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A General Equilibrium Approach To Monetary Theory

I WILL TAKE THE OPPORTUNITY provided by the first issue of a journal devoted to monetary economics to set forth and illustrate a general framework for monetary analysis. It is not a new approach, but one shared at least in spirit by many monetary economists. My purpose here is exposition and recapitulation.[1]

1. *The capital account.*—The approach focuses on the capital accounts of economic units, of sectors of the economy, and of the economy as a whole. A model of the capital account of the economy specifies a menu of the assets (and debts) that appear in portfolios and balance sheets, the factors that determine the demands and supplies of the various assets, and the manner in which asset prices and interest rates clear these interrelated markets. In this approach, monetary assets fall into place as a part, but not the whole, of the menu of assets; and the commercial banking system is one sector, but not the only one, whose balance sheet behavior must be specified.

Treatment of the capital account separately from the production and income account of the economy is only a first step, a simplification to be justified by convenience rather than realism. The strategy is to regard income account

[1] Among my many debts, I will acknowledge here a special one to my colleague and, on occasion, collaborator, William C. Brainard, who has helped to develop and clarify the approach here expounded. He is not responsible, however, for errors and confusions that may remain in this particular exposition. See also Tobin and Brainard, "Financial Intermediaries and the Effectiveness of Monetary Controls," *American Economic Review*, 53 (May, 1963), pp. 383–400 and Brainard, "Financial Intermediaries and a Theory of Monetary Control," *Yale Economic Essays*, 4 (Fall, 1964), pp. 431–482. These papers are reprinted as Chapters 3 and 4 in *Financial Markets and Economic Activity*, ed. Hester and Tobin. Cowles Foundation Monograph 21, (New York: Wiley, 1967).

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variables as tentatively exogenous data for balance-sheet behavior, and to find equilibrium in the markets for stocks of assets conditional upon assumed values of outputs, incomes, and other flows. Of course the linkages run both ways. Some of the variables determined in asset markets affect the flows of spending and income. In a complete equilibrium the two sides of the economy —one is tempted to call them "financial" and "real"—must be mutually consistent. That is, the financial inputs to the real side must reproduce the assumed values of the real inputs to the financial side.

A familiar and simple example of this strategy is the "*LM* curve." Macroeconomics texts and lectures have immortalized Hicks's decomposition of the Keynesian system into sub-models. One of these tells what asset stock equilibrium corresponds to any tentative assumption about aggregate real income and the commodity price level. In this conditional equilibrium "the" interest rate equates the demand and supply of money and clears the markets for other assets. Of the many *LM* equilibria, only one is in general consistent with the other relationships in the complete system.

The key behavioral assumption of this procedure is that spending decisions and portfolio decisions are independent—specifically that decisions about the accumulation of wealth are separable from decisions about its allocation. As savers, people decide how much to add to their wealth; as portfolio managers, they decide how to distribute among available assets and debts the net worth they already have. The propensity to consume may depend upon interest rates, but it does not depend *directly* on the existing mix of asset supplies or on the rates at which these supplies are growing.

Figure 1 illustrates schematically the approach just sketched.

2. *Accounting framework.*—The general accounting framework for a theory of the capital account is indicated in Table 1. Rows represent assets or debts. A row might be labeled "money" or "physical capital," or in a finer classification "demand deposits" or "producers' durable equipment." Columns represent sectors of the economy: for example, commercial banks, central government, nonbank financial institutions, public. Entries in cells, in general, can be positive, negative, or zero. A negative entry means that the sector in question is a debtor in the kind of asset indicated by the row. All holdings must be valued in the same numéraire, e.g., either in the monetary unit of account or in terms of purchasing power over consumer goods. The sum across a row is the net exogenous supply of the asset to the economy as a whole. For stocks of goods, this exogenous supply is the economy's inheritance from the past. For internally generated financial assets the net exogenous supply is, of course, zero. If from the sums in the final column the central government's holdings of an asset are subtracted (or its debt added), the net holdings of the private economy result. The sum of a column represents the net worth of a sector. The sum of the final column is national wealth. As indicated, private wealth differs from this total by the amount of the government's net worth. If the

