sacrifice an equal amount of utility).[142] Although the doctrine of proportional sacrifice (each taxpayer to sacrifice an equal proportion of his utility) leads to progressive taxation with the Bernoulli utility function,[143] the minimum sacrifice doctrine (which insured progression if the marginal utility of income diminished) soon triumphed.

Two Italian writers used the logarithmic law in quantitative work: Gini, in the analysis of demand;[144] del Vecchio, in the analysis of budgetary data.[145] These studies belong in the history of demand theory, however; and we shall not discuss them here.

Max Weber's famous essay on the Weber-Fechner law is commonly, and perhaps properly, interpreted as a final demonstration that economists can ignore this law. Weber had three main points. First, the law does not hold in all cases ("Tiffany-Vasen, Klosettpapier, Schlackwurst, Klassiker-Ausgaben, Prostituierten . . ."). Second, the law refers to psychical reactions to external stimuli, whereas economics deals with observable behavior in response to subjective needs. Third, economics can get along with the empirical fact that man has limited means to satisfy competing ends and can allocate these means rationally to maximize the fulfilment of the ends.[146] This pungently written essay is hardly conclusive, however, on whether economists should adopt the law. This turns on whether it yields fruitful hypotheses concerning economic behavior. Since it does not,[147] it should not be used.

V. THE MEASURABILITY OF UTILITY

The first careful examination of the measurability of the utility function and its relevance to demand theory was made by Fisher.[148] He solved the measurability

142 If $U = k \log R$, a tax of T involves a sacrifice of

$$k \log \frac{R}{R-T}.$$

On the equal sacrifice doctrine,

$$k \log \frac{R}{R-T} = \text{constant} = c$$

$$\frac{R}{R-T} = e^{c/k}$$

so

$$\frac{T}{R} = e^{-c/k}\left(e^{c/k} - 1\right) = \text{constant}.$$

143 Using the notation of the previous footnote, the doctrine requires that

$$\frac{k \log \dfrac{R}{R-T}}{k \log R} = \text{constant} = m\ ,$$

or

$$\frac{R}{R-T} = R^{m},$$

whence

$$\frac{T}{R} = 1 - R^{-m}.$$

144 "Prezzi e consumi," *Giornale degli economisti*, Series 3, XL (1910), 99–114, 235–49.

145 "Relazioni fra entrata e consumo," *Giornale degli economisti*, Series 3, XLIV (1912), 111–42, 228–54, 389–439.

146 "Die Grenznutzlehre und das 'psychophysisches Grundgesetz'" (1908), reprinted in *Gesammelte Aufsätze zur Wissenschaftslehre* (Tübingen: Mohr, 1922). The fundamental argument is in the third paragraph (pp. 361–68).

147 As applied to commodities, it puts unrealistic limitations on the income elasticities; as applied to income, it implies that there will be no gambling.

148 Walras had already pointed out that only the ratios of the marginal utilities enter into demand analysis:

"What are v_a, v_b, v_c, . . . [the exchange values]? They are absolutely nothing but the indeterminate and arbitrary terms of which the ratios represent the common, identical ratios of the marginal utilities of all the commodities for all the exchangers in a state of general equilibrium of the market, and of which consequently only the ratios (taking the commodities two at a time)—equal to the ratios of the marginal utilities for every exchanger,—can be measured numerically. Thus value in exchange is essentially relative, being always based upon marginproblem quite satisfactorily for the case in which the marginal utilities of the various quantities are independent of one another.[149] His procedure was as follows:

Select arbitrarily a quantity of any commodity, say, 100 loaves of bread. Let the marginal utility of this quantity of commodity be the unit of utility (or util). Grant the ability of the individual to order the utilities of specified amounts of two goods, i.e., to indicate a preference (if one exists) or indifference between the two quantities. Then it is possible to construct the utility schedule of (say) milk. Start with no milk, and find the increment of milk (Δm_1) equivalent to the hundredth loaf of bread, i.e., the minimum amount of milk the individual would accept in exchange for the hundredth loaf of bread. Find a second increment (Δm_2), given the possession of Δm_1, equivalent to the hundredth loaf, etc. We obtain thus a schedule (or function) such as that given in Table 1. This function gives the amounts of milk necessary to obtain equal increments of utility; by interpolation we determine the amounts of utility obtained from equal increments of milk (Table 2).

TABLE 1

Symbol	Increment of Milk		
	Quantity (Cubic Inches)	Utility of Increment of Milk	Total Utility of Milk
Δm_1..........	3	1	1
Δm_2..........	4	1	2
Δm_3..........	5	1	3
Δm_4..........	6	1	4
Δm_5..........	7	1	5

TABLE 2

Milk (Cubic Inches)	Total Utility of Milk	Marginal Utility of Milk*
3..............	1.0000	....
6..............	1.7667	.7667
9..............	2.4333	.6667
12..............	3.0000	.5667
15..............	3.4667	.4667

* Per 3 cubic inches.

This initial choice of a unit is arbitrary, but this is not objectionable:

> Any unit in mathematics is valuable only as a divisor for a second quantity and constant only in the sense that the quotient is constant, that is, independent of a third quantity. If we should awaken tomorrow with every line in the universe doubled, we should never detect the change, if indeed such can be called a change, nor would it disturb our sciences or formulae.[150]

Suppose now that the marginal utility of milk depends not only upon the quantity of milk but also upon the quantities of bread and beer—more generally, suppose the generalized utility function of Edgeworth holds. We could proceed as before in finding the quantities of milk, Δm_1, Δm_2, . . . , whose utilities equaled that of the hundredth loaf of bread. Let us now shift to the marginal utility of (say) 60 bottles of beer as our unit and proceed in identical fashion to find $\Delta m_1'$, $\Delta m_2'$, . . . , and thus measure the utility of milk in terms of beer. We shall find the new increments of milk, $\Delta m_1'$, $\Delta m_2'$, . . . , are not proportional to the old,[151] because the marginal utilities of beer and of bread will vary differently as the quantity of milk increases. Hence the total utility

al utility, which alone is absolute" (*Éléments*, pp. 139–40). He dropped the discussion at this point.

An early analysis of utility functions was made by C. [G.?] B. Antonelli, *Sulla teoria matematica della economia politica* (Pisa: Folchetto, 1886); the librarian of Columbia University has not been able to find a copy in the United States.

[149] *Op. cit.*, pp. 11 ff.

[150] *Ibid.*, p. 18.

[151] That is, $\Delta m_1 : \Delta m_2 : \Delta m_3 : \ldots$ will not equal $\Delta m_1' : \Delta m_2' : \Delta m_3' : \ldots$.

curve of milk will take on an entirely new shape, and not merely differ by a proportionality factor, when we change the commodity in terms of which it is measured. Thus we can no longer use this procedure to measure utility.[152]

Fisher concludes his brilliant dissertation with the argument that the total utility function cannot in general be deduced from the indifference curves and that, for purposes of explaining consumers' reactions to prices and income changes, there is no occasion to introduce total utility:

> Thus if we seek only the causation of the *objective facts of prices and commodity distribution* four attributes of utility as a quantity are entirely unessential, (1) that one man's utility can be compared to another's, (2) that for the same individual the marginal utilities at one consumption-combination can be compared with those at another, or at one time with another, (3) even if they could, total utility and gain might not be integratable, (4) even if they were, there would be no need of determining the constants of integration.[153]

Fisher's statement of the difficulty of constructing total utility functions from differential equations of the indifference curves was extremely concise,[154] and we shall elaborate it in connection with Pareto. We may note in passing that thirty-five years later Fisher qualified much of this argument. He was now willing to assume independence of utilities (at least for broad categories such as food and housing) and comparability of utilities of different persons—in order, apparently, to achieve concrete results applicable to income taxation.[155]

[152] Fisher, *op. cit.*, p. 67.

[153] *Ibid.*, p. 89.

[154] *Ibid.*, pp. 74–75, 88–89.

