

A New and Superior Process for Making Social Choices

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This paper describes and elaborates a process first discovered by Edward H. Clarke that motivates individuals to reveal their true preferences for public goods. The essence of the process is that each individual is offered a chance to change the outcome that would occur without his vote by paying a special charge equal to the net cost to others that results from including his vote in the decision. Because the special charge on any one person is not paid to any other person, a very small budget surplus results. Applications to both discrete and continuous decisions are illustrated.

This paper describes a new process for making social choices, one that is superior to other processes that have been suggested. The method is immune to strategic maneuvering on the part of individual voters. It avoids the conditions of the Arrow theorem by using more information than the rank orders of preferences, and selects a unique point on or “almost on” the Pareto-optimal frontier, one that maximizes or “almost maximizes” the consumer surplus of society. Subject to any given distribution of wealth, the process may be used to approximate the Lindahl equilibrium for all public goods.¹

These are strong claims, and it is therefore only sensible to begin this paper by pointing out that the process will not cure cancer, stop the tides, or, indeed, deal successfully with many other problems. As far as we know, all existing social-choice processes are subject to exploitation by suitably designed coalitions. This process is no exception. In addition, as in all democratic voting processes, voters are undermotivated to

¹ The method is also applicable to decisions about income and wealth redistribution, instead of leaving that issue aside in the conventional manner. It can be used to ensure the competitiveness of markets, and it provides a welfare criterion superior, in our opinion, to Pareto optimality. All of these matters must be deferred for later publication.

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invest time and effort in a comparative evaluation of alternatives. The motives they are given for making sensible decisions are somewhat stronger than those for persons engaging in voting under majority rule, but voters will be asked to do more in the way of expressing their preferences than simply saying yes or no. Therefore, it is not clear whether the lack of an incentive to vote is more or less of a handicap for this process than it is for ordinary voting processes.

The process may be described most generally as a demand-revealing process. It relies on what might be called an incomplete compensation mechanism that appears to have been first described by Vickrey (1961) in the context of optimum counterspeculation policy for a socialist economy. The essence of the mechanism is that each person is paid for the benefits (or pays the costs) of his actions, but no one is charged (or credited) as required for budget balance. Vickrey showed that it would be possible to motivate individuals to reveal their true supply and demand schedules for a private good by paying each person the net increase in the sum of the producer and consumer surpluses of other persons in the market that resulted from the supply or demand schedule that the one person reported. Vickrey noted that there would be a problem of financing such a system, since it would generate a deficit. He did not discuss the potential application of such a system to public goods.

Two persons who were unaware of each other's work or Vickrey's discovered the applicability of a similar compensation mechanism to the problem of motivating individuals to reveal their true demands for public goods. The first to publish was Edward Clarke (1971, 1972), whose papers until now have made very little impact on the economics profession. The lack of impact can be attributed partly to the nature of the idea that Clarke put forward, which is counterintuitive to almost any welfare economist, and partly to Clarke's difficult writing style. The second person was Theodore Groves. In one paper (Groves 1973) he offered a mathematically rigorous treatment of a procedure like Vickrey's for allocating scarce private goods within an organization. More recently Groves and Loeb (1975) published a procedure isomorphic to Clarke's for selecting the optimal quantities of public goods. Our objectives in this paper are to provide a clear explanation of the demand-revealing process as it applies to public goods and to extend the understanding of the process on several fronts.

While, as Bowen (1943) showed, majority rule is efficient if the intensity of voters' preferences is distributed symmetrically, the demand-revealing process does not require for its efficiency any restriction on the distribution of intensities of voters' preferences. Unlike the voting processes proposed by Thompson (1965), Dreze and de la Vallee Poussin (1971) and Tideman (1972), the demand-revealing process requires no special beliefs on the part of voters. Basically, it provides an environment in

which each voter is motivated to reveal his preferences correctly. This is accomplished by the use of a special—indeed, bizarre—tax mechanism which rewards truthful presentation of preferences and penalizes concealment or falsification.