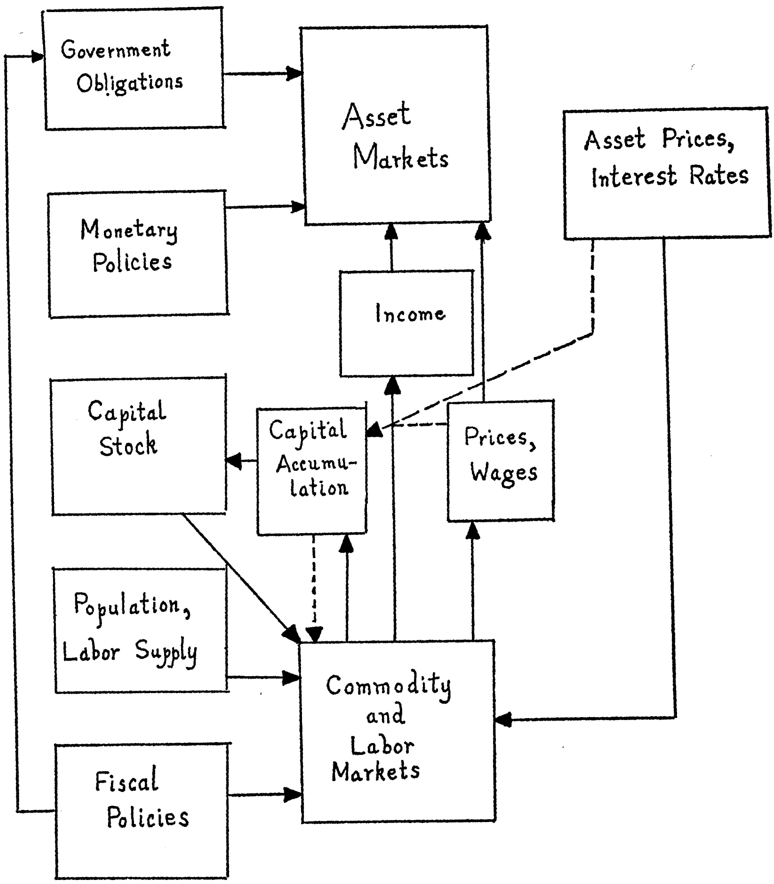

FIG. 1. Capital Account Approach (Schematic).

TABLE 1

GENERAL ACCOUNTING FRAMEWORK

Assets	Sectors of the Economy 1	2	3	.	.	. m	Central Government	Net Total Holdings = Exogenous Supply
1								
2								
3								
.								
.								
.								
n								
Net Worth	Total Private Wealth (= National Wealth less Government Net Worth)						Government Net Worth	National Wealth

government is a net debtor, as will typically be the case, at least if its stocks of goods are ignored, then private wealth exceeds national wealth. The framework illustrated by Table 1 is intended for a closed economy, although it could be extended to include capital-account relations with the rest of the world.

3. *The analytical framework.*—The accounting framework of Table 1 can be brought to life as a framework for monetary analysis by (a) assigning to each asset a rate of return r_i, $(i = 1, 2, \cdots n)$ and (b) imagining each sector $j(j = 1, 2, \cdots m)$ to have a net demand for each asset, f_{ij}, which is a function of the vector r_i and possibly of other variables as well. Of course in practice many of the cells are empty; certain sectors are just not involved with certain assets, either as holders or as debtors.

Each sector is, at any moment of time, constrained by its own net worth. Its members are free to choose their balance sheets—the entries in the columns of Table 1—but not to choose their net worth—the sum of the column entries. This is determined by their past accumulations of assets and by current asset prices. The individual economic unit can neither change the legacy of the past nor, it is assumed, affect by his own portfolio choices the current market valuations of his assets. Of course, as time passes the individual may save and may make capital gains or losses. A year later his net worth will be different, but it will be once again a constraint on his portfolio behavior.

This adding-up requirement has certain obvious and simple implications. For any sector, the sum over all assets of responses to a change in any rate of return r_k is zero:

$$\sum_{i=1}^{n} \frac{\partial f_{ij}}{\partial r_k} = 0.$$

This is also true for any other variable that enters the sector's asset demand functions. The exception is the sector's net worth itself; clearly the sum of asset changes due to a change in wealth is equal to one:

$$\sum_{i=1}^{n} \frac{\partial f_{ij}}{\partial W_j} = 1.$$

These same properties will hold for demand functions aggregated over sectors, that is for $f_i = \sum_{j=1}^{m} f_{ij}$.

Each row in Table 1 corresponds to one market-clearing equation, by which the net demands of the m private sectors add up to the available supplies, whether issued by the government or otherwise exogenous. But these n equations are not independent. Whatever the values of the determining variables, the left-hand sides (net private demands) of these n asset equations sum to the same value as the right-hand sides (supplies), namely to aggregate private wealth. Therefore, contrary to superficial first impression, the n equations will not determine n rates of return but only $n - 1$ at most.