[155] See "A Statistical Method of Measuring 'Marginal Utility' and Testing the Justice of a Progressive Income Tax," in *Economic Essays Contributed in Honor of John Bates Clark* (New York: Macmillan, 1927), pp. 157 ff.

Pareto was the great proponent of doubts on the existence of unique utility functions and of the relevance of such functions to economic behavior. Apparently independently of Fisher, Pareto noticed the problem of the existence of a utility function as early as 1892.[156] Soon thereafter most of his basic mathematical theory was developed.[157] The import of the theory was realized only slowly, however: in the *Cours* (1896 and 1897) he was still willing to accept the interpersonal comparison of utilities for welfare pur-

[156] "Considerazioni . . . ," *Giornale degli economisti*, Series 2, IV (1892), 415. He refers casually to the fact that when the differential equation of the indifference curve is of the form

$$Q\,(x,\,y)\,dx + R\,(x,\,y)\,dy\ ,$$

"it may happen that $P[R]$ and Q are not partial derivatives of the same function and then the function will not exist." This was not quite correct: in the two-commodity case there always exists an integrating factor.

[157] "Considerazioni . . . ," *Giornale degli economisti*, Series 2, VII (1893). He introduces the index functions (p. 297), recognizes that it is always possible to integrate the differential equations when the marginal utilities are independent, and presents the integrability condition for the three-commodity case (p. 300). Let the differential equation of the indifference surface be

$$dx_1 + R\,dx_2 + S\,dx_3 = 0\ .$$

Then Pareto gives the integrability condition:

$$\frac{\partial R}{\partial x_3} = \frac{\partial S}{\partial x_2}.$$

He should have given,

$$\frac{\partial R}{\partial x_3} - \frac{\partial S}{\partial x_2} = S\,\frac{\partial R}{\partial x_1} - R\,\frac{\partial S}{\partial x_1}.$$

He also corrected the statement in the last footnote: "If there are only two economic goods, equation (52) is always integrable" (p. 299 n.). Subsequently he forgot this again (*Manuale di economia politica* [Milan: Piccola Biblioteca Scientifica, 1919—first published in 1906], pp. 499 ff.). He was gently reminded of it by V. Volterra, "L'Economia matematica," *Giornale degli economisti*, Series 2, XXXII (1906), 296–301.

poses.[158] In the *Manuel* (1909), however, measurable utility had fallen into the background—of his theory, if not of his exposition. For Pareto, two questions on measurability were at issue.

The first, and to Pareto the major, problem is this: We can deduce the slopes of indifference curves at (in principle) all possible combinations of goods from budgetary data, because the slopes of the price lines equal the ratios of the marginal utilities (slopes of indifference curves). Thus we obtain empirically the differential equation of the indifference curves. Can we integrate it to obtain the equation of the indifference curves?

Before we look at the mathematics, we may present the problem verbally. Will the choices that an individual makes between combinations of goods differing by infinitesimal amounts be consistent with the choices he makes between combinations differing by finite amounts? For example, the individual starts with the combination $100X_1$, $100X_2$, $100X_3$. By infinitesimal steps we obtain an infinite number of combinations, each equivalent to the preceding, reaching ultimately the combination $90X_1$, $85X_2$, $120X_3$. Will the individual consider this last combination equivalent to the first? The intuitive answer usually is: Yes, he is consistent in his preferences. The mathematical answer is equivalent: If the preference system displays a proper continuity, the equation is integrable. If we postulate indifference surfaces, there is no problem: then by hypothesis the infinitesimal comparisons are consistent with discrete comparisons. Economists have usually been willing to admit that the individual can well display this type of consistency. Pareto at times did likewise.[159]

Mathematically, the issue is: Does the line integral of

$$f(x_1, x_2, x_3, \ldots)\,dx_1 + g(x_1, x_2, x_3, \ldots)\,dx_2 + h(x_1, x_2, x_3, \ldots)\,dx_3 + \ldots = 0\,,$$

exist independently of the path between the beginning and end points? Pareto's first two answers are Fisher's: (1) Yes, if f is a function only of x_1, g only of x_2. . . .[160] (2) Yes, if there exists an integrating factor, that is, if the integrability conditions are fulfilled.[161] He adds: (3) If the integrability conditions are not fulfilled, the integral depends on the order of integration, and if this is known the equation can be integrated.[162]

Pareto displayed a peculiar literalness of mind when he tried to translate this third case into economic terms. He identified the order of integration with the order of consumption of the goods.[163] This was absurd for precisely the same reason that dinner-table demonstrations of diminishing marginal utility are objectionable; they do not bear on the problems economics is interested in. Acts of consumption are of little concern; the purpose of the theory of consumption is to explain the pattern of consumption, not its episodes. Economics is usually in-

[158] *Cours d'économie politique* (Lausanne: Rouge, 1897), II, 47–48. The comparisons were limited to types or classes of people to avoid personal idiosyncrasies. The measurability problem was referred to only incidentally (*ibid.*, I, 10 n.).

[159] *Manuel*, pp. 169 n., 264.

[160] *Ibid.*, pp. 545–46, 555; "Économie mathématique," *Encyclopédie des sciences mathématiques* (Paris: Gauthier-Villars, 1911), I, iv, 614.

[161] *Manuel*, pp. 545 ff.; "Économie mathématique," *op. cit.*, pp. 598 ff. The equations are

$$f\left(\frac{\partial h}{\partial x_2} - \frac{\partial g}{\partial x_3}\right) + g\left(\frac{\partial f}{\partial x_3} - \frac{\partial h}{\partial x_1}\right) + h\left(\frac{\partial g}{\partial x_1} - \frac{\partial f}{\partial x_2}\right) = 0\,,$$

and similarly for all triplets of goods.

[162] *Manuel*, pp. 553 ff.

[163] *Ibid.*, pp. 251, 270, 539 ff.

terested only in the time rates of purchase and consumption of goods, and it is not interested in whether the soup precedes the nuts, or whether the consumer drinks three cups of coffee at breakfast or one after each meal, or pours them down the sink. The correct translation of the integrability problem was in terms of the consistency of consumer preferences, not of the temporal sequence of consumption.[164] Pareto indicated elsewhere that economics is interested in repetitive patterns of behavior, and we may view this discussion as a minor aberration.[165]

Given the indifference curves, we come to the second issue: Can we deduce a unique total utility surface? In general, "No." There are in general an infinite number of total utility surfaces whose contours constitute these indifference curves. If we construct one utility surface, we can get another by squaring the amounts of utility, another by taking the logarithm of utility, etc. So far as observable behavior is concerned, one utility surface will do as well as another. We shall return to this, Pareto's basic answer.

He gave also an introspective reply. We can construct a unique total utility function if the consumer can tell us the magnitude of the utility gained by moving from one indifference curve (I_1) to a second (I_2) relative to the utility gained by a move from I_2 to I_3. If he can tell us that the move from I_1 to I_2 gains (say) three times as much utility as the move from I_2 to I_3, then utility is "measurable." That is, if we have one utility surface, we may no longer submit it to transformations such as squaring the amount of utility—then we should have increased the utility of the move from I_1 to I_2 to *nine* times the utility of the move from I_2 to I_3. We can still take the utility function (U) and write it as ($aU + b$), but this merely says that the origin and unit of measurement are arbitrary for utility just as they are for length and other measurements.[166] But Pareto believed the consumer could not rank utility differences.