In order to explain the process, we start not with the problem with which Clarke's first paper dealt, the choices of the optimal amount of a single-dimensional public good, but with the simpler case of a choice among discrete options, which Clarke's second paper discussed cryptically. For simplicity, we shall start with two alternatives, which may be conceived of as two policies or two candidates. We shall then show how the process can be extended to more than two options. Having introduced the subject with these simple examples, we shall then turn to the choice of the optimal amount of a public good.

Choice between Two Options

Suppose that a collective choice must be made between two options, designated A and B. The rule we describe involves asking each individual to state which option he prefers and the amount of money he is willing to pay to secure his preferred option instead of the other. We shall show shortly why he would have an incentive to respond truthfully. In table 1, we show the "votes" of each of three voters for the two options. Option A is worth a total of \$70 to the persons who prefer it and is chosen by the rule because B is worth less to its proponent.

We now turn to why the voter is motivated to correctly state his preferences. There is a "Clarke tax" to be levied, and, as we said before, it is a bizarre tax. We inquire with respect to each voter what the outcome would have been if he had not voted. For example, if voter 1 had not voted, then the outcome would have been that alternative A would have received \$40 total and alternative B \$60; hence, alternative B would have won. We charge voter 1 \$20, the amount necessary to bring the "votes" for A up to equality with the "votes" for B. By the same line of reasoning, voter 3 pays a tax of \$30. Voter 2 pays no tax because his vote did not change the outcome. Note that, had voter 1 understated his preference for A by an amount less than \$10, he would have paid exactly the same tax as he did. If he had understated his preference for A by more than \$10, B would have been selected. And voter 1 would prefer having A at the price of \$20 to having B. Similarly, if a voter overstates his preferences, either the overstatement makes no difference in what is selected or what he pays or else (e.g., if voter 2 said B was worth \$100 to him) he changes the result by his action and pays more for his choice than it is worth to him.

To describe the decision rule generally, define S_A as the sum over all voters who state a preference for A over B of the amounts they offer to

TABLE 1

AGGREGATING PREFERENCES FOR TWO OPTIONS (IN DOLLARS)

VOTER	DIFFERENTIAL VALUES OF OPTIONS	
	A	B
1	30	0
2	0	60
3	40	0
Total.....	70	60

pay to have A instead of B. Define S_B similarly. The collective-choice rule will be to choose A if $S_A > S_B$, choose B if $S_B > S_A$, and flip a coin to decide if $S_A = S_B$. The incentive to respond truthfully is generated by a "Clarke tax," a rule that a voter must pay a portion of his offer if and only if his vote changes the outcome. Any voter who changes the outcome must pay $|S_A - S_B|$, calculated without his vote. In the case of a tie that is decided by a coin toss, every voter on the side that wins the toss is regarded as having changed the outcome. If the result without a person's vote is a tie, he pays nothing.

In effect, this rule gives each voter the choice of (1) leaving the outcome where it would be without his vote or (2) changing it at a price of the reported net loss to other voters. If the value to a voter of his preferred outcome is less than the net value of the alternative to others, then he prefers (1), which occurs if he responds truthfully. If his value is greater than the aggregate net value to others, then he prefers (2), which again is what occurs upon a truthful response. If his value exactly balances the net value reported by others, then he is indifferent between the two possibilities, and we flip a coin if he responds truthfully. A nontruthful response cannot benefit the respondent, and it carries a risk of making him worse off than he would have been with the truth. If he understates his value, he may pass up an opportunity to obtain the result he desires at an attractive price. If he overstates his value, he may wind up paying more than it is worth to him to have his choice.

To characterize the rule in terms of property rights, one might call it "entitlement to the consequence of one's abstention," since the result that occurs if a voter abstains costs him nothing; while if his vote changes the collective choice, he must pay. This has a certain family resemblance to majority rule, where the voter's only entitlement is to what a majority of persons other than himself want, except that, if all others tie, he can decide the issue.