The value of aggregate or sectoral wealth may depend on asset prices, which are themselves related to the r_i, the market rates of return, determined by the system of equations. This will be true of all assets whose life exceeds the length of the assumed period of portfolio choice. For example, the outstanding supplies of durable physical capital and of long-term government bonds change in value as their market rates of return change. Consequently, the $n - 1$ market-clearing equations actually include rates of return in two roles, as arguments in the asset demand functions and as determinants of the values of existing asset supplies and total wealth.

In some applications of the analysis there are fewer than $n - 1$ rates of return free to be determined. There are fewer endogenous rates of return than there are independent market-clearing equations. Some rates are institutionally or legally fixed—consider the conventional zero own-rate of interest on currency, the prohibition of interest on demand deposits, effective ceilings on interest paid on time and savings accounts. Some are constrained, at least in the long run, by real factors—for example, by the technological marginal productivity of physical capital assets. In these cases the capital account equations cannot be satisfied unless some asset supplies are not exogenous but adjust to clear the markets, or unless some relevant variables from the real side of the economy—income, price level, price expectations—assume appropriate values. I will return to these problems in the illustrations that follow.

4. *A money-capital economy.*—I turn now to some simple applications of the approach just described. First, consider an economy with only one private sector and only two assets: money issued by the government to finance its budget deficits, and homogeneous physical capital. Let p be the price of currently produced goods, both consumer goods and capital goods. I shall, however, allow the value of existing capital goods, or of titles to them, to diverge from their current reproduction cost—let qp be the market price of existing capital goods. Let r_M and r_K be the real rates of return available from holding money and capital respectively. Let $\rho_p{}^e$ be the expected rate of change in commodity prices, let $r_M{}'$ be the nominal rate of interest on money (generally, zero), and let R be the marginal efficiency of capital relative to reproduction cost. Let W be wealth and Y income, both measured in commodity prices.

Model I is as follows:

Wealth definition:

$$W = qK + M/p \tag{I.0}$$

Balance equations:

$$f_1(r_K, r_M, Y/W)W = qK \quad \text{capital } (r_K) \tag{I.1}$$

$$f_2(r_K, r_M, Y/W)W = M/p \quad \text{money } (r_M) \tag{I.2}$$

Rate-of-return equations:

$$r_K q = R \quad \text{capital} \tag{I.3}$$

$$r_M = r_M' - \rho_p^e \quad \text{money} \tag{I.4}$$

The two portfolio behavior functions have been written in a special form. They are homogeneous in wealth; the proportions held in the two assets are independent of the absolute scale of wealth. The "adding-up" requirement tells us that $f_1 = 1 - f_2$; therefore, one of the two balance equations, let it be I.1, can be omitted. It is natural to assume the own-rate derivatives $\partial f_1/\partial r_K$ and $\partial f_2/\partial r_M$ to be positive and the cross-derivatives therefore to be negative.

The ratio of income to wealth appears in both asset demand functions; if it appears in one, it must be in the other one too. The conventional assumption is that more money will be "needed for transactions purposes" at higher income levels. The implication is that the demand for capital will, other things equal, be reduced by a rise in income. However, "other things" will not be equal if on the real side of the economy there is a positive connection between Y and R, and therefore between Y and r_K.

Whether income falls with wealth constant or wealth rises with income constant, a smaller fraction of wealth is needed to meet transactions requirements. The demand for money will fall relative to the demand for capital. I shall make the usual Keynesian assumption that the partial elasticity of demand for money with respect to income is positive but does not exceed one. The reasoning is that transactions demand is, at most, proportional to income (elasticity equal to one), but transactions balances are only part of money holdings. The assumption is, then, that

$$0 < \frac{\partial(f_2 W)}{\partial Y} \Big/ \frac{f_2 W}{Y} = \frac{\partial f_2}{\partial(Y/W)} \Big/ \frac{f_2}{Y/W} \leqq 1.$$

Equation I.3 expresses an inverse relation between the market valuation of capital equity and the market rate of return upon it. Suppose that the perpetual real return obtainable by purchasing a unit of capital at its cost of production p is R. If an investor must pay qp instead of p, then his rate of return is R/q. The consol formula of I.3 applies strictly only for perfectly durable capital. For depreciating capital, or physical assets of finite life, the relation of r_K and q will not be so simple or so pronounced. But there will still be an inverse relation.

