

## American Economic Association

---

The Single Transferable Vote

Author(s): Nicolaus Tideman

Source: *The Journal of Economic Perspectives*, Vol. 9, No. 1 (Winter, 1995), pp. 27-38

Published by: [American Economic Association](#)

Stable URL: <http://www.jstor.org/stable/2138352>

Accessed: 05/10/2010 15:07

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=aea>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).



American Economic Association is collaborating with JSTOR to digitize, preserve and extend access to *The Journal of Economic Perspectives*.

<http://www.jstor.org>

## The Single Transferable Vote

Nicolaus Tideman

**T**he single transferable vote (STV) is a method of proportional representation based on rankings of candidates by voters. The basic idea of proportional representation is that the distribution of views among those who are elected to representative bodies should resemble the distribution of views in the electorate. STV is the predominant form of proportional representation in countries with a heritage of English influence. It is used to elect the Dáil (Assembly) in Ireland, the Senate in Australia and the House of Representatives in Malta. In the United States, STV is used to elect the City Council and School Committee in Cambridge, Massachusetts, and to elect Community School Boards in New York City. It is used as well by dozens of unions and religious, charitable, and professional organizations in many countries.

As the name “single transferable vote” suggests, STV is a system in which each voter casts one vote, and under prescribed conditions the vote is transferred from one to another of a voter’s ranked list of candidates. STV is actually an evolving family of vote-counting rules rather than a single rule. The different varieties of STV share the following features: a quota of votes is established, and any candidate who attains the quota is elected; surplus votes of elected candidates are transferred to other candidates favored by those who voted for the elected candidates; candidates are eliminated sequentially and their votes transferred to other candidates, with the candidate eliminated at each stage generally being the one with the fewest current votes. Differences arise with respect to the computation of the quota, whether the surplus votes

■ *Nicolaus Tideman is Professor of Economics, Virginia Polytechnic and State University, Blacksburg, Virginia. His e-mail address is [ntideman@vtvm1.cc.vt.edu](mailto:ntideman@vtvm1.cc.vt.edu).*

are ones picked at random or fractions of all votes, the way in which ties are resolved, and whether account is taken of already-elected candidates to whom surplus votes would be transferred if they were not already elected.

The central attraction of the single transferable vote compared to other voting procedures is that voters are able to rank candidates in whatever ways they wish, and the vote-counting process sorts the voters into equal-sized groups that are suitably represented by particular candidates (with the possibility that some voters will be split among two or more groups).<sup>1</sup> This paper discusses the history and characteristics of the single transferable vote.

## Comparisons with Other Voting Procedures

The single transferable vote selects a set of candidates that replicates the diversity of opinion within the electorate. Systems of proportional representation based on party lists are also supposed to do this. In these systems, voters vote for parties, positions are awarded to parties in proportion to the votes they receive, and individual candidates are then declared elected based on position on party lists. The deficiencies of these systems compared to STV are that they presuppose that what voters care about is captured in party definitions, and they give tremendous power to party officials.

When systems of proportional representation are not used, democracies usually select multimember bodies through single-member districts with plurality voting. This election method tends to create uniformity within the elected body, with diversity arising primarily from whatever variation exists in the characteristics of districts. This method also produces great conflict over the drawing of district boundaries.

Sometimes the individual members of multimember bodies are elected “at large,” with each voter usually given more than one vote. If each voter has as many votes as there are positions to be filled, then a majority coalition is able to secure all of the positions. If each voter has only one vote, then there is significant potential for diversity in the elected body, but there is a great premium on organizing among voters, so that votes are not wasted on candidates who will not be elected. When voters have an intermediate number of votes, the system has characteristics between the two polar cases.

<sup>1</sup>In more formal terms, the attraction of the single transferable vote is expressed by the condition that Dummett (1984, p. 282) has called “proportionality for solid coalitions” (PSC). The solid coalition for the set of candidates  $C$  consists of those voters who rank all candidates in  $C$  ahead of all other candidates. If a vote counting rule satisfies “proportionality for solid coalitions,” then for any set of candidates  $C$ , whatever percentage of the voters are in the solid coalition for  $C$ , at least as large a percentage of the elected candidates (rounded down to an integer) will be from  $C$ , as long as  $C$  contains at least that many candidates. It is the fact that STV satisfies PSC that justifies describing STV as a system of proportional representation.