Any money collected from voters in this system must be wasted or given to nonvoters to keep the incentives correct. If voters received the money collected, the possibility of increasing their shares would distort

their incentives. However, if the revenue were simply divided equally among all voters, the effective distortion would be minimal if there were more than 100 or so voters; with a large number of voters, it is most likely that no one vote will change the outcome, so that in most cases no taxes for voting will be collected. We will discuss the significance of the lack of budget balance in more detail in the context of decisions about continuous variables.

A serious problem in all voting systems is the weakness of the incentive to vote. The demand-revealing process is no exception. The only wholly instrumental reason for a person to vote is the possibility that his vote will be decisive. Failure to vote carries a risk of passing up a chance to alter the outcome at a favorable price, but the probability of being decisive is usually small enough so that people might still reasonably conclude that voting is not worth the effort. Even if people do decide to vote, they are normally not motivated to give any serious study to their vote in collective decision processes, because the probable gain from acquiring further information or simply reflecting on the information already at hand is usually less than the cost. Thus, ill-informed voting is to be expected. The demand-revealing process is no exception to this general rule put forward by Downs (1957).

It may seem that a person who sustains a large loss when his preference is not followed deserves compensation, but this cannot be given without motivating an excessive statement of differential value. If a voter expected to lose, an offer to compensate his loss would motivate a statement from him of a larger loss. In regard to the uncompensated losses that are produced, the demand-revealing process is similar to majority rule. In the latter, every voter must live with the choice of the majority. His only opportunity to be decisive on the issue is if there is an equal number of other voters on each side. Similarly, in the demand-revealing process, every voter must live with a finding that the aggregate value placed by others on the alternatives is opposed to his own interest, provided that he is not willing to pay enough to give his preference the higher aggregate value.

It might be objected that the demand-revealing process would permit confiscatory action. If there is a proposal to tear down one person's house and make the site a park, and if others report a greater gain from the park than the occupant's loss, then the latter loses his property. The demand-revealing process would indeed have this confiscatory characteristic if there were no constitutional limits on the proposals that could be considered. In this respect, the system is again like majority rule, which has a similar confiscatory potential. It is reasonable to expect that people making collective choices by the demand-revealing process would desire constitutional restrictions that would limit the potential for overt redistribution. For instance, a proposal that a person's property be taken

for a public purpose might be admissible only if he would be given reasonable compensation.

Choices among Several Discrete Options

We now show how a demand-revealing process operates when there are more than two discrete options. In table 2 there are three voters, indicated by numbers, and three options, indicated by letters. The numbers in table 2 have been obtained by simply adding a third option, C, to the two shown in table 1, while leaving the differential the voter is willing to pay for a choice between A and B the same as it was in table 1. The difference between the numbers associated with any two options is interpreted as the amount of money the voter is willing to pay to have the one option with the higher number instead of the other. Whether or not this is a legitimate interpretation will be discussed below.

It may be noted that the rank-order preferences generate cyclic choices when majority rule is used. To determine the collective choice by the demand-revealing process, we simply sum the columns and select the option with the highest total, which in this case is A. There is no cycle, nor could there be, although a tie would be possible.

The tax for each voter is calculated from the lower portion of table 2. For the tax on voter 1, add up the sum of the other individuals' votes. Option C would have been chosen with \$70. The tax on voter 1 would be \$70 minus \$40, or \$30, and voter 1 is better off by \$20 ($\$50 - \30) than he would have been by abstaining. Note that if he had understated his preferences enough to avoid being taxed, for instance, if he had reported that A only benefited him \$25, C would have been selected and voter 1 would have been worse off than he was by correctly presenting his preferences. In the case of voter 2, there is no tax because his vote does not change the outcome; and in the case of voter 3, there is a tax of \$30 ($\$80 - \50), and he obtains a net benefit of \$10 ($\$40 - \30). These taxes are fairly substantial, but that is because we have only a small number of voters. With many voters, the probability is high that the total tax would be relatively miniscule if not zero.