He did not adhere to these views with consistency. The *Manuel* is strewn with passages that are meaningful only if utility is measurable. Two examples will suffice: First, Pareto's definitions of complementary and competing goods were dependent on the measurability of utility.[167] Second, the marginal utility of income was discussed at length.[168]

Yet much of the foregoing discussion is a digression from the viewpoint of Pareto's mature theory of utility. This digression reflects the heavy hand of the past, and it is justified (rather weakly) chiefly on expository grounds.[169] Fundamentally, Pareto argued that the differential equation of the indifference surface is given by observation and that this is all that is necessary to derive the demand functions:

> The entire theory . . . rests only on a fact of experience, that is to say, on the determination of the quantities of goods which constitute combinations which are equivalent for the individual. The theory of economic science thus acquires the rigor of rational mechanics; it deduces its results from experience, without the intervention of any metaphysical entity.
>
> [Edgeworth] assumes the existence of utility (ophelimity) and from it he deduces the indifference curves; I instead consider as empirically given the curves of indifference, and I deduce

[164] Pareto might equally well have debated how one consumer can consume all goods at once, since the equality of marginal utilities divided by prices is a set of simultaneous equations.

[165] *Manuel*, p. 262.

[166] *Ibid*., pp. 264–65.

[167] See below, Sec. VI.

[168] *Manuel*, pp. 579 ff.

[169] *Ibid*., p. 160.

from them all that is necessary for the theory of equilibrium, without having recourse to ophelimity.[170]

Observations on demand consistent with any utility function φ will also be consistent with an arbitrary utility index-function $F(\varphi)$ so long as the order of preference among the combinations is preserved $[F'(\varphi) > 0]$.[171]

Two mathematicians consolidated this position, that all notions of measurable utility could be eliminated from economics. W. E. Johnson demonstrated that the variation of quantity purchased with price and income was independent of the measurability of utility:

This impossibility of measurement does not affect any economic problem. Neither does economics need to know the marginal (rate of) utility of a commodity. What is needed is a representation of the ratio of one marginal utility to another. In fact, this ratio is precisely represented by the *slope* of any point of the utility-curve [indifference curve].[172]

Johnson thereafter dealt only with ratios of marginal utilities.

Two years later E. E. Slutsky published his magnificent essay on the equilibrium of the consumer.[173] To put economics on a firm basis, "we must make it completely independent of psychological assumptions and philosophical hypotheses."[174] His utility function was accordingly an objective scale of preferences. Slutsky did not deny the interrelations of "economic" utility and "psychological" utility but sought to deduce empirical tests of any psychological hypotheses. If introspection suggests that the marginal utilities of commodities are independent, we can test the hypothesis by the equation it implies.[175] Slutsky assumes that the increment of utility obtained by moving from one combination to another is independent of the path of movement and offers an empirical test of its validity.[176] Conversely, he shows that a full knowledge of demand and expenditure functions is not sufficient in general to determine whether marginal utility diminishes.[177] The beauty and power of the essay are unique.

With Slutsky's development, intro-

[170] *Ibid.*, pp. 160, 169 n.; see also pp. 539–44.

[171] *Ibid.*, p. 542.

[172] "The Pure Theory of Utility Curves," *Economic Journal*, XXIII (1913), 490. Of course the first sentence is too strong. See M. Friedman and L. J. Savage, "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy*, LVI (1948), 279–304.

[173] "Sulla teoria del bilancio del consumatore," *Giornale degli economisti*, Series 3, LI (1915), 1–26.

E. E. Slutsky was born in 1880 in Novom, Yiaroslavskoi Gubernii, and died in Moscow on March 10, 1948. As a student of mathematics at the University of Kiev in 1901, "because of his participation in an illegal meeting he was drafted as a soldier, and only a large wave of protests by students in the big cities of the country forced the government to return him to the University in the same year. At the beginning of the next year, 1902, E. E. was dismissed from the University without the right to study in any institution of higher education. Only after 1905 was he able to return to the University of Kiev, but this time he entered the law school.

"This choice was dictated by E. E.'s desire to prepare himself for scientific work in the field of mathematical economics, an interest which he had developed from a thorough study of works of Ricardo, Marx, and Lenin. He finished at the law school in 1911, and received a gold medal for his final paper. However, because of his reputation for being 'unreliable' he was not asked to continue his academic career at the University." Thereafter he worked intensively in probability and mathematical statistics, teaching at the Institute of Commerce at Kiev from 1912 to 1926, when he went to Moscow "to work in a number of scientific research institutions of the capital."

This information is from N. Smirnov's obituary notice, *Izvestiya Akademiia Nauk SSSR* ("Mathematical Series"), XII (1948), 417–20, a translation of which was kindly made for me by Dr. Avram Kisselgoff.

[174] *Op. cit.*, p. 1.

[175] *Ibid.*, p. 25.

[176] *Ibid.*, pp. 3, 15–16. That is, the integrability condition is fulfilled.

[177] *Ibid.*, pp. 19–23.

spection no longer plays a significant role in utility theory. There is postulated a function which the consumer seeks to maximize, and the function is given the characteristics necessary to permit a maximum. This is perhaps subjective in origin: the notion of maximizing behavior was probably derived from introspection, although it need not be. Slutsky posits such a function merely because it contains implications that observation can contradict, and hence yields hypotheses on observable behavior. We shall return later to the question whether this is an efficient method of obtaining hypotheses.

We have been marching with the vanguard; we retrace our steps now and examine the views of the other leading economists of the period on measurability.

CONTEMPORARY PRACTICE

None of the other leading economists of this period rejected the measurability of utility; we may cite Wicksteed,[178] Wicksell,[179] Barone,[180] Edgeworth,[181] and Pigou.[182] It is true that by the end of the period the leading economists were realizing that measurability of utility was not essential to the derivation of demand curves, but they were loath to abandon the assumption. In part this reluctance was based on the desire to employ utility theory in welfare analysis; in part it was psychological theorizing. Yet with the passage of time, caution increased, as Marshall's evolution will illustrate.

Marshall was at first unqualified in his acceptance of the measurability of utility:

> Thus then the desirability or utility of a thing to a person is commonly measured by the money price that he will pay for it. If at any time he is willing to pay a shilling, but no more, to obtain one gratification; and sixpence, but no more, to obtain another; then the utility of the first to him is measured by a shilling, that of the second by sixpence; and the utility of the first is exactly double that of the second.
>
> The only measurement with which science can directly deal is that afforded by what a person is willing to sacrifice (whether money, or some other commodity, or his own labour) in order to obtain the aggregate of pleasures anticipated from the possession of the thing itself.[183]

Moreover, he fully accepted the intergroup comparisons of utility:

> Nevertheless, if we take averages sufficiently broad to cause the personal peculiarities of individuals to counterbalance one another, the money which people of equal incomes will give to obtain a pleasure or avoid a pain is an *extremely accurate* measure of the pleasure or the pain.[184]

Indeed, as we have already noticed, he believed that one can even compare the utilities of groups with different incomes, by using Bernoulli's hypothesis.

We need not trace in detail the growth of Marshall's caution and reticence in this area. He became unwilling to attribute precision to interpersonal comparisons.[185] The discussion of consumer surplus becomes increasingly defensive.

[178] *Common Sense of Political Economy*, I, 148 ff.; II, 470, 473, 661.

[179] *Lectures*, I, 29 ff., 221; he apparently did not fully understand the Pareto analysis (see his review of the *Manuel*, *Zeitschrift für Volkswirtschaft, Sozialpolitik, und Verwaltung*, XXII [1913], 136 ff.).

[180] *Principi di economia politica* (Rome: Bertero, 1908), pp. 12–13, 22–24.

[181] *Papers*, II, 473 n., 475.

[182] *Wealth and Welfare* (London: Macmillan, 1912), *passim*.

[183] *Principles* (1st ed.), pp. 151, 154 n.

[184] *Ibid.*, p. 152. (My italics.) See also *ibid.*, p. 179.

[185] The Bernoulli hypothesis is no longer applied to social classes. The "extremely accurate" comparison of groups with equal incomes becomes "there is not in general any very great difference between the amounts of the happiness in the two cases [two events with equal money measures]" (*Principles* [8th ed.], p. 131).