Note that the commodity price level p does not affect the real rate of return on capital, calculated either on reproduction cost or on market value. However, the expected rate of inflation of commodity prices does enter portfolio behavior, as one of the constituents of the real rate of return on money in I.4.

With I.1 omitted as redundant, Model I consists of four equations. The interpretation of the model depends on which four variables are taken as endogenous.

5. *Short-run interpretation of the money-capital model.*—One interpretation (IA) is the following:

Endogenous variables: r_K, r_M, W, q

Exogenous variables: K, M, Y, p, R, $\rho_p{}^e$, $r_M{}'$

Then, by (I.4) r_M is, in effect, exogenous. By various substitutions the model can then be expressed as a single equation in q:

$$f_2\left(R/q, r_M, \frac{Y}{qK + M/p}\right)(qK + M/p) = \frac{M}{p} \tag{I.5}$$

The assumptions made in the previous section are sufficient, not necessary, to assure that $\partial q/\partial M > 0$, in words that an increase in the quantity of money is expansionary, causing a rise in the valuation of existing capital and stimulating investment. The same conditions assure that $\partial q/\partial R > 0$, i.e., that an increase in the marginal efficiency of capital pulls up its price; that $\partial q/\partial r_M < 0$, i.e., that an increase in the real rate of interest on money diminishes the valuation of capital; and that $\partial q/\partial Y < 0$, i.e., that asset equilibrium requires a lower valuation of capital the higher the level of income relative to asset stocks.

This last result leads to the observation that, as part of a short-run model of income determination, equation (I.5) can be interpreted as a species of the standard Keynesian *LM* curve. That is, it tells what combinations of real income Y and the rate of return on capital equity, r_K or R/q, are compatible with equilibrium in asset markets (Figure 2). Like the textbook *LM* curve, this relationship shifts to the right when M increases or p diminishes. The difference is that the interest rate on the vertical axis here is the return on capital equity rather than Keynes's long-term bond rate. However, Keynes was assuming the two rates to be equal, or to differ only by a constant risk premium. If this assumption is dropped, R/q is the appropriate variable for the diagram, which needs to be completed by an *IS* curve. The *rate* of investment—the speed at which investors wish to increase the capital stock—should be related, if to anything, to q, the value of capital relative to its replacement cost.

The *LM* curve of Figure 2 was drawn on the assumption of a fixed marginal efficiency of capital, R. If R rises with Y, $\partial q/\partial Y$ will be smaller than with R constant, and may even be negative. In Keynesian theory there has always been ambivalence on this point, between the apparent view of Keynes himself that investors' estimate of the marginal efficiency of capital is related to a future largely independent of the current level of income and the view that

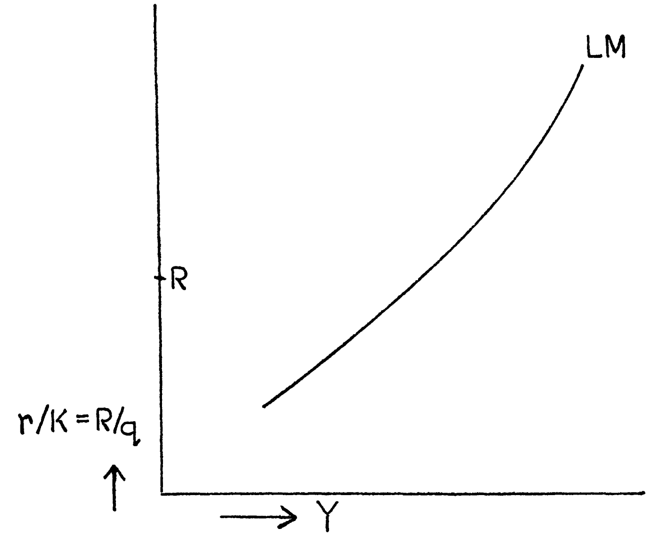

FIG. 2. *LM* Curve Drawn from Equation (I.5).