We next inquire whether the proposed method produces results that are "independent of irrelevant alternatives." If option C is dropped from the example in table 2, what difference would it make? Consider voter 1 first. He reports a differential value of \$30 for A over B in table 2. If C were dropped from consideration, voter 1 would no longer have to offer \$50 for A instead of C. He would be richer, and he might spend some of his additional wealth to increase his offer with respect to A instead of B, say, from \$30 to \$32. Such wealth effects could conceivably change the result.

TABLE 2
AGGREGATING PREFERENCES FOR THREE OPTIONS (IN DOLLARS)

VOTER	DIFFERENTIAL VALUES OF OPTIONS			TAX	NET BENEFIT OF VOTING
	A	B	C		
1	50	20	0	30	20
2	0	60	20	0	0
3	40	0	50	30	10
Total.....	90	80	70
Total without Indicated Votes					
For 1: 2 + 3....	40	60	70
For 2: 1 + 3....	90	20	50
For 3: 1 + 2....	50	80	20

We do not, however, think that this is what is normally meant by a dependence on irrelevant alternatives. Option C is relevant because its presence or absence affects the wealth of voter 1. If A owns a pizza den and is negotiating with B for its sale and C builds another pizza restaurant directly across the street, this will clearly affect the bargain between A and B. However, we do not think that it would be proper to say that this was a situation which "lacked independence of irrelevant alternatives." In view of the general controversies surrounding this particular criterion of the Arrow theorem, however, we should like to simply discuss the wealth effect in the demand-revealing process rather than attempt to clear up the linguistic problem.

To put the matter another way, when we insist that each voter arrange the options on a linear scale, so that the difference between the numbers on the scale for any pair of options represents what he is willing to pay to have one option instead of the other, we leave no room for wealth effects. It may be that voter 1's true willingness to pay is \$22 for B instead of C and \$32 for A instead of B, but only \$50 (rather than \$54) for A instead of C, because if he has to pay \$22 to get from C to B, he is poorer than if he starts at B. With his lower wealth, it is not irrational for him to be willing to spend only \$28 rather than \$32, at that point, to get from B to A. The linear scale does not permit voter 1 to report these wealth effects, so he compromises by reporting the values in table 2.

It might be proposed that voters be asked to report their preferences among all pairs so that wealth effects could be taken into account in the decision process. However, to do that would be to reintroduce a possibility of cycles. Consider the two voters shown in table 3.

Voter 1 has the preferences described earlier. Voter 2 has preferences in the opposite order, of approximately the same magnitude but with

TABLE 3

THE POSSIBLE CYCLE IN THREE OPTIONS (IN DOLLARS)

	A against B		B against C		A against C	
1	32	0	22	0	50	0
2	0	31	0	21	0	51

less nonlinearity from wealth effects. When the preferences are summed, we find a collective choice for A over B and B over C, but also for C over A. This problem of intransitivity might be resolved by applying some analytic device such as the "tournament matrix" described by Moon and Pullman (1970), but it is not clear how the Clarke tax would then be calculated. Furthermore, as long as there were cycles, there would be incentives for strategic misstatements. Therefore, it may be best to require each voter to submit a linearized statement of his preferences, letting him make the necessary approximations there. Then if he can guess which option would be selected without his vote, it will be in his interest to present comparisons with respect to that option truthfully.

A Simple Continuous Application

We now proceed with the specific case Clarke presented in his *Public Choice* article (1971). Assume there is some public good which can be purchased in any desired quantity. For the purpose of graphic ease, we assume that it is sold in units which cost \$1, no matter how many are purchased, so that the line at \$1/unit on figure 1 represents the social cost schedule for purchasing different quantities. The first stage in Clarke's process is to assign to each voter his share of the total cost. Let us temporarily assume this share is assigned arbitrarily, and for the i th voter the share is the line shown at P_i . Later we will discuss how it is possible to approximate the Lindahl condition in the allocation of these shares.