The single transferable vote removes much of the need for discipline and strategy in voting. It allows voters to classify candidates in terms of whatever characteristics they believe to be relevant. And it selects a group of elected candidates that reflects the diversity of views in the electorate.

### **The Early History of the Single Transferable Vote**

The earliest known proposal in the spirit of the single transferable vote is the rule that Thomas Hill developed in 1819 for electing a Committee of the Birmingham (England) Society for Literary and Scientific Improvement (Hill, 1988, p. 252). Under Thomas Hill's rule, voters wrote their names on their ballots and each voted for a single representative. Any candidate who received five or more votes was elected. If any candidate received more than five votes, ballots equal to the excess above five votes were selected at random, and the voters who had submitted them were invited to vote for alternative representatives. After the process had iterated until there were no surplus votes, the voters whose votes had elected no one were invited to vote again, until there were fewer than five votes that had not been used to elect anyone.

The idea of having voters rank candidates was first put forward in 1855 by Carl Andrae, a Danish mathematician and politician. Without knowing about Hill's work, Andrae had the idea of establishing a "quota" equal to the largest integer in  $N/K$ , where  $N$  is the number of votes and  $K$  is the number of candidates to be elected. In Andrae's procedure, ballots are examined in a random order, and any candidate who achieves a quota of votes is elected. If the candidate at the top of a voter's ranking is already elected, the vote goes to the first unelected candidate in the voter's ranking. After all ballots have been examined, the remaining positions are filled by candidates with the most votes. At Andrae's instigation, his system was used for some Danish elections (Hoag and Hallett, 1926, pp. 172–74).

But it was a London barrister named Thomas Hare who gave the idea of ranking-based proportional representation the publicity it needed to become a permanent part of political thinking. Hare reinvented Andrae's system and publicized it in his 1857 pamphlet, *The Machinery of Representation* (Hoag and Hallett, 1926, p. 175), and then elaborated the idea further in a treatise (Hare, 1859). In consequence, the single transferable vote is sometimes called the "Hare system" of proportional representation. In 1865 Hare revised his proposal, incorporating the provision that after all surplus votes have been transferred, if the prescribed number of positions has not been filled, candidates are eliminated successively beginning with the one with the fewest votes. As each candidate is eliminated, each of the candidate's votes is transferred to the unelected and uneliminated candidate ranked highest by the voter who

supplied that vote.<sup>2</sup> With the introduction of the transfer of the votes of eliminated candidates, Hare recovered an important virtue of Hill's proposal (Hoag and Hallett, 1926, pp. 175–77).

### The Choice of a Quota

After Hare's idea of transferring votes of eliminated candidates, the next significant improvement in the single transferable vote came in 1868 when another London barrister, H. R. Droop, proposed that the quota be reduced from the integer part of  $N/K$  to the integer part of  $[N/(K + 1)] + 1$  (Hoag and Hallett, 1926, pp. 177, 378–80). The Droop quota is the smallest integer quota such that no more than  $K$  candidates can have a quota of votes.

The value of the Droop quota can be seen by considering the following example. Suppose that two Democrats,  $D$  and  $E$ , and two Republicans,  $R$  and  $S$ , are competing for three positions. There are 100 votes, distributed as follows: 24 rank the candidates  $DE$ , 23  $ED$ , 32  $RS$ , and 21  $SD$ . Notice that the first two groups of voters rank both Democrats ahead of both Republicans, while the second two groups rank both Republicans first. Since a majority of the electorate ranks both Republicans ahead of both Democrats, one would expect the Republicans to be awarded two of the three positions.

But if the quota is set at 33 in accordance with Hare's proposal, then no candidate receives a quota of first-place votes, and the candidate with the fewest votes, namely  $S$ , is eliminated, so that Democrats are awarded two of the three seats even though only 47 percent of the electorate favor the Democrats. On the other hand, if Droop's suggestion is employed, then the quota is 26.  $R$  is elected with a surplus of 6, which is transferred to  $S$ , securing  $S$ 's election, and the faction with a majority of the vote is awarded the majority of the seats.