Probably because of the growing criticism of hedonism, many terminological changes are made: "benefit" for "pleasure"; "satisfaction" for "utility"; etc. Bentham's dimensions of pleasure were approved at first;[186] they lose their sponsor and place in the text.[187] The distinction between desires and realized satisfactions becomes prominent.[188] Yet Marshall seems never to have been seriously skeptical of the measurability of utility, and the changes in his exposition were not accompanied by any change in the fundamentals of his theory.

VI. COMPLEMENTARITY

Jevons had noticed the case of "equivalent" (substitute) commodities and implicitly defined them by the constancy of the ratio of their marginal utilities.[189] In this he was inconsistent, for he treated the marginal utility of X_1 as dependent only on the quantity of X_1 in his general theory, whereas if X_1 and X_2 are "equivalent," the marginal utility of X_1 depends also on the quantity of X_2. One cannot define the usual relationships among the utilities of commodities with an additive utility function, so the utility theory of complementarity had to wait for Edgeworth's generalization of the utility function. In fact, it had to wait a little longer, for Edgeworth glossed over this problem in the *Mathematical Psychics*.

The first formal definition of the relationship between utilities of commodities was given by the remarkable Viennese bankers, Auspitz and Lieben:

> The mixed differential quotient,
> $$\frac{\partial^2\phi}{\partial x_a \partial x_b},$$
> indicates what influence (if any) an algebraic increase in x_b—a larger purchase or a smaller sale of B—has on the utility of the last unit of A purchased or not sold. If we consider the simplest case, in which only A and B are consumed,
> $$\frac{\partial^2\phi}{\partial x_a \partial x_b} \gtreqless 0,$$
> according as B complements the satisfaction derived from A, has no influence on it, or competes with A.[190]

Fisher repeated this definition and illustrated certain limiting cases by indifference curves. He defined two commodities to be perfect substitutes if the ratio of the marginal utilities of the amounts "actually consumed" was absolutely constant; they were perfect complements if the quantities consumed were in a constant ratio.[191] Edgeworth gave the same criterion in 1897.[192]

Let us illustrate the use of this criterion with a numerical example. We may construct a table of total utilities as a function of the quantities of X_1 and X_2 and from it calculate the marginal utilities of X_1 (Table 3). Our example has

TABLE 3

Quantity of X_2	Total Utility Quantity of X_1: 1	2	Marginal Utility of X_1
1	3.0	5.4	2.4
2	5.4	9.0	3.6

[186] *Principles* (1st ed.), p. 153.

[187] *Principles* (8th ed.), p. 122 n.

[188] *Ibid.*, p. 92.

[189] *Theory of Political Economy*, p. 134.

[190] *Untersuchungen über die Theorie des Preises* (Leipzig: Duncker & Humblot, 1889), p. 482; see also pp. 154 ff., 170 ff.

[191] *Mathematical Investigation*, pp. 65–66, 69, 70–71. The definitions of these limiting cases are independent of the existence of a unique utility function.

[192] He was so punctilious in acknowledging predecessors that his tone suggests independence of discovery. See "The Pure Theory of Monopoly," reprinted in *Papers*, I, 117 n. His criterion differed in one detail—ϕ was the utility function in terms of money and hence involved the marginal utility of money (the complicating effects of which were not discussed). This was not inadvertent; he desired symmetry with the definition of complementarity of products in production (*ibid.*, I, 127; II, 123). The Auspitz and Lieben definition was given later (*ibid.*, II, 464).

been so chosen that the marginal utility of a given quantity of X_1 increases when the quantity of X_2 increases, hence X_1 and X_2 are complements.

Now let us construct a new table, in which total utility is equal to the logarithm of the total utility in Table 3. This is the kind of transformation we may make if utility is not measurable; it does not preserve the relative differences between utilities, but it preserves their order. We now find (Table 4) that by the

TABLE 4

Quantity of X_2	Total Utility Quantity of X_1 1	2	Marginal Utility of X_1
1.........	.4771	.7324	.2553
2.........	.7324	.9542	.2218

same criterion, X_1 and X_2 are substitutes. We have shown that the criterion is ambiguous if utility is not uniquely measurable.[193]

Perhaps Fisher was so casual on this point because he saw the dependence of the definition on the measurability of utility, and Edgeworth was unconcerned because he believed utility was measurable. But Pareto was inconsistent; he made extensive use of this definition at the same time that he was rejecting the measurability of utility.[194]

Marshall displayed greater inconsistency than Pareto, for he implicitly followed the Auspitz-Lieben definition even though he employed an additive utility function which did not permit of complementarity. Thus he speaks of "rival commodities, that is, of commodities which can be used as substitutes for it."[195] In the third edition this definition in terms of utility becomes reasonably explicit.[196] I suspect that Marshall was led into the inconsistency by his preoccupation with the role of rival and completing goods in production. That Pareto and Marshall adhered to the criterion is weighty testimony for its intuitive appeal.

W. E. Johnson supplied a definition of complementarity in terms of utility that was independent of the measurability of utility.[197] His criterion turned on the behavior of the slope of the indifference curve when one quantity was increased. That is to say, X_1 and X_2 are complements if the more of X_1 the individual possesses, the larger the increment of X_1 he will give up to obtain a unit of X_2.[198] For the fairly broad classes of commodities usually dealt with in budget studies, all commodities are probably complements on the Johnson definition. Slutsky

193 Equivalently, let φ be a utility function, $F[\varphi]$ a transformation of it such that $F' > 0$. Then

$$U = F[\varphi(x_1, x_2)]$$
$$U_1 = F'\varphi_1$$
$$U_{12} = F'\varphi_{12} + F''\varphi_1\varphi_2$$

so F'' must be zero—the transformation must be linear—if the sense of the definition is to be preserved.

194 *Manuel*, chap. iv, pp. 576 ff.

195 *Principles* (1st ed.), p. 160; see also pp. 438 and 178 n., with its accompanying Mathematical Note VI referring to "several commodities which will satisfy the same imperative want. . . ."

196 "The loss that people would suffer from being deprived both of tea and coffee would be greater than the sum of their losses from being deprived of either alone: and therefore the total utility of tea and coffee is greater than the sum of the total utility of tea calculated on the supposition that people can have recourse to coffee, and that of coffee calculated on a like supposition as to tea" (*loc. cit.*, p. 207 n. [131–32 n.]).

197 *Op. cit.*, p. 495. See also Henry Schultz, *The Theory and Measurement of Demand* (Chicago: University of Chicago Press, 1938), pp. 608–14.

198 The commodities are complements if both of the following inequalities hold:

$$\frac{\partial\left(\frac{\varphi_1}{\varphi_2}\right)}{\partial x_1} < 0\,, \qquad \frac{\partial\left(-\frac{\varphi_1}{\varphi_2}\right)}{\partial x_2} < 0\,.$$

They are substitutes if one of the inequalities is reversed; the stability condition (convex indifference curves) inhibits the reversal of both inequalities.

offered no definition of complementarity.[199]

It is difficult to see the purpose in Johnson's definition of complements, or, for that matter, in more recent versions such as that of Hicks and Allen. They cannot be applied introspectively to classify commodities (as the Auspitz-Lieben definition could be), so they offer no avenue to the utilization of introspection. Hence no assumption concerning their magnitude or frequency is introduced into the utility function—except for the condition that their frequency and magnitude be consistent with the assumption of stability.[200] As a result, such criteria can be applied concretely only if one has full knowledge of the demand functions. If one has this knowledge, they offer no important advantage over simple criteria such as the cross-elasticity of demand; if one does not have this knowledge, the simple criteria are still often applicable. The chief reason for presenting criteria in terms of utility, I suspect, is that, when familiar names are given to unknown possibilities, an illusion of definiteness of results is frequently conferred.

VII. THE DERIVATION OF DEMAND FUNCTIONS

Walras' derivation of the demand curves from utility functions was complete and correct for the generalized utility function of Edgeworth as well as for the additive utility function. But Walras passed from utility to demand intuitively and failed to demonstrate that any limitations on demand curves followed from the assumption of diminishing marginal utility.