The voters are now asked to state their demand curves for the public good. Voter i 's curve is shown as D_i . These curves are then summed vertically to get the aggregate demand (aggregate willingness to pay) curve AD . The point where the sum crosses the cost curve, that is, the \$1/unit line, is the efficient quantity of public good to purchase. This is, of course, the Samuelson equilibrium and has many fine properties, although not as many as the Lindahl equilibrium, toward which we shall move shortly.

How do we motivate voter i , and indeed all of the other voters, to correctly reveal their true demand curves? The answer is by telling each voter he will be subject to a Clarke tax, calculated as follows. When all the ballots are received, the tax for voter i will be calculated by summing

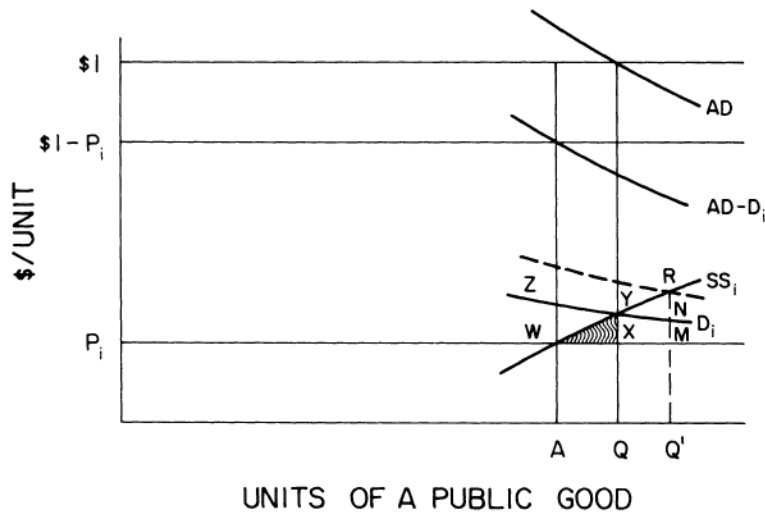


FIG. 1.—The tax on a person whose benefit exceeds his assigned tax share

(vertically) the demand curves of all of the voters other than i , generating the curve $AD - D_i$, and finding the intersection between that curve and the line $\$1 - P_i$, which is the share of the tax cost that all voters other than i will pay. The intersection in figure 1 occurs at quantity A . This is the quantity of the public good that would be purchased if i reported a perfectly elastic (i.e., horizontal) demand schedule, identical to his cost share. Such a vote, offering to pay one's assigned share of whatever quantity others desire to purchase, is the analogue for continuous choices of abstaining in discrete choices. With voter i "abstaining," the revealed demand (of others) would intersect their share of cost at A , and i 's payment would be the rectangle to the left of A and below P_i . In order to compute i 's tax when he does not "abstain," we determine from the curve $AD - D_i$ the amount of compensation that voter i would have to pay to keep all other voters indifferent to any change from quantity A . The required compensation per unit at any quantity is the difference at that quantity between the total cost and the aggregate willingness of others to pay. We call the schedule of such amounts, calculated as $\$1 - (AD - D_i)$, a synthetic supply schedule. It can be thought of as the net marginal social cost of supplying i with additional units of the public good after allowing a credit against the gross cost for the value of the good to others. This schedule is shown as line SS_i in figure 1. The schedule SS_i is a mirror image of $AD - D_i$. In this example, we assume that i has a higher willingness to pay than his cost share at A . This implies that the effect of including his demand is to increase the quantity. The intersection of the synthetic supply curve and i 's demand is at the quantity Q , and that

is the optimal amount of the public good, because that is also the quantity where AD intersects the $\$1$ line.

The amount that would have to be paid to individuals other than i in order to make them indifferent to the move from A to Q is represented by the area under SS_i , while the gain to voter i is the area under his demand curve. Voter i pays a composite tax which is the standard payment if he abstained plus the Clarke tax area under SS_i from A to Q . The total tax is equivalent to his assigned share of the cost of Q units of the public good (the rectangle to the left of line Q and below P_i) plus the shaded triangle WXY . The sum of such rectangles for all voters is enough to pay the total cost of the public good; the shaded triangle and corresponding amounts for other voters must be wasted or given to nonvoters to keep all the incentives correct.²

Suppose that voter i had misstated his demand curve in an effort to increase his net benefit. The benefit he has obtained from voting is the triangle WYZ . Clearly, stating his demand as less than it actually is would reduce the size of his triangle. On the other hand, if he stated his demand schedule as higher than it actually is, so that the quantity chosen would be, say, Q' , his additional taxes would be $QQ'RY$ while his additional benefits would be only $QQ'NY$. He is best off correctly presenting his demand curve.