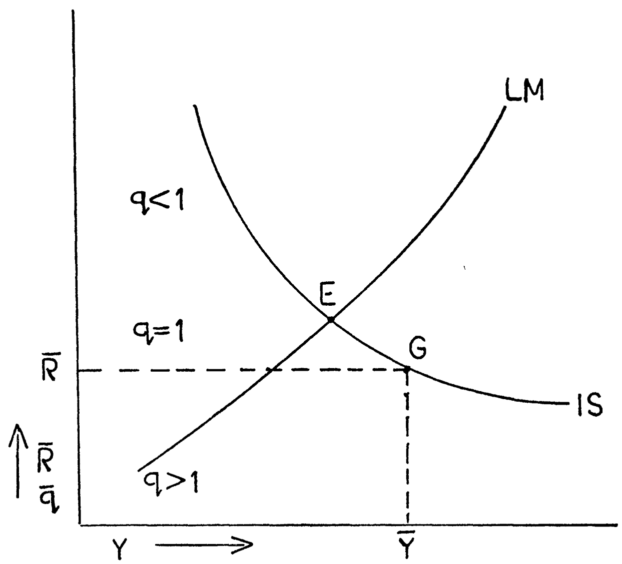

FIG. 3. *LM* Curve Plotted with *IS* Curve.

investors simply expect the current rate of profit on capital to continue. If, in line with the second view, some dependence of R on Y is built into the LM curve, there is no one-to-one relation between r_K and q.

Consequently, Figure 3 plots the LM curve against $\bar{R}/q$, where $\bar{R}$ is the marginal efficiency of the existing capital stock K at a standard real income $\bar{Y}$. This standard income $\bar{Y}$ is the level at which saving would just suffice to increase the capital stock at the natural rate of growth of the economy. For example, let this growth rate be g and the saving ratio s; then $gK = s\bar{Y}$. Investment at this rate will, under the usual assumptions of neo-classical growth theory, keep $\bar{R}$ unchanged. Consequently, investment at this rate is compatible with $q = 1$, $rK = \bar{R}$. In other words, the *IS* curve goes through the point $(\bar{R}, \bar{Y})$.[2] At an income lower than $\bar{Y}$, this normal rate of investment will exceed saving; therefore, investment-saving equality requires a q less than one. The short-run equilibrium—for a given real money supply M/p—is shown as E in Figure 3; in this illustration it occurs at a lower income level and equity valuation than the steady growth position G.

6. *Long-run equilibrium in the money-capital model.*—An alternative interpretation of Model I(IB) requires that capital be valued at its reproduction cost, i.e., that $q = 1$. This may be regarded as a condition of equilibrium in the long run. In a long-run growth equilibrium, E and G in Figure 3 must coincide; moreover this income $\bar{Y}$ must also represent equilibrium of labor supply and demand. Then if M/p, R, Y, and K are given, they determine r_K and W. Equation I.2 must then determine r_M, the real rate of interest on money. That is, either expectations of price change $\rho_p{}^e$ or the own-rate on money r_M' must be market-determined rather than institutionally or legally fixed. Otherwise, there is no way of reconciling wealth-owners to the supplies of capital and real balances that history and policy have determined.

Alternatively, if r_M is fixed, the supplies of capital and money, measured in real terms, must be free to adjust to public portfolio preferences. Models of the role of outside money in long-run growth show how this adjustment can occur.[3] One mechanism is flexibility in the price level p, which assures that any nominal supply of money M can be turned into the real supply that the public wants at the prevailing set of real interest rates. Another possible mechanism is fiscal policy itself, adjusting the size and rate of expansion of the government debt so as to achieve equilibrium.

7. *A money-securities-capital model.*—In Model I there is no monetary policy as this term is generally understood. The supply of money is identical with the government debt. It is not possible to increase money by a dollar without simultaneously increasing private wealth by a dollar. They rise together in money value when the government runs a budget deficit and prints money to cover it, or in real value when the price level falls. An increase in the nominal money stock is a monetary consequence of fiscal policy rather than monetary policy in the usual sense. The closest conceivable thing to monetary policy in model IA is variation of r_M', the institutionally determined rate of interest on money.

[2] Jerome Stein has insisted on this property of the short-run investment schedule. See his paper, "Money and Capacity Growth," *Journal of Political Economy*, 74 (October, 1966), 451–65.

[3] See Tobin, "Money and Economic Growth," *Econometrica*, Vol. 33, No. 4 (October, 1965), pp. 671–84.