Pareto was the first to make this logical extension of utility theory. Working with the simple additive utility function, he showed in 1892 that diminishing marginal utility rigorously implies that the demand curves have negative slopes.[201] A year later he partially solved the problem when the marginal utilities of the commodities are interdependent.[202] He could no longer deduce any meaningful limitation on the slope of the demand curve, and dropped the analysis. In the *Cours* he went further and argued that the demand curve for wheat may have a positive slope.[203]

A corresponding derivation of the effect of a change in income on the consumption of a commodity was presented in the *Manuel*, but Pareto gave no explicit mathematical proof and the analysis has generally been overlooked:

> If we assume that the ophelimity of a commodity depends only on the quantity of that commodity that the individual consumes or has at his disposal, the theoretical conclusion is that, for such commodities, consumption increases when income increases; or, at the limit, that the consumption is constant when income exceeds a certain level. Consequently, if a peasant subsists only on corn, and if he becomes rich, he will eat more corn, or at least as much as when he was poor. He who has only one pair of sabots

[199] His compensated variation of price is intimately related to the later definition of Hicks and Allen.

[200] Thus, in the two-commodity case, both commodities cannot be substitutes on Johnson's definition; however, neither need be.

[201] "Considerazioni . . . ," *Giornale degli economisti*, Series 2, V (1892), 119 ff. His demonstration is equivalent to ours (above, Sec. III). He also suggested the analysis of the problem of the simultaneous variation of all prices—which can be made equivalent to an income variation—but did not solve the problem explicitly (*ibid.*, p. 125). As we have noticed (Sec. IV), under the less stringent assumption of a convex utility function, one commodity can have a positively sloping demand curve.

[202] "Considerazioni . . . ," *Giornale degli economisti*, Series 2, VII (1893), 304–6. This is equivalent to our illustration (Sec. IV).

[203] *Cours*, II, 338. The discussion was hypothetical, employing the same argument that Marshall used for the Giffen case.

a year because they are too expensive, may when he becomes rich use a hundred pairs, but he will always use one pair. All this is in manifest contradiction to the facts: our hypothesis must therefore be rejected. . . .[204]

Despite this admirable test of the hypothesis of independent utilities, Pareto continued to find some use for the additive utility function.

Pareto also made a number of minor applications of utility theory to demand analysis. He showed that the demand and supply curves cannot be linear when there are three or more commodities and that the demand curve of a commodity cannot have constant elasticity when there are three or more commodities. Both demonstrations rested on the independence of the marginal utilities of the commodities.[205] We shall notice later his analysis of the constancy of the marginal utility of money.

Fisher had shown graphically in 1892 that if the utility function is not additive, an increase in income may lead to decreased consumption of a commodity.[206] The compatibility of negatively sloping income curves with convex indifference curves was first shown mathematically by W. E. Johnson.[207] Johnson also demonstrated that a rise in price may lead to an increase in the quantity of the commodity purchased.[208] Moreover, Johnson was first to carry through the explicit analysis of utility with the use only of the ratios of marginal utilities. His exposition was concise and peculiar, however, and was slow to receive attention.[209]

The complete and explicit analysis of the general case was given in lucid form by Slutsky.[210] We may illustrate his general logic with a numerical example. Let the individual consumer buy

100 units of X_1 at \$1.00, a cost of \$100 ,
60 units of X_2 at \$0.75, a cost of \$ 45 ,

exactly equaling his income of \$145. Let now the price of X_1 rise to \$1.10. Then the apparent deficiency of income, in Slutsky's language, is 100 times \$0.10 = \$10, for this is the amount that must be added to the individual's income to permit him to purchase the former quantities. If, simultaneously with the rise in the price of X_1, we give the individual \$10, Slutsky calls it a compensated variation of price. Although the individual experiencing a compensated rise in the price of X_1 can still buy the same quantities, he will always substitute X_2 for X_1, because X_2 is now relatively cheaper: Slutsky demonstrated that this is a consequence of the convexity of the indifference curves.[211] The individual will move to perhaps

86.36 units of X_1 at \$1.10, a cost of \$95 ,
80.00 units of X_2 at \$0.75, a cost of \$60 .

[204] *Manuel*, pp. 273–74.

[205] "Économie mathématique," *Encyclopédie*, I, iv, 616 ff.

[206] *Mathematical Investigations*, pp. 73–74.

[207] *Op. cit.*, p. 505.

[208] *Ibid.*, p. 504.

[209] A good discussion was given by Edgeworth, *Papers*, II, 451 ff.

[210] It is summarized by Schultz, *op. cit.*, chap. i, xix; R. G. D. Allen, "Professor Slutsky's Theory of Consumers' Choice," *Review of Economic Studies*, February, 1936. Slutsky takes the equation,

$$d^2\varphi = \varphi_{11} d x_1^2 + \varphi_{22} d x_2^2 + \dots + 2\varphi_{12} d x_1 d x_2 + \dots$$

and by a linear transformation puts it in the canonical form,

$$d^2\varphi = A_1 d a^2 + A_2 d b^2 + A_3 d c^2 + \dots .$$

He carries through two analyses, one for all $A_i < 0$, called the normal case, and a second for one $A_i > 0$, called the abnormal case. If two or more A_i are positive, $d^2\varphi$ will not be negative along the budget constraint (*op. cit.*, pp. 4–5).

[211] More precisely, he demonstrated that it is a consequence of the stability of the maximum the consumer has achieved (Slutsky, *op. cit.*, p. 14, Eq. 52).

The changes in quantities

$$86.36 - 100 = -13.64 \text{ units of } X_1 ,$$
$$80.00 - 60 = 20.00 \text{ units of } X_2 ,$$

were called the residual variabilities. If now we withdraw the $10 of income used to compensate for the variation in price, the individual may move to, say,

80 units of X_1 at $1.10, a cost of $88 ,
76 units of X_2 at $0.75, a cost of $57 .

In our example the individual reduces the quantities of both goods when income falls; Slutsky calls such goods relatively indispensable. Had X_1 been relatively dispensable, the decline in income of $10 would have led to a rise in the quantity purchased, conceivably sufficient to offset the residual variation. We have thus the laws of demand:

1. The demand for a relatively indispensable good is necessarily normal, that is to say, it diminishes when its price increases and rises when the price diminishes.
2. The demand for a relatively dispensable good may in certain cases be abnormal, that is to say, it increases with the increase of price and diminishes with its decrease.[212]

In addition, he deduced the integrability equations connecting the effects of the price of X_1 on X_2 and the price of X_2 on X_1:

$$\frac{\partial x_1}{\partial p_2} + x_2 \frac{\partial x_1}{\partial R} = \frac{\partial x_2}{\partial p_1} + x_1 \frac{\partial x_2}{\partial R} .$$ [213]

And so we have fulfilled the historian's wish: the best has come last.

MARSHALL

Marshall constructed a demand curve superior to Walras' for empirical use but related it to utility by an exposition less than masterly. This demand curve was of the form

$$x_i = f(p_i, R, I) ,$$

where I is an index number of all prices. Marshall assumed, of course, that tastes are fixed.[214] The constancy of the "purchasing power of money" (the reciprocal of our I) is an assumption governing the entire *Principles*, and it is specifically reaffirmed in the discussion of demand.[215] The role of money income is clearly recognized.[216]

I interpret I in Marshall's equation as an index number representing the average price of all commodities excluding X_i. Then his demand curve differs from the Walrasian demand curve in that he holds constant the average of other prices rather than each individual price. Changes in I may be measured by an index number embracing all commodities (including X_i), as in effect Marshall proposes, but only at the cost of inconsistency: when all prices except p_i are constant, I will vary with p_i. Unless the expenditure on X_i is large relative to income, and unless its price varies greatly, however, the quantitative error will be small.[217] We could eliminate this inconsistency (and certain ambiguities too) in Marshall's treatment by interpreting I as the average of all prices, so real income is held constant along the demand

[212] *Ibid.*, p. 14.