It must be mentioned that there is a very slight conceptual problem in the specification of these demand curves. Quantity demanded depends on income as well as price, and one determinant of a person's income is the Clarke tax he must pay. Since one person's Clarke tax depends on the demand curves specified by others, each person could logically say that he could not specify his demand curve until all others had done so. This is not a practical problem, however, because the Clarke tax, as we show below, is very small, and in most cases the uncertainty in the Clarke tax is very small, since it depends only on the elasticity of the aggregate willingness to pay of other voters, and in any event people can simply be directed to report demand curves that reflect their best guesses about their incomes.

In figure 2, we assume that j , given his tax share, wants less than that amount of the public good which the other voters would choose. As in figure 1, D_j represents his true demand for the public good, and quantity A represents the amount which would be purchased if he chose to abstain, that is, the point where the sum of the demand curves of all the other voters intersects the sum of their shares of the tax price. Line SS_j in this case, as in figure 1, represents the rate of compensation per unit which it would be necessary to pay the other voters to compensate them for any

² One possibility for avoiding waste would be for pairs of communities to agree to exchange their collections of these excess revenues.

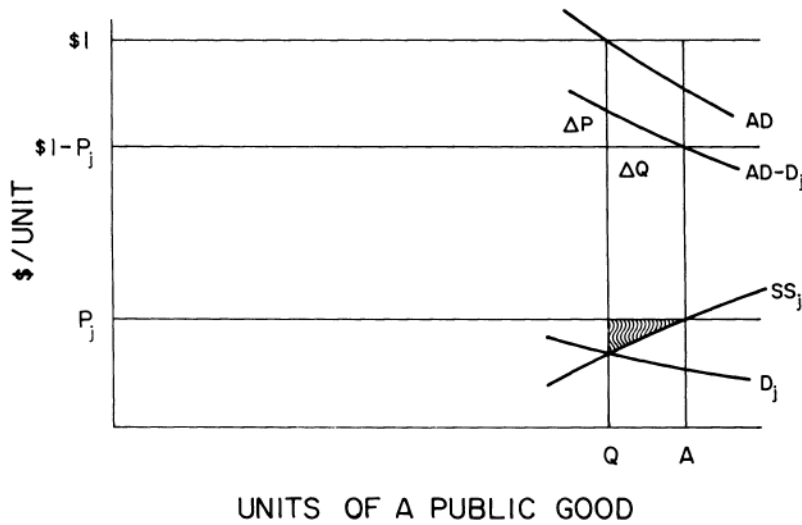


FIG. 2.—The tax on a person whose benefit is less than his assigned tax share

change from point *A* in the amount of the public good. To the left of point *A*, SS_j may be interpreted as the rate of reduction in taxes for *j* that can be granted while reducing the quantity of the public good and reducing taxes for others by the full amount of the loss of income that they experience. As in figure 1, this compensation is not actually going to be paid, but voter *j* will be taxed this amount.

Once again, the point of intersection between *j*'s demand curve and the synthetic supply curve represents the optimal quantity of the public good, *Q*, which is also the quantity at which *AD* intersects the \$1/unit line. In this case, *Q* is less than would be chosen if voter *j* abstained. Voter *j* then pays a tax which is equal to the rectangle to the left of line *Q* and below his tax share, plus the shaded triangle. The rectangle is enough to pay his share of the cost of provision of the public good; the shaded triangle, once again, is wasted or given to nonvoters. We will leave to the reader the demonstration that the correct presentation of his demand curve will maximize his welfare under these circumstances. It is essentially the same as the demonstration for figure 1.