Monetary policy can be introduced by allowing some government debt to take non-monetary form. Then, even though total government debt is fixed at any moment of time, at least in terms of its original money value, its composition can be altered by open market operations—or by debt management operations, which are really the same thing. Model II makes this emendation of Model I: (Let $\hat{r}$ stand for the vector of real rates of return (r_K, r_M, r_S)). Wealth definition:

$$W = qK + \frac{M + S}{p} \tag{II.0}$$

Balance equations:

$$f_1(\hat{r}, Y/W)W = qK \quad \text{capital } (r_K) \tag{II.1}$$

$$f_2(\hat{r}, Y/W)W = M/p \quad \text{money } (r_M) \tag{II.2}$$

$$f_3(\hat{r}, Y/W)W = S/p \quad \text{gov't. securities } (r_S) \tag{II.3}$$

Rate-of-return equations:

$$r_K q = R \quad \text{capital} \tag{II.4}$$

$$r_M = r_M' - \rho_p{}^e \quad \text{money} \tag{II.5}$$

$$r_S = r_S' - \rho_p{}^e \quad \text{gov't. securities} \tag{II.6}$$

Here it is assumed for simplicity that securities are short-term, so that their market value is independent of their interest rate r_S'. Otherwise a relationship between the two could be introduced, playing the same role as II.4 for capital, and allowed for in the calculation of wealth.

An interpretation analogous to IA takes as exogenous Y, M, S, K, R, r_M', $\rho_p{}^e$, and p, leaving q, W, r_K, r_S, r_M, r_S' to be determined by the six independent equations. Consolidation gives the following two equations, along with the definition of W, to determine q and r_S:

$$f_2(R/q, r_M, r_S, Y/W)W = M/p \tag{II.7}$$

$$f_3(R/q, r_M, r_S, Y/W)W = S/p \tag{II.8}$$

It is assumed, as before, that the own derivatives of the f_i

$$\left(\frac{\partial f_1}{\partial r_K}, \frac{\partial f_2}{\partial r_M}, \frac{\partial f_3}{\partial r_S}\right)$$

are positive, and that all the cross-derivatives are non-positive. (It will be remembered also that $\sum_i \partial f_i/\partial x = 0$ for any x that appears as an argument in the functions f_i.) In other words, the assets are gross substitutes: the demand for each asset varies directly with its own rate and inversely with other rates.

It is also assumed, as before, that the partial elasticity of demand for money with respect to income is positive but does not exceed one. Moreover, now that government securities are available, it is assumed that they, rather than capital, absorb changes in transactions requirements for money. That is,

$$\frac{\partial f_3}{\partial(Y/W)} = -\frac{\partial f_2}{\partial(Y/W)} \quad \text{and} \quad \frac{\partial f_1}{\partial(Y/W)} = 0.$$

These assumptions lead to the conclusions presented in Table 2.

The first two columns represent increases in government debt taking one form or the other. The third column represents monetary policy in the shape of open market purchases. Here, unlike Model IA, it is possible to shift the *LM* curve of Figures 2 and 3 to the right by monetary policy in the usual sense. The fourth column represents monetary policy in the guise of an increase in the legally-determined interest rate on money.

What is the feature of money that leads to the results tabulated in the first three columns? That is, why does an increase in government debt in monetary form have a more expansionary effect than increase in government debt in the form of securities? And why is substitution of money for securities via open market purchases expansionary?

It is not because asset No. 1 has been called "money" and asset No. 2 "securities." It is not because asset No. 1 is a means of payment or has any other intrinsic properties asset No. 2 lacks. It is not that asset No. 1 bears no interest—it may or may not. These properties have nowhere entered the analysis, except in the general sense that they explain why the assets are not perfect substitutes for each other.

The essential characteristic—the only distinction of money from securities

TABLE 2

EFFECTS ON ENDOGENOUS VARIABLES OF INCREASE IN SPECIFIED EXOGENOUS VARIABLES, WITH ALL OTHERS HELD CONSTANT

Endogenous Variables	Exogenous Variables							
	M	S	M at expense of S	r_M'	Y	R	p	ρ_p^e
q	+	?	+	−	−	+	−	+
r_S	−	+	−	+	+	?	?	−
r_K	−	?	−	+	+	+	+	−

that matters for the results given above—is that the interest rate on money is exogenously fixed by law or convention, while the rate of return on securities is endogenous, market-determined. If the roles of the two assets in this respect were reversed, so also would be the economic impacts of changing their supplies. Conceivably the government could fix the interest rate on its time obligations and let the rate on its demand debts be determined in the market. Then the way for the central bank to achieve an expansionary monetary impact would be to buy money with securities!