[213] *Ibid.*, p. 15.

[214] *Principles* (1st ed.), p. 155 [94]: "If we take a man as he is, without allowing time for any change in his character. . . ."

[215] "Throughout the earlier stages of our work it will be best . . . to assume that there is no change in the general purchasing power of money" (*ibid.*, p. 9 [62]).

[216] In addition to a reference discussed below (*ibid.*, p. 155 [95]), we may cite Book III, chap. iii [iv], with its discussion of rich and poor buyers and the "disturbing cause." "Next come the changes in the general prosperity and in the total purchasing power at the disposal of the community at large" (*ibid.*, p. 170 [109]).

[217] It is sufficient, Marshall says, to "ascertain with tolerable accuracy the broader changes in the purchasing power of money" (*ibid.*, p. 170 [109]); elsewhere he proposes to do this with an index number of wholesale prices (*Memorials*, pp. 207–10).

curve.[218] But then we should encounter new inconsistencies.[219]

Marshall insists that the prices of rival goods be held constant.[220] This proviso is troublesome to reconcile with his utility theory but not to explain. The reconciliation is troublesome because rival goods are defined in terms of utility and cannot exist with an additive utility function.[221] (We can of course eliminate this difficulty by generalizing the utility function or shifting to a definition of rival products in terms of demand cross-elasticities.) The purpose of the proviso is obvious, however; when p_i rises, consumers will shift to close rivals, and their prices will tend to rise even if the price level is stable, so the effect of changes only in p_i on purchases of X_i will be obscured.[222]

This Marshallian demand curve can be derived by the conventional Walrasian technique simply by grouping together all commodities except the one under consideration and identifying their price with the price level.[223] But then what is the role of that famous assumption, the constancy of the marginal utility of money (income)? The answer is that this additional assumption is quite indispensable to his textual instruction on how "to translate this Law of Diminishing Utility into terms of price."[224] Marshall moves directly and immediately from marginal utility to demand price by the (implicit) equation,

$$MU_i = \text{constant} \times p_i ,$$

and adds, "so far we have taken no account of changes in the marginal utility to [the buyer] of money, or general purchasing power."[225] The assumption of constancy of the marginal utility of money is essential to his exposition of the relationship between utility and demand curves, and essential also to the substance of the apparatus of consumers' surplus. But it is not essential to the Marshallian demand curve if expositional simplicity is sacrificed.

Precisely what does Marshall mean by the constancy of the marginal utility of income? He tells us (in Book V!):

> There is a latent assumption which is in accordance with the actual conditions of most markets; but which ought to be distinctly recognized in order to prevent its creeping into those cases in which it is not justifiable. We tacitly assumed that the sum which purchasers were willing to pay, and which sellers were willing to take for the seven hundredth bushel would not be affected by the question whether the earlier bargains had been made at a high or a low rate. We allowed for the diminution in the marginal utility of corn to the buyers as the amount bought increased. But we did not allow for any appreciable change in the marginal utility of money; we assumed that it would be practically

[218] See M. Friedman, "The Marshallian Demand Curve," *Journal of Political Economy*, LVII (1949), 463–95.

[219] Examples are the Giffen paradox and the statement that, in cases of multiple equilibria, consumers prefer to buy the quantity at the largest intersection of the supply and demand curves (*Principles* [1st ed.], p. 451 n. [472 n.]).

[220] "One condition which it is especially important to watch is the price of rival commodities . . ." (*ibid.*, p. 160 [100]). Complements' prices were added in the second edition (*loc. cit.*, p. 158 [100 n.]).

[221] See Sec. VII.

[222] Marshall also assumes in effect that the anticipated future price equals the present price (*Principles* [1st ed.], p. 161).

[223] No explicit derivation was given along these lines, but one can be read into Mathematical Note III [II].

[224] The phrase, but not the thought, dates from the second edition (*loc. cit.*, p. 151 [94]).

[225] *Principles* (1st ed.), p. 155 [95]. In the first edition this was the only explicit statement of the assumption in the book on demand; but see also Mathematical Note VI with its cross-reference to pp. 392–93 [334–35]. After the quoted sentence, Marshall discusses the effect of income on the marginal utility of money but is eloquently silent on the effect of price changes.

the same whether the early payments had been at a high or a low rate.

This assumption is justifiable with regard to most of the market dealings with which we are practically concerned. When a person buys anything for his own consumption, he generally spends on it a small part of his total resources; while when he buys it for the purposes of trade, he looks to re-selling it, and therefore his potential resources are not diminished. In either case the marginal utility of money to him is not appreciably changed. But though this is the case as a rule, there are exceptions to the rule.[226]

It seems beyond doubt that Marshall treated the marginal utility of money as approximately, and not rigorously, constant, and fairly clear that it is constant with respect to variations in the price of a commodity whose total cost is not too large a part of the budget.

The large volume of writing on Marshall's assumption adds an ironical overtone to our phrase "expositional simplicity." Some of the studies have been concerned with the implications of strict constancy.[227] Pareto and Barone gave such interpretations in our period.[228] The approximate constancy of the marginal utility of income has also been discussed.[229] Pareto skirted such an interpretation;[230] it can be elaborated to show that approximate constancy has no implications beyond those already implicit in the additive utility function.[231] The assumption looms large in economic literature but marks a fruitless digression from the viewpoint of the progress of utility theory.

THE ABANDONMENT OF UTILITY

Demand functions, as we have already noticed, had been treated as empirical data in the classical economics and in the work of economists such as Cournot.[232] Gustav Cassel was the first of the modern theorists to return to this approach. His theory was developed in 1899 and never changed thereafter in essentials.[233] He attacked the utility theory along two lines.

His first and constructive thesis was that one can employ demand functions directly, without a utility substructure:

The individual has a value scale in terms of money, with which he can not only classify his needs but also express numerically their intensities. . . . If I adopt the fiction that the needs of individuals A and B are of the same intensity, if both value a given need at one mark, then I have extracted from the psychological assump-

[226] *Ibid.*, pp. 392–93 [334–35]; see also [p. 132]

[227] See M. Friedman, "Professor Pigou's Method for Measuring Elasticities of Demand from Budgetary Data," *Quarterly Journal of Economics*, L (1935), 151–63; P. A. Samuelson, "Constancy of the Marginal Utility of Income," in Oscar Lange *et al.* (eds.), *Studies in Mathematical Economics and Econometrics* (Chicago: University of Chicago Press, 1942), pp. 75–91.

[228] In 1892 Pareto argued that the assumption implied that each demand curve has unitary elasticity; "Considerazioni . . . ," *Giornale degli economisti*, Series 2, IV (1892), 493. In 1894 Barone made a more elaborate analysis and reached a similar conclusion; *Le Opere*, I, 48. A few months later he offered a second interpretation: when p_i varies, money income varies by an amount equal to the change in expenditure on X_i (*ibid.*, pp. 59 ff.).

[229] N. Georgescu-Roegen, "Marginal Utility of Money and Elasticities of Demand," *Quarterly Journal of Economics*, L (1936), 533–39.

[230] *Manuel*, pp. 582 ff.; "Économie mathématique," *op. cit.*, p. 631.

[231] Let X_1 be the commodity, X_2 all other commodities. I interpret Marshall to mean that the rate of change of the marginal utility of X_2 is small relative to the rate of change of the marginal utility of X_1, or—introducing prices to eliminate the units in which commodities are measured—that

$$\frac{\varphi_{22}p_1^2}{\varphi_{11}p_2^2}$$

is approximately zero.

[232] A. A. Cournot, *Mathematical Principles of the Theory of Wealth* (New York: Macmillan, 1929), esp. chap. iv.