In view of examples of this sort, most proponents of the single transferable vote recommend that the number of voters be divided by  $K + 1$  in computing the quota. But there is some sentiment against dividing by  $K + 1$ , because this

<sup>2</sup>The proof that Hare's proposal satisfies the "proportionality for solid coalitions" condition described in the previous note is as follows: Suppose that there is a solid coalition, with a size greater than or equal to  $J$  quotas, for the set of candidates  $C$ , and that there are at least  $J$  candidates in  $C$ . The number of voters not in the coalition is at most  $N(K - J)/K$ , which is enough to provide at most  $(K - J)$  quotas. Therefore the number of candidates that can be elected without using votes from the coalition is at most  $K - J$ . The votes of voters in the coalition are assigned initially to candidates in  $C$ , and since the coalition ranks all of the candidates in  $C$  ahead of all other candidates, these votes remain assigned to candidates in  $C$  when candidates are eliminated, as long as there are candidates in  $C$  that remain unelected and uneliminated. Since the coalition has enough members to elect  $J$  candidates, it is not possible for the last unelected candidate in  $C$  to be eliminated until  $J$  candidates from  $C$  have been elected. Since those outside the coalition can elect at most  $K - J$  candidates without votes from the coalition, the election cannot end before  $J$  candidates from  $C$  have been elected.

means that the vote-counting process creates  $K$  equal-sized groups that are represented and one group of the same size that is not represented. However, dividing by  $K + 1$  can be regarded as a generalization of the principle that barely more than half the votes are needed to win when one person is elected. And there seems to be no other way to avoid embarrassing cases of a majority coalition receiving a minority of the positions.

While the introduction of the Droop quota improves the performance of the single transferable vote, in some examples a majority coalition is still awarded a minority of the positions. Suppose for example that four Democrats,  $D$ ,  $E$ ,  $F$ , and  $G$ , and four Republicans,  $R$ ,  $S$ ,  $T$  and  $U$ , are contesting an election in which there are 80 voters and seven positions to be filled. The votes are distributed as follows:

10	<i>DEFGIRSTU</i>	11	<i>RSTUDEF</i>
10	<i>EFGDSTUR</i>	11	<i>STUREFGD</i>
10	<i>FGDETURS</i>	11	<i>TURSF</i>
9	<i>GDEFURST</i>	8	<i>URSTGDEF</i>

This example has a pattern similar to the previous example, in that the four groups of voters on the left rank all of the Democrats ahead of all of the Republicans, while those on the right rank the Republicans ahead of the Democrats. Since 39 voters rank the Democrats ahead of the Republicans while 41 rank the Republicans ahead of the Democrats, one would expect a majority of the positions to go to the Republicans. However, the Droop quota is  $\lceil 80/(7 + 1) \rceil + 1 = 11$ . Therefore the three Republicans with 11 votes are elected, and then the fourth Republican is eliminated, resulting in the election of the four Democrats. The desired result is achieved with a quota of  $N/(K + 1)$ , along with a tie-breaking rule to deal with the possibility that  $K + 1$  candidates receive quotas. Such a quota was proposed by Robert Newland and Frank Britton (1973) and will therefore be called an NB quota.

The difficulty with the NB quota is that it does not treat certain ties fairly. For example, suppose that four candidates are competing for two positions and there are 12 votes distributed as follows: 4 for *DFGE*, 4 *EFGD*, 3 *FGED*, and 1 *GFDE*. The NB quota is 4, so candidates  $D$  and  $E$  are elected. But since there are four voters who rank  $F$  and  $G$  ahead of all other candidates, it seems unfair that both  $F$  and  $G$  are invariably eliminated. A mechanism for achieving a fairer result has been offered by Irwin Mann (1973). Mann's rule is that a candidate is not declared elected until he or she has *more* than an NB quota of votes, with all of the excess above the NB quota treated as surplus. Thus, for this example, no one is elected upon the distribution of the first-place votes,  $G$  is eliminated, that vote is transferred to  $F$ , and in view of the resulting tie, one of  $D$ ,  $E$ , and  $F$ , is chosen by lot to be eliminated.

## Determining Which Votes to Transfer

Hill's, Hare's and Droop's systems for transferring votes all entail a stochastic component in the choice of which votes to transfer and therefore a stochastic component in the final outcome. It is more attractive for a vote-counting rule to yield the same outcome every time a set of votes is counted, unless there is a true tie.

One way to reduce the stochastic component in outcomes, used in Irish elections, is to transfer not a random sample of all of the votes of a candidate with a surplus, but rather a stratified random sample, after sorting the ballots according to who is named next on them. But because these ballots might be transferred again, a stochastic component remains (Hill, 1988, p. 254).