What is true for voters *i* and *j* is true for all voters. They are motivated by this peculiar tax procedure to present accurately their true demand curves. The motivation, however, represented by triangle *WYZ* in figure 1, would normally be very small, about the same as the area of the shaded triangle.

To see how small the Clarke taxes would be, note that the shaded triangle in figure 2 is a mirror image of the one with sides labeled ΔP and ΔQ . If the elasticity of $AD - D_j$ is η , then $\Delta Q = \eta Q \Delta P / (1 - P_j)$, so

that the area of the triangle is $1/2\eta Q(\Delta P)^2/(1 - P_j)$. The denominator approaches 1 as the number of voters increases, so that if η is on the order of magnitude of 2, then each voter's Clarke tax is roughly $Q(\Delta P)^2$. The values of ΔP would be related to the number of voters (N); it would be implausible for the average value of $(\Delta P)^2$ to be greater than $1/N^2$. Thus, the typical voter, whose share of the resource cost is Q/N , has a Clarke tax on the order of magnitude of $1/N$ times his resource cost, and the sum of all Clarke taxes is on the order of magnitude of one voter's taxes. Thus, if the citizens of the United States were voting on the annual federal budget, the grand total of all the Clarke taxes charged would be in the neighborhood of \$2,000, or about one-thousandth of a penny per person.

As the triangles go to zero, the motivation for taking the trouble to present one's demand curve goes to zero. The method is cheat-proof and generates the socially optimal quantity of the public good when every voter maximizes his self-interest, but when N is large the Downs paradox is present: voters have almost no incentive to vote.

Since the excess revenues generated by the process are so very small, and certainly less than the administrative cost for any situation with more than a very small number of persons, the excess revenues deserve to be ignored. This may be rather untidy, but it is normal in welfare economics to ignore the cost of reaching a decision. If the Clarke tax is considered as part of the cost of making the decision, then it should be ignored. Contrarily, if it is not ignored, the cost of the processes of reaching decision rules by other processes should also be included. We feel that our suggestion of simply wasting the extra revenue rather than searching for some complex budget-balancing process which might or might not achieve the same result is an important contribution. It is also one of the reasons why it is so hard for welfare economists (ourselves included) to feel at home with the process.

So far, we have generated the Samuelson equilibrium; we now indicate how the Lindahl equilibrium may be approximated. To this point, we have simply assigned the base share of the total expenditure for the individual in an arbitrary manner. Suppose that, instead of assigning it arbitrarily, we appoint someone to do this, with the stipulation that from his pay we are going to subtract some multiple of the sum of the triangles for all of the voters.³ The person assigning the fixed shares would be motivated to try to minimize the triangles. In the limit, if he were able to perfectly achieve his goal, there would be no triangles and no loss; we would have a perfect Lindahl equilibrium, with each voter paying for public goods according to his marginal evaluation.

³ Probably the best way of selecting the "tax setter" would be to solicit bids. Precautions against bribery would, of course, be necessary.

It is unlikely that the official allocating the shares could do this perfectly, but he might be able to do quite well with advanced econometric methods. It should be emphasized, however, that there is one piece of information he cannot use in assigning the tax share of any individual: that individual's performance on previous choices. The voter, in making his choices on each individual decision, must not be able to offset against the optimality conditions for that particular choice the prospect of changing his base tax share in the future, because this would motivate him to misstate his demand curve.