[233] "Grundriss einer elementaren Preislehre," *Zeitschrift für die gesamte Staatswissenschaft*, LV (1899), 395 ff.; cf. *The Theory of Social Economy* (New York: Harcourt, Brace, 1932), esp. pp. 80 ff., where the tone is much more gentle and conciliatory.

tions everything that is relevant to the economic side of the matter.[234]

> The subjective element which we seek to isolate is the relationship between valuation and external factors [income and prices]. In order to discover this relationship, we must allow the external factors to vary; then the value the individual attributes to the good in question will also vary. This value is therefore a function of the external factors, and in this functional relationship we have the complete and pure expression of the subjective element, that is, of the nature of the individual so far as it affects the formation of prices.[235]

But Cassel made no studies of the properties of the demand functions.

No doubt it was psychologically inevitable that Cassel had also a second thesis: that the utility theory was full of error. This theory, he charged, required a unit of utility that no one could define;[236] it required unrealistic divisibility of commodities and continuity of utility functions;[237] it required, or at least always led to, meaningless interpersonal comparisons of utility;[238] the assumption of constancy of the marginal utility of money is meaningless or objectionable;[239] etc.

Wicksell quickly replied for the utility theorists and with sufficient vigor to estrange Cassel for life.[240] He properly pointed out the weaknesses in Cassel's criticisms of the marginal utility theory: that it did not require measurability of utility or interpersonal comparisons except for welfare analyses; that Cassel's discontinuity objections were unrealistic and in any event did not affect the substance of the theory; etc. Wicksell also properly pointed out the considerable use of utility language in Cassel's positive theory and his implicit use of utility to reach welfare conclusions. And, finally, Wicksell criticized Cassel for his rough treatment of predecessors on the rare occasion when he recognized them at all—a charge that was exaggerated but not unfounded.[241]

But Wicksell did not meet the substantive claim of Cassel that it was possible to start directly with demand functions and that the utility theory added no information on the nature of these functions. He seemed content at this point merely to argue that the utility theory incorporated reliable psychological information into economics.[242]

Barone employed the same empirical approach to demand in his famous article on collectivist planning:

> There is no need to have recourse to the concepts of *utility*, of the *final degree of utility*, and the like; and neither is it necessary to have recourse to Pareto's concept of the *Indifference Curve*. . . .
>
> . . . the *tastes* of the various individuals. On these last we will make no presupposition, no preliminary inquiry, limiting ourselves simply to assuming the fact that at every given series of prices of products and productive services, every single individual portions out the income from his services between consumption and saving in a certain manner (into the motives of which we will not inquire) by which, at a given series of prices, the individual makes certain de-

[234] "Grundriss . . . ," pp. 398–99.

[235] *Ibid.*, p. 436.

[236] *Ibid.*, pp. 398 ff.

[237] "The fact is, that every person who is even moderately well off buys the greater part of the articles he uses for much less than the value they have for him" (*ibid.*, p. 417).

[238] *Ibid.*, p. 402.

[239] *Ibid.*, pp. 428–29.

[240] "Zur Verteidigung der Grenznutzenlehre," *Zeitschrift für die gesamte Staatswissenschaft*, LVI (1900), 577–91; amplified in some respects in "Professor Cassel's System of Economics," reprinted in *Lectures*, I, 219 ff. Cassel replied in an appendix to "Die Produktionskostentheorie Ricardos," *Zeitschrift für die gesamte Staatswissenschaft*, LVII (1901), 93–100.

[241] Cassel was not the equal of Pareto in this respect (see especially the latter's "Économie mathématique").

[242] "Zur Verteidigung . . . ," p. 580.

mands and certain offers. These quantities demanded and offered vary when the series of prices vary.

Thus we disengage ourselves from every metaphysical or subtle conception of utility and of the functions of indifference, and rely solely on the authenticity of a fact.[243]

Yet Barone is not an important figure in the movement to abandon utility. He employed this approach only in the one article,[244] and there perhaps chiefly to bring out the analogies between competitive and collectivist economies. What is more important, he did not discuss the crucial problem: Can one say more about the demand functions if they are derived from utility functions?

One final theorist of the period consistently ignored utility in his work on demand—Henry L. Moore. It was Moore's program to join economic theory with the then recent developments of statistical theory to quantify the important economic functions. In this lifelong task he has found no assistance in utility theory and paused only briefly to criticize it:

In the closing quarter of the last century great hopes were entertained by economists with regard to the capacity of economics to be made an "exact science." According to the view of the foremost theorists, the development of the doctrines of utility and value had laid the foundation of scientific economics in exact concepts, and it would soon be possible to erect upon the new foundation a firm structure of interrelated parts which, in definiteness and cogency, would be suggestive of the severe beauty of the mathematico-physical sciences. But this expectation has not been realized. . . .

The explanation is to be found in the prejudiced point of view from which economists regarded the possibilities of the science and in the radically wrong method which they pursued. . . . Economics was to be a "calculus of pleasure and pain," "a mechanics of utility," a "social mechanics," a "*physique sociale*." . . . They seemed to identify the method of physical sciences with experimentation, and since, as they held, scientific experimentation is impossible in social life, a special method had to be devised. The invention was a disguised form of the classical *caeteris paribus*, the method of the static scate.[245]

This is not the place to quarrel with certain aspects of Moore's methodological views, nor is it the place to discuss the deficiencies in his statistical work on demand, nor is it the place to give him his due as a major figure in the history of demand theory. It is a suitable place, however, to conclude our history of the theory of utility.

VIII. A THEORY OF ECONOMIC THEORIES

We have before us a fairly complete account of the major developments in one branch of economic analysis. I wish now to review this history with a view to isolating the characteristics of successful (and hence of unsuccessful) theories, where success is measured in terms of acceptance by leading economists. (It would require a different history to answer the interesting question: To what extent, and with what time interval, do the rank and file of economists follow the leaders?) The bases on which economists chose between theories may be summarized under the three headings of generality, manageability, and congruence with reality.

A. THE CRITERION OF GENERALITY

The successful theory was always more general than the theory it supplanted. The marginal utility theory was more general than the classical theory of

[243] "The Ministry of Production in the Collectivist State" (1908), translated in F. A. Hayek, *Collectivist Economic Planning* (London: Routledge, 1938), pp. 246, 247.

[244] Conventional utility analysis is used in his *Principi di economia politica*, Part I.

[245] *Economic Cycles: Their Law and Cause* (New York: Macmillan, 1914), pp. 84–86.

value (with its special cases of producible and nonproducible goods); the generalized utility function was more general than the additive utility function; the nonmeasurable utility function was more general than the measurable utility function. On the other hand, the Bernoulli hypothesis was rejected as arbitrary (i.e., particularizing). There was no important instance in which a more specific theory supplanted a more general theory, unless it was Marshall's assumption of the constant marginal utility of money, and this assumption had little vogue outside Cambridge circles.

What does generality mean here? Occasionally it is simply an application of Occam's razor, of using a weaker assumption that is sufficient to reach the conclusion in which one is interested. The nonmeasurable utility function was the leading instance of this kind of generality, although I shall argue below that perhaps logical elegance was not the major reason for abandoning measurability. Very seldom has Occam's razor beautified the face of economic theory.

More often, generality meant the encompassing of a wider range of phenomena. The marginal utility theory enabled economists to analyze the values of nonproducible goods and the short-run values of producible goods. The generalized utility function allowed the analysis of interrelationships of the marginal utilities of commodities, which previously had been outside the domain of utility theory.

Yet we must note that generality is often only verbal, or at least ambiguous. The Walrasian theory was more general than the Ricardian theory in that the former applied to both producible and nonproducible goods, but it was less general in that it took the supply of labor as given. Cassel's empirical demand curves seemed more general in that they were valid even if every element of utility theory was banished;[246] but the utility theorist Wicksell could reply that the utility theory was more general because it permitted welfare judgments. Unless one theory encompasses all the variables of the others, their order of generality will vary with the question in hand.

Generality, whether formal-logical or substantive, is a loose criterion by which to choose among theories. It is always easy and usually sterile to introduce a new variable into a system, which then becomes more general. Yet a more general theory is obviously preferable to a more specific theory if other things are equal, because it permits of a wider range of prediction. We turn now to the other things.