The first version of the single transferable vote to eliminate the stochastic component in the outcome, apart from the true ties, was the "Senatorial rules" developed in about 1880 by J. B. Gregory of Tasmania, Australia (Hill, 1988, p. 254). Under the Senatorial rules, every voter has 100 votes. When there is a surplus, the same number of votes from each voter in the group that generated the surplus is transferred. Later rules accomplished the same result by transferring the same fraction of each vote that was in the group to be transferred, rounding up to the nearest hundredth.

There is no controversy about the way fractional transfers ought to be applied to the transfer of a single surplus. However, there is a sharp difference of opinion among advocates of the single transferable vote regarding what procedure is appropriate when more than one surplus is to be transferred. Consider the following example of 80 voters and five candidates competing for three positions:

25	<i>RSVUT</i>	6	<i>TURSV</i>
15	<i>RSTUV</i>	7	<i>UTSRV</i>
9	<i>SRTUV</i>	9	<i>VTURS</i>
9	<i>SUTRV</i>		

While the Senatorial rules use the Droop quota, I will use the NB quota here, to facilitate comparisons with other rules that use the NB quota.

In this case, the NB quota is  $80/(3 + 1) = 20$ . Candidate *R* has 40 first-place votes, all of which name *S* as the second choice. Since *R* needs only 20 of these to be elected, each vote for *R* is transferred to *S* as half a vote, which when combined with *S*'s 18 first-place votes, give *S* 38 votes, so that *S* is elected. But *S* needs only two votes (or 5 percent of a vote from each of 40 half-votes) to be elected. So under the Senatorial rules, each of the 40 half-votes for *S* is now transferred as 45 percent of a vote to the voter's third choice, which is *V* for 25 voters and *T* for 15 voters. The 11.25 votes thereby transferred to *V* raise *V*'s vote total to 20.25, and *V* secures the third position.

Notice that in the Senatorial rules, the only votes that are transferred are those from the “bundle” that created the original surplus—in this case, the votes that went from  $R$  to  $S$ . While this procedure is tolerable when simplicity is highly valued (for example, when the votes are counted by hand), fairness suggests that the power to have one’s vote transferred should be shared among all who vote for candidates with surpluses. In this example, those who named  $S$  as their first choice should, after their candidate has exceeded the quota, have some portion of their votes transferred to their second choices.

The first rule that allowed all votes for a candidate with a surplus to participate in the distribution of the surplus was offered by Brian Meek (1969). Meek’s proposal was that for each candidate with a surplus, a fraction between 0 and 1 be calculated, such that when the candidate retains that fraction of every vote or fractional vote that comes his way, he ends up with exactly a quota. Applying Meek’s proposal to the example above, the fractions  $f_R$  and  $f_S$  must satisfy the equations,

$$40f_R + 9(1 - f_S)f_R = 20;$$

$$18f_S + 40(1 - f_R)f_S = 20.$$

With a bit of algebra, the solution to this pair of equations can be derived to be (approximately)  $f_R = .4494$  and  $f_S = .4997$ .

The implication of these numbers is that each of the 25 votes with ranking  $RSVUT$  is transferred to  $V$  as  $(1 - .4494)(1 - .4997)$ , or .2754 of a vote, each of the 15 votes with ranking  $RSTUV$  is transferred to  $T$  as .2754 of a vote, each of the 9 votes with ranking  $SRTUV$  is transferred to  $T$  as .2754 of a vote, and each of the 9 votes with ranking  $SUTRV$  is transferred to  $U$  as  $1 - .4997$ , or .5003 of a vote. After these transfers, the vote totals (upon calculation with greater precision) are:  $R = 20.000$ ,  $S = 20.000$ ,  $T = 13.611$ ,  $U = 10.503$ , and  $V = 15.886$ . Since there are no further surpluses to be transferred, the candidate with the fewest votes,  $U$ , is eliminated, and all of  $U$ ’s votes are transferred to  $T$ , which gives  $T$  the position that went to  $V$  under the Senatorial rules. In other words, allowing those who voted for  $S$  to have some power to transfer votes changes the winner of the third position.

If there are more than two surpluses, one would almost certainly want to use a computer and a method of successive approximations to implement Meek’s proposal, as the degree of the equations is equal to the number of candidates with surpluses. A computer program implementing Meek’s proposal has been published by Hill, Wichmann, and Woodall (1987).