When the supply of any asset is increased, the structure of rates of return, on this and other assets, must change in a way that induces the public to hold the new supply. When the asset's own rate can rise, a large part of the necessary adjustment can occur in this way. But if the rate is fixed, the whole adjustment must take place through reductions in other rates or increases in prices of other assets. This is the secret of the special role of money; it is a secret that would be shared by any other asset with a fixed interest rate.

As observed above, an n-asset economy will provide no more than $n - 1$ independent market-clearing equations. The system will determine, therefore, no more than $n - 1$ real rates of return. If the rate on one asset, "money," is fixed, then the market rate of return on capital can, indeed must, be among the $n - 1$ rates to be determined. This enables the monetary authority to force the market return on physical capital to diverge from its technological marginal efficiency—or, what is the same thing, to force the market valuation of existing capital to diverge from its reproduction cost. By creating these divergences, the monetary authority can affect the current rate of production and accumulation of capital assets. This is the manner in which the monetary authority can affect aggregate demand in the short run—diagrammatically, by moving the *LM* curve of Figure 3 to the left or right and changing its intersection with the *IS* curve.

If the interest rate on money, as well as the rates on all other financial assets, were flexible and endogenous, then they would all simply adjust to the marginal efficiency of capital. There would be no room for discrepancies between market and natural rates of return on capital, between market valuation and reproduction cost. There would be no room for monetary policy to affect aggregate demand. The real economy would call the tune for the financial sector, with no feedback in the other direction. As previously observed, something like this occurs in the long run, where the influence of monetary policy is not on aggregate demand but on the relative supplies of monetary and real assets, to which all rates of return must adjust.

8. *A model with bank deposits and loans.*—As a third and final illustration of the approach, consider an economy with two sectors rather than one. Model III has a banking system as well as a general public sector and adds two new assets—deposits and private loans—to the economy's menu of assets. There are also two new real rates of interest to be determined, r_D on deposits

and r_L on loans, and two new nominal rates, r_D' on deposits and r_L' on loans, to be established either exogenously or endogenously. A new interest rate relevant to the banks, the central bank discount rate d', (d in real terms) can also be introduced; this is another instrument of monetary control.

Let $\hat{r}$ be the vector of real interest rates (r_K, r_M, r_S, r_D, r_L, d). For convenience, both bank and public portfolio choices will be written as functions of $\hat{r}$. But it will be understood that the discount rate d is irrelevant to the public, and that the market rate on capital r_K is irrelevant to the banks, which are assumed not to hold such equity. For the same reason, the banks' asset demands could be expressed equally well in money values and related to money interest rates rather than real. The legal reserve requirement enters as k.

Asset No. 2 is still the demand debt of the government, inclusive of the central bank. The size of this debt, net of the banks' borrowings from the central bank at the discount window, is the supply of currency and unborrowed reserves to the banks and the public. But of course M no longer corresponds to the quantity of money as conventionally defined. Rather it represents "high-powered" money. The money stock would include the public's share of M plus bank deposits (or perhaps only demand deposits if, as is not done here, time deposits were distinguished from them). Thus the money stock would be an endogenous quantity.

Wealth definition:

$$W = qK + \frac{M + S}{p} \tag{III.0}$$

Balance equations:

Sector:

Banks *Public*

$$f_{1P}(\hat{r}, Y/W)W = qK \text{ (capital } (r_K)) \tag{III.1}$$

$$kD + f_{2B}(\hat{r})D(1 - k) + f_{2P}(\hat{r}, Y/W)W = M/p \text{ (currency and reserves) } (r_M, d) \tag{III.2}$$

$$f_{3B}(\hat{r})D(1 - k) + f_{3P}(\hat{r}, Y/W)W = S/p \text{ (government securities } (r_S)) \tag{III.3}$$

$$f_{4B}(\hat{r}) + f_{4P}(\hat{r}, Y/W)W = 0 \text{ (deposits } (r_D)) \tag{III.4}$$

$$D = f_{4P}(\hat{r}, Y/W)W \text{ (definition of } D) \tag{III.4a}$$

$$f_{5B}(\hat{r})D(1 - k) + f_{5P}(\hat{r}, Y/W)W = 0 \text{ (loans } (r_L)) \tag{III.5}$$

Rate-of-return equations:

$$r_K q = R \text{ (capital)} \tag{III.6}$$

$$r_M = r_M' - \rho_p^e \text{ (currency and reserves)} \tag{III.7}$$

$$r_S = r_S' - \rho_p^e \text{ (government securities)} \tag{III.8}$$

$$r_D = r_0' - \rho_p^e \text{ (deposits)} \tag{III.9}$$

$$r_L = r_L' - \rho_p^e \text{ (loans)} \tag{III.10}$$

$$d = d' - \rho_P^e \text{ (discount rate)} \tag{III.11}$$

The equity of bank shareholders is ignored, so that the items in the bank column sum to zero, just as the items in the public column sum to private net worth W.