As in all voting methods, there is a possibility for coalitions to distort the result. In particular, consider the strategy for a coalition of N persons whose equal benefits are greater than their equal tax shares. In figure 3, the demand schedule of voter i , D_i , is shown as a horizontal line because changes in the height would generally be negligible over the range of potential effects he and his coalition could have. The higher line, D_c , represents the demand that i would express taking account of the benefit of $D_i - P_i$ that each member of his coalition would receive for each unit increase in the quantity chosen. The distance from A to S is N times as far as the distance from A to Q , where the outcome would have been moved by an honest vote. With N persons in the coalition voting this way, the effect is to move the choice $N(N - 1)$ times the distance from A to Q , compared with honest voting. The gross benefit of the coalition activity to each member, in terms of benefits not paid for by his standard tax share, is $N(N - 1)(D_i - P_i)(Q - A)$, which is the area of the shaded rectangle. Each member's extra tax from coalition activity, apart from his standard tax share, is that portion of the shaded rectangle that is below the synthetic supply schedule. Thus, the net benefit of coalition activity to each member is a triangle like that in the upper left corner of the shaded rectangle, the area of which is proportional to $(N - 1)^2$ and to $(D_i - P_i)^2$. Thus, the benefit of forming coalitions varies with the square of the errors in tax shares and with the square of the number of members minus one. Voters whose tax shares overstate their benefits have a similar opportunity to form coalitions that multiply the understatements of their demands.

In this example, we have assumed that the only thing chosen is the unidimensional quantity of one public good. One of the convenient characteristics of the demand-revealing process is that it is not necessary to restrain voting to one issue at a time. A multidimensional public good or several public goods or public goods *plus* candidates can all be dealt with simultaneously. In general, it is much harder to organize coalitions in cases where the choice is not unidimensional. This is not to say that it is impossible. Still, we suspect that the demand-revealing process is rather less susceptible to coalition distortion than most voting methods.

The extension of the voting method to choices for more than one good

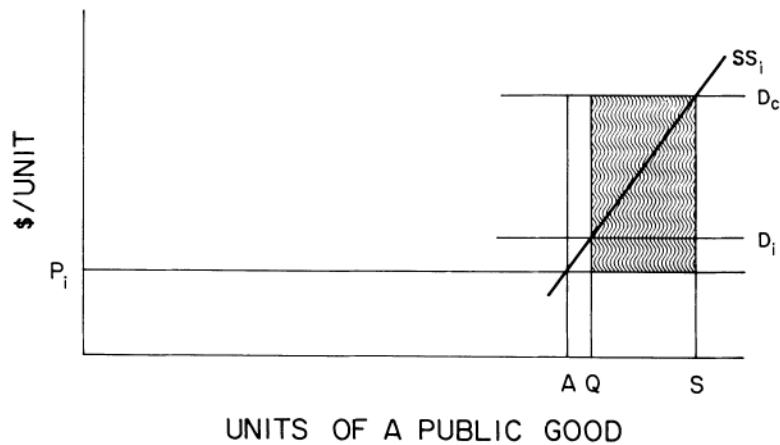


FIG. 3.—The strategic calculation of a coalition

is straightforward if the chosen quantity of one public good has no impact on demands for other public goods. However, if the chosen quantity of some goods affects the demands for other goods, a simultaneous solution is needed. One could ignore the interactions in the choice procedure and rely on individuals to make estimates of the quantities of other goods that would be chosen in reporting their demand schedules, but any mis-estimates by voters would lead to unnecessary inefficiencies.

At a conceptual level, one could ask all voters to report their marginal valuation schedules for each good at every combination of quantities of other public goods, although the data problem if this were really attempted would be unmanageable. If it were not impossible to obtain and operate on the data, the identification of an equilibrium where the appropriate marginal conditions were all satisfied simultaneously would be essentially no different from calculating a competitive equilibrium for private goods. Groves and Ledyard (1975) developed the theoretical foundations of such a system in detail.

In later publications,⁴ we propose to apply the process to a number of other problems such as income redistribution, badly behaved demand curves, and use as a welfare indicator. We will also discuss its practical application in realistic government structure. The purpose of this paper, however, has been to explain the system and to demonstrate that it solves a number of problems previously thought to be unsolvable. The process does not violate the Arrow theorem, but it avoids the problems of the Arrow theorem by not meeting Arrow's assumptions. However, it seems to us that, if the Arrow theorem is considered as a result that suggests that a good voting process cannot be devised, then the real problem raised by Arrow is solved by this process.

⁴ Mimeographed preliminary drafts are available on request.

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