The Meek proposal has been criticized by C. H. E. Warren (1983) for the way that it allocates the “cost” of electing a candidate among the candidate’s supporters. Warren argues that it is inappropriate for the voters whose rankings begin  $RS$  to be “charged” less (0.2751 of a vote per person) for the election of  $S$ , than the voters who ranked  $S$  first (0.4997 of a vote per person).



Warren suggests that the counting rule ought to charge every supporter of each elected candidate the same price (percentage of a whole vote), as long as there is enough of the voter's vote left to pay the price. Thus in the example above, since there are 49 voters whose votes end up supporting *R* (that is, the first three groups of voters), each is charged  $20/49$ , or 40.82 percent, of a vote toward the election of *R*. Similarly, each of 58 voters who end up supporting *S* (those who have *S* ranked first or second) is charged  $20/58$ , or 34.48 percent, of a vote toward the election of *S*.

Then each of the 25 votes with ranking *RSVUT* is transferred to *V* as  $(1 - .4082 - .3448)$ , or .2470 of a vote, each of the 15 votes with ranking *RSTUV* is transferred to *T* as .2470 of a vote, each of the 9 votes with ranking *SRTUV* is transferred to *T* as .2470 of a vote, and each of the 9 votes with ranking *SUTRV* is transferred to *U* as  $1 - .3448$ , or .6552 of a vote. After these transfers, the vote totals (upon calculation with greater precision) are  $R = 20.000$ ,  $S = 20.000$ ,  $T = 11.928$ ,  $U = 12.897$ , and  $V = 15.175$ . Thus, under the Warren rule, the low-vote candidate *T* is eliminated, those votes are transferred to *U*, and *U* is awarded the third position. I agree with Warren's views on this issue.

A second controversy over the way surpluses are distributed arises with respect to the treatment of votes that cannot be transferred, because the voters chose to rank only some of the candidates, leaving the remainder rankings blank. The rules proposed by Newland and Britton (1973) provide that when votes that cannot be transferred are encountered in the transfer of a surplus, the weight of the votes that *can* be transferred is increased to offset the nontransferable votes, but not above what their weight had been as votes for the candidate with a surplus. The beneficial effect of this rule is that it limits the fall in the total number of votes allocated to all candidates who have not been eliminated. This number falls in any case when the votes of an eliminated candidate cannot be transferred, but keeping the number higher reduces the extent to which, at the end of an election, there are candidates who are elected by default with less than a quota because all other candidates have been eliminated. Thus the incorporation of this provision into the Newland and Britton rules is understandable.

Some members of the Technical Committee of the Electoral Reform Society argue that a similar provision should be incorporated in any computer program for counting votes. However, Meek takes a different approach in his program. Instead of increasing the weight of the votes that can be transferred, he compensates for the nontransferability of some votes by reducing the quota. In effect, the Meek procedure says that decisions by voters to submit incomplete rankings, and thus to forego some of the voting power available to them, should be treated as reductions in the number of votes, thereby reducing the quota needed to win.

I agree with Meek. A voter who submits an incomplete ranking is saying, in effect, "If the count reaches a point where it is not possible for all or part of my

vote to be allocated to any of the candidates I have ranked, then I desire that the remaining part be allocated to no one." The fact that one voter declines to use some of the voting power that is available to him means that some other voters must be accorded additional power in a relative sense. The practice of increasing the weight of transferable votes gives this additional relative power to the other voters whose votes were in the same bundle as that of the partial abstainer, while Meek's practice of lowering the quota accords the additional relative power to all voters.

If the goal of the election procedure were to devise a statistical estimate of how partial abstainers would have voted if they had chosen to complete their votes, then some variation on the NB procedure would be appropriate. However, in view of the fact that it is not general practice in elections to search for a proxy for each voter who abstains, Meek's practice of reducing the quota, and thereby giving the additional power to all voters, seems more appropriate.

While there are many variations in vote-counting rules under the single transferable vote, to the extent that standards do exist, they are those of the Electoral Reform Society of Great Britain and Ireland. This organization was formed in London in 1884 as the The Proportional Representation Society. When its initial efforts were unsuccessful it fell moribund for about 20 years, but since 1906 it has been continuously active in promoting the single transferable vote. It was primarily through the efforts of the Proportional Representation Society that STV was introduced in Ireland in the 1920s. In 1959 the society's present name was adopted to avoid the possibility that people would think that it promoted party-list systems of proportional representation, which are used by many European countries. The Electoral Reform Society has endorsed the rules of Newland and Britton (second edition, 1976). Meek's rules, as specified by the computer program of Hill, Wichmann, and Woodall (1987), are used by the Royal Statistical Society.