B. THE CRITERION OF MANAGEABILITY

The second criterion employed in choosing between theories has been manageability. Economists long delayed in accepting the generalized utility function because of the complications in its mathematical analysis, although no one (except Marshall) questioned its realism. They refused to include in the individual's utility function the consumption of other individuals, although this extension was clearly unimportant only in the social life of Oxford. The nonintegrable differential equation of the indifference curves was similarly unpopular. In these cases manageability was the prime consideration: economists tacitly agreed that it is better to have a poor, useful theory than a rich, useless one.

Of course, this is true, although the choice is not really this simple as a rule.

[246] Actually he put sufficient conditions on his demand functions to make them logically equivalent to those derived from indifference curves (see H. Wold, "A Synthesis of Pure Demand Analysis," *Skandinavisk Aktuarietidskrift*, XXVII [1944], 77 ff.).

Manageability should mean the ability to bring the theory to bear on specific economic problems, not ease of manipulation. The economist has no right to expect of the universe he explores that its laws are discoverable by the indolent and the unlearned. The faithful adherence for so long to the additive utility function strikes one as showing at least a lack of enterprise. I think it showed also a lack of imagination: no economic problem has only one avenue of approach; and the non- and semimathematical utility theorists could have pursued inquiries suggested by theories beyond their powers of mathematical manipulation.[247] The investigator in his science is not wholly dissimilar to the child in his nursery, and every parent has marveled at how often unreasoning obstinacy has solved a problem.

C. THE CRITERION OF CONGRUENCE WITH REALITY

The criteria of generality and manageability are formal; the empirical element entered through the criterion of congruence with reality. It was required of a new theory that it systematize and "explain" a portion of the empirical knowledge of the times. It must perform tasks such as accounting for the fact that often goods sold for less than their costs of production (which the marginal utility theory did) or for liking bread more when there was butter on it (which the generalized utility function did).

The reality with which theories were required to agree was one of casual observation and general knowledge. It was composed of the facts and beliefs that the men of a time mostly share and partly dispute and of the observations of men who earned and spent incomes and watched others do so. Of course the type and amount of such information varied widely among economists. Some, like Marshall, had a deep knowledge of their economies; others, like Edgeworth and Pareto, were more worldly scholars; still others, like Walras and the young Fisher, kept the world at a distance.

This casual knowledge was loose and relatively timeless with respect to utility theory; these economists knew little more about utility and not a great deal more about demand than their ancestors. In this respect utility theory is not wholly representative of economic theory; in population theory, for example, casual knowledge changed radically with the times and exercised a decisive influence on the comparative acceptabilities of various population theories. The one changing element in the general knowledge was the growing skepticism of hedonism in academic circles. Economists were surely (if improperly) more susceptible to the proposal to abandon the measurability of utility when the psychologists chided them:

> Important as is the influence of pleasures and pains upon our movements, they are far from being our only stimuli. . . . Who smiles for the pleasure of smiling, or frowns for the pleasure of the frown? Who blushes to avoid the discomfort of not blushing?[248]

[247] E.g., the generalized utility function suggested studies of the interrelations of prices in demand; the effect of other people's consumption on one's utility suggested the use of relative income status rather than absolute income in demand analysis; etc.

[248] William James, *Psychology* (New York: Holt, 1893), p. 445. William McDougall was more emphatic and pointed (as well as absurd and illogical):

"Political economy suffered hardly less from the crude nature of the psychological assumptions from which it professed to deduce the explanations of its facts and its prescriptions for economic legislation. It would be a libel, not altogether devoid of truth, to say that the classical political economy was a tissue of false conclusions drawn from false psychological assumptions. And certainly the recent progress in economic science has largely consisted in, or resulted from, the recognition of the need for a less inadequate psychology" (*An Introduction to Social Psychology* [3d ed.; London: Methuen, 1910], pp. 10–11).

The sieve of casual knowledge was broad in its gauge. It could reject the notion (of Cassel) that consumers do not equate marginal utilities divided by prices because they do not know the prices, or the notion (of the abstemious Fisher) that the marginal utility of liquor increases with quantity. But it could not reject even the imaginary Giffen paradox. Casual knowledge is better calculated to detect new error than to enlarge old truth.

This third criterion of congruence with reality should have been sharpened—sharpened into the insistence that theories be examined for their implications for observable behavior, and these specific implications compared with observable behavior. The implication of the diminishing marginal utility of money, that people will not gamble, should have been used to test this assumption, not to reproach the individuals whose behavior the theory sought to describe.

Not only were such specific implications not sought and tested, but there was a tendency, when there appeared to be the threat of an empirical test, to reformulate the theory to make the test ineffective. Thus, when it was suggested that there might be increasing marginal utility from good music, as one acquired a taste for it, this was interpreted as a change in the utility function.[249] Yet if in the time periods relevant to economic analysis this phenomenon is important, it is a significant problem—the defenders had no right to rush to the dinner table. When it was suggested that the marginal utility of the last yard of carpet necessary to cover a floor was greater than that of fewer yards, the theory was modified to make the covering of the entire floor the unit of utility analysis.[250] They did not anxiously seek the challenge of the facts.

In this respect Pareto was the great and honorable exception. Despite much backsliding and digression, he displayed a constant and powerful instinct to derive the refutable empirical implications of economic hypotheses. He was the first person to derive the implications of the additive utility function with respect to demand and income curves. It was left for Slutsky to carry out this task for the generalized utility function, but Pareto—and he alone of the economists—constantly pressed in this direction.

But exception he was. The ruling attitude was much more that which Wieser formulated:

> Any layman in economics knows the whole substance of the theory of value from his own experience, and is a layman only in so far as he does not grasp the matter theoretically,—i.e., independently, and for and by itself,—but only practically,—that is to say, in some given situation, and in connection with its working out in that situation. If this be true, how else shall be better proved our scientific statements than by appealing to the recollection which every one must have of his own economic actions and behavior?[251]

That this criterion was inadequate was demonstrated by the slowness with which utility theory progressed. The additive utility function was popularized in the 1870's; it was 1909 before the implication of positively sloping income curves was derived. The generalized utility function was proposed in 1881; it was 1915 before its implications were derived. The chief of these implications is that, if consumers do not buy less of a commodity when their incomes rise, they will surely buy less when the price of the

[249] Marshall, *Principles* (8th ed.), p. 94; Wicksteed, *Common Sense*, I, 85.

[250] Marshall, *Principles* (8th ed.), p. 94; Wicksteed, *Common Sense*, I, 83; Pareto, *Manuel*, p. 266.

[251] *Op. cit.*, p. 5.

commodity rises. This was the chief product—so far as hypotheses on economic behavior go—of the long labors of a very large number of able economists. These very able economists, and their predecessors, however, had known all along that demand curves have negative slopes, quite independently of their utility theorizing.

Had specific tests been made of the implications of theories, the unfruitfulness of the ruling utility theory as a source of hypotheses in demand would soon have become apparent. Had these economists sought to establish true economic theories of economic behavior—that is, to isolate uniformities of economic events that permitted prediction of the effects of given conditions—they would not long have been content with the knowledge that demand curves have negative slopes. They would have desired knowledge on the relative elasticities of demand of rich and poor, the effects of occupation and urbanization on demand, the role of income changes, the difference between short- and long-run reactions to price changes, and a whole host of problems which we are just beginning to study. They would have given us an economic theory which was richer and more precise.

These remarks shall have been completely misunderstood if they are read as a complaint against our predecessors' accomplishments. It would be purposeless as well as ungracious to deprecate their work. They improved economics substantially, and, until we are sure we have done as much, we should find gratitude more fitting than complaint. But we should be able to profit not only from their contributions to economics but also from their experiences in making these contributions. That such able economists were delayed and distracted by the lack of a criterion of refutable implications of theories should be a finding as useful to us as any of the fine theoretical advances they made.