There are eleven independent equations. As before, Y, M, S, K, R, r_M', ρ_P^e, p, d', and K may be taken as exogenous and the system solved for the eleven variables q, W, r_K, r_M, r_S, r_S', r_0, r_D', r_L, r_L', and d. In this interpretation of model III, the interest rate paid on deposits is endogenous, market-determined. The banks' deposit supply function f_{4B} tells, for given values of other interest rates, the quantity of deposits banks wish to accept at any given deposit rate. In equilibrium this must be equal to the quantity of deposits the public wishes to hold at this same set of rates.

As before, the effects of various instruments of monetary policy and of other exogenous variables on the key variable q represents their impact on aggregate demand. With the same assumptions about asset substitution, and about income-elasticity of demand for high-powered money, the results will be qualitatively the same as in the other models. They will be quantitatively very different, of course. Fractional reserve banking means that a bigger reshuffling of portfolios and larger changes in rates of return are needed to absorb a given increase in the supply of high-powered money. To the extent the banks are not induced to add the new supply to their excess reserves, the public must be induced to hold some multiple of it as deposits. The change in rates of return necessary to accomplish either of these results, or any combination of them, may be very large in comparison with the 100 per cent money regime depicted in models I and II.

An alternative interpretation is to take the deposit rate r_D' as institutionally or legally fixed. Adding it to the list of exogenous variables means that one equation must be deleted. The one to delete, of course, is III.4. With an effective ceiling on the interest banks are allowed to pay, banks fall short of their supply curve $(-f_{4B})$. They accept all the deposits the public is willing to leave with them at the prevailing set of interest rates, and they would gladly accept more. Thus III.4 becomes an inequality: $f_{4B} + f_{4P} > 0$. The remaining equations in the model, including III.4a, still apply.

This interpretation is the one customarily made. It accords with United

States institutions—prohibition of interest on demand deposits and a ceiling on time deposit interest. Once again the effects on q of policy measures and other exogenous changes can be analyzed. Here, however, there is a new possible source of abnormal results. The "gross substitutes" assumption may be violated in the market as a whole even though it is satisfied by each sector—banks and public—separately. For example, an increase in the deposit rate or a reduction in the securities rate might increase rather than diminish the net demand for currency or government securities. While the public's direct demands fall as they shift into deposits, the banks' demands may increase simply because they have more deposits.[4]

This formulation adds the deposit rate ceiling to the list of monetary policy instruments and permits analysis of the question whether an increase in the ceiling is expansionary or contractionary.

9. *Concluding remarks.*—The models discussed here were meant to be illustrative only, and to give meaning to some general observations about monetary analysis. The basic framework is very flexible. It can be extended to encompass more sectors and more assets, depending on the topic under study. Other financial intermediaries can be introduced. More distinctions can be made among categories of government debts and types of private debts. Equally important, the assumption that physical capital is homogeneous can be dropped, and a number of markets, prices, and rates of return for stocks of goods introduced—distinguishing among houses, plant, equipment, consumers' durables, etc.

According to this approach, the principal way in which financial policies and events affect aggregate demand is by changing the valuations of physical assets relative to their replacement costs. Monetary policies can accomplish such changes, but other exogenous events can too. In addition to the exogenous variables explicitly listed in the illustrative models, changes can occur, and undoubtedly do, in the portfolio preferences—asset demand functions—of the public, the banks, and other sectors. These preferences are based on expectations, estimates of risk, attitudes towards risk, and a host of other factors. In this complex situation, it is not to be expected that the essential impact of monetary policies and other financial events will be easy to measure in the absence of direct observation of the relevant variables (q in the models). There is no reason to think that the impact will be captured in any single exogenous or intermediate variables, whether it is a monetary stock or a market interest rate.[5]

[4] These problems are analyzed in the Tobin-Brainard and Brainard papers cited above.

[5] This point has been illustrated in simulation of a numerical model on the order of Model III above. See Brainard and Tobin, "Pitfalls in Financial Model Building," *American Economic Review*, 58 (May, 1968), pp. 99–122.