## **Limitations of the Single Transferable Vote**

Like all vote-counting rules, the single transferable vote is subject to limitations described by the Arrow theorem (Arrow, 1963, pp. 96–100) and the Gibbard-Satterthwaite theorem (Gibbard, 1973; Satterthwaite, 1975). The central implication of the Arrow theorem is that STV and other vote-counting rules are subject to inconsistency as the agenda changes, and therefore to agenda manipulation: the introduction of a new candidate, even one that is not chosen, can change the collective ranking of previous candidates (Bordes and Tideman, 1991). The Gibbard-Satterthwaite theorem implies that no (non-dictatorial) ranking-based vote-counting rule for more than two candidates can motivate voters to respond truthfully in all circumstances. All vote-counting rules have these limitations in some circumstances.

Certain other limitations of STV do not apply to all vote-counting rules. In particular, STV lacks the properties of “nonnegative responsiveness” and “Condorcet consistency.” Nonnegative responsiveness is the condition that an incremental upward movement of a candidate on one voter’s ballot cannot adversely affect that candidate’s winning status. That STV lacks nonnegative responsiveness is shown by an example with 11 voters and three candidates: 3 choose *ABC*, 4 *BCA*, 3 *CAB*, and 1 *CBA*. If one candidate is to be elected, then *A* is eliminated in the first round under STV (because of the fewest first-place votes), and *B* wins in the second round. But if the last voter were to move *B* ahead of *C*, then *A* and *C* would tie for fewest votes. If the tie were resolved in favor of *A*, then *A* would beat *B* in the second round of voting, so that *B*’s incremental upward movement cost it the election.

Condorcet consistency is the condition that if a candidate beats all other candidates in head-to-head comparisons, then that candidate is elected. It is not clear that Condorcet consistency is an appropriate condition when more than one candidate is to be chosen, but STV does not satisfy Condorcet consistency even when just one candidate is chosen. If some candidate is everyone’s second choice, that candidate can beat all others in head-to-head comparisons while having no first-place votes, and therefore be eliminated first under the single transferable vote.

### **A Further Refinement of the Single Transferable Vote**

It is possible to design a rule in the spirit of the single transferable vote that possesses Condorcet consistency when one candidate is elected, if the sequential eliminations of STV are replaced with a system of simultaneous comparisons of all possible pairs of outcomes (sets of candidates of the prescribed size). I call the resulting rule CPO-STV, for comparison of pairs of outcomes by the single transferable vote.

The comparison of two outcomes in CPO-STV proceeds somewhat like plain vanilla STV, beginning with the calculations of a quota. However, it is then necessary to list every set of potential winners and then compare these sets two at a time. In each such comparison of two sets of potential winners, each vote is allocated to the first candidate in that voter’s ranking that is in at least one of the two sets being compared. However, votes are transferred only from those candidates (if any) that have more than quota *and* are in *both* sets. The count is finished as soon as all such surpluses have been transferred. The result is given by the difference between the sum of the votes of the candidates in one set and the sum of the votes of candidates in the other set. When all pairs of outcomes have been compared in this fashion, the winning set is the set, if there is one, that beats all other sets in these head-to-head comparisons. If there is none, the winner is the set whose worst loss is least bad.<sup>3</sup>

<sup>3</sup>A more extensive discussion of CPO-STV is available from the author on request.

CPO-STV is computationally tedious, and for an election with several winners and many candidates it may not be feasible. Still, the task is not as difficult as it might seem. If a set is found that beats all others, no other comparisons are needed. Furthermore, the growing speed of computers and especially the advent of parallel processing make computational cost not nearly as serious a problem for CPO-STV as it would have been just a few years ago. Nevertheless, because the number of possible outcomes can be exceedingly large (6,435 for selecting 7 of 15 options; over 184,000 for selecting 10 of 20 options), it will be important to achieve computational efficiencies if CPO-STV is to be feasible for elections involving more than a few candidates. Further work will be needed to determine whether there are sufficiently fast algorithms for finding CPO-STV winners.

For the election of a single candidate, CPO-STV reduces to the min-max rule—that is, the winner is the candidate whose worst loss in paired comparisons is least bad. For the election of  $K$  out of  $K + 1$  candidates, it turns into ordinary STV. The reason for this is that when there is only one candidate who will not be elected, the distribution of all surpluses under STV leaves all candidates with exactly a quota of votes. The candidate who is not elected (assuming that there is no tie) will be the only one who has not achieved a surplus to distribute. Each comparison by CPO-STV of the set that wins under STV with an alternative set will involve picking one of the candidates who wins under STV, along with the candidate who loses under STV, to not have their surpluses distributed. The selected winning candidate will accumulate a positive surplus while the defeated candidate will accumulate less than a quota, so that each such comparison will show the set that wins under STV defeating the selected alternative set, and the set of winners under STV will be the winners under CPO-STV. Thus CPO-STV can be regarded as a synthesis of STV and the min-max rule.

It may be that along with Condorcet consistency, the CPO-STV rule also possesses nonnegative responsiveness. But I have neither a proof nor a counterexample.

## **The Refinement-Comprehensibility Trade-Off**

Each refinement of the single transferable vote answers an objection to an earlier version, but at some cost in computations or in making the rule less comprehensible. Are such refinements worth their costs? Some members of the Electoral Reform Society who are concerned with spreading the acceptance of the single transferable vote—and have considerable experience with trying to explain it—believe that any rules more sophisticated than those introduced in the early 1970s for elections in Northern Ireland (a variation on the Senatorial rules) would be unacceptable to a general electorate. Others would stop at the Newland and Britton rules or the Meek rules. However, whatever the tolerance of computational cost and complexity may be, the more sophisticated rules

provide valuable insights into the cost of overcoming limitations of the simpler rules. Also, experimenting with sophisticated rules can reveal the losses, if any, from using simpler rules.

In experimenting with alternative rules for the single transferable vote, I have found that, when more sophisticated rules make differences in real elections, they generally produce outcomes that the simpler rules would have produced with changes of very few votes. Still, the more sophisticated rules generally do yield outcomes that are more defensible when there is a difference. Thus it is sensible to use the most sophisticated STV rule that engenders no significant opposition for its sophistication, and rest assured that no great harm is done.

■ *I am grateful to David Hill, Timothy Taylor, Irwin Mann, Carl Shapiro and Alan Krueger for comments on earlier drafts.*

## References

- Arrow, Kenneth**, *Social Choice and Individual Values*. 2nd ed., New Haven: Yale University Press, 1963.
- Bordes, Georges, and Nicolaus Tideman**, "Independence of Irrelevant Alternatives in the Theory of Voting," *Theory and Decision*, March 1991, 30, 163–86.
- Dummett, Michael**, *Voting Procedures*. Oxford: Clarendon Press, 1984.
- Gibbard, Allan**, "Manipulation of Voting Schemes: A General Result," *Econometrica*, July 1973, 41, 587–601.
- Hare, Thomas**, *Treatise on the Election of Representatives, Parliamentary and Municipal*. London: Longman, Green, Reader, and Dyer, 1859.
- Hill, I. D.**, "Some Aspects of Elections: To Fill One Seat or Many," *Journal of the Royal Statistical Society, Series A*, 1988, 151:2, part 2, 243–75.
- Hill, I. D., B. A. Wichmann, and D. R. Woodall**, "The Single Transferable Vote by Meek's Method," *Computer Journal*, 1987, 30:3, 277–81.
- Hoag, Clarence, and George Hallett**, *Proportional Representation*. New York: MacMillan, 1926.
- Mann, Irwin**, STV computer program written in 1973, available from the author at 26 W. 10th St., New York, NY 10011.
- Meek, Brian**, "Une Nouvelle Approche du Scrutin Transférable," *Mathématiques et Sciences Humaines*, 1969, 7:25, 13–23.
- Newland, Robert, and Frank Britton**, *How to Conduct an Election by the Single Transferable Vote*. 1st ed., 2nd ed., London: Electoral Reform Society of Great Britain and Ireland, 1973, 1976.
- Satterthwaite, Mark**, "Strategy-proofness and Arrow's conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions," *Journal of Economic Theory*, April 1975, 10, 187–217.
- Warren, C. H. E.**, "Counting in STV Elections," unpublished paper, 19 Rectory Road, Farnborough, Hants GU14 7BU, England. 1983.