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AN ESSAY ON THE PURE THEORY OF COMMODITY MONEY*

By JOHN K. WHITAKER

CLASSICAL authors like J. S. Mill (1848) and early neoclassical authors like W. S. Jevons (1875, 1884), L. Walras (1874, 1886) or A. Marshall (1871, 1887, 1899, 1923) premised monetary theory on an endogenously-supplied commodity money produced under free competition. This essay views their problem in modern dress, eschewing any attempt at exegesis or criticism of their writings. A clearer appreciation of the conceptual issues certainly assists understanding of nineteenth century monetary thought and phenomena. But it also illumines matters of perennial interest for monetary theory and proposals for monetary reform. Commodity money has been involved in several modern discussions, such as Friedman (1949), Buchanan (1962), Burstein (1963, pp. 97–100), Pesek and Saving (1967), Johnson (1967, 1969), Niehans (1969), Chen (1972), Fischer (1972), Luke (1975), all dealing with fundamental monetary issues, and a general treatment seems desirable.

The subsequent analysis attempts to establish a framework within which the logical implications of a simple commodity currency can be explored rather than on normative issues as to the practicability, desirability and optimal design of a commodity-currency or commodity-reserve scheme. For fuller consideration of such issues see particularly Friedman (1949), Graham (1962), Johnson (1967) and Barro (1979). The discussion must also be sharply distinguished from recent treatments of the competitive production of bank money by Klein (1974), Gramm (1975) and others.

Section I constructs a simple macroeconomic model, incorporating the production of commodity money into an otherwise-orthodox setting, with given factor supplies and demand conditions. The properties of this model are explored in Section II, while Section III contrasts the use of commodity money with the use of fiat money. It is shown, contrary to common belief, that commodity-money production might lower economic activity and prices for a given money stock, or make them more sensitive to exogeneous disturbance. Section IV develops a limiting partial-equilibrium approach, which is used to explore the implications of previously omitted non-monetary uses for the monetary commodity and a distributed-lag response of its production. Brief concluding remarks are in Section V. The more involved proofs are relegated to Appendices.

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I. *A simple macroeconomic model*

To avoid inessential complications assume

(i) a closed economy with fixed factor supplies and stationary technology,
(ii) no economic activity by the government,
(iii) the expected inflation rate always remains at its long-run or "natural" rate, which is zero in the models considered here,
(iv) 100%-reserve banking,
(v) no non-monetary uses for the money commodity.

Assumptions (i) and (ii) call for no comment beyond observing that an open economy would be subject to the usual specie-flow mechanism and that non-zero investment can be admitted only by invoking the usual *approximate* short-run constancy of the capital stock. The latter can be justified only as a pedagogic simplification, as there is no reason to suppose that the ratio of new production to existing stock is of a smaller order of magnitude for real capital than commodity money, yet accumulation of the latter is considered explicitly. Assumption (iii) is certainly a serious restriction, but any attempt at a more satisfactory treatment of expectations would only distract attention from the central issues by greatly complicating the arguments.

Competitive pressures must generate fractional-reserve banking unless prohibited, but the forced restriction to 100%-reserve banking merely sets the ratio between the total and commodity money at unity rather than some larger multiplier. There might in consequence be some saving in the resource cost of the banking system, but this cost will be disregarded here.[1] Finally, although it is doubtless the norm historically for money commodities to have been in earlier widespread use as commodities simpliciter, this is not a logical necessity.[2] Non-monetary use is admitted in Section IV, where it does not change the results crucially.

Denote production of the money commodity by x and production of a single aggregate representing all other commodities (hereafter referred to as "goods") by y. This goods aggregate can be devoted equally well to consumption or investment, as in orthodox one-sector models, and investment is assumed to suffer no depreciation. The physical characteristics of the money commodity are not of concern. It must be assumed to meet some of the standard prerequisites, but circulation could be by exchange of warehouse receipts. It cannot be either consumed or productively invested.

[1] The resource cost of banking differs in two essential ways from that of a commodity money. (i) It depends on the total stock of money rather than the addition to it. (ii) It depends on real rather than nominal balances (adding zeros to all accounts and transactions increases cost only negligibly). See Gramm (1974).

[2] See Einzig (1949) for a comprehensive survey of actual commodity moneys.

Assume that the money commodity is produced by labour and land alone, whereas goods are produced by labour, land and capital. Labour, but not land, is transferable between sectors. With given endowments of land, and approximate constancy of the capital stock, the production functions for new money and goods are

$$x = f_m(n_m) \tag{1}$$

$$y = f_g(n_g) \tag{2}$$

where n_m, n_g are the amounts of labour used in the two sectors. Assume for $i = m, g$ that $f_i'(0) = \infty$, $f_i'(\infty) = 0$, and that $f_i' > 0 > f_i''$ when $0 < n_i < \infty$. (The primes denote differentiation.) If each output is produced competitively, profits in terms of either goods or money are maximized when

$$f_m'(n_m) = w \tag{3}$$

$$f_g'(n_g) = w/p \tag{4}$$

where w is the money wage rate and p is the money price of goods ("the price level"). The peculiarity of producing money is that the marginal *physical* product has to equal the *money* wage rate. Conditions (3) and (4) define labour-demand functions $n_m = h_m(w)$ and $n_g = h_g(w/p)$, which in turn give rise to the supply functions

$$x = f_m(h_m(w)) \equiv S_m(w) \tag{5}$$

$$y = f_g(h_g(w/p)) \equiv S_g(w/p) \tag{6}$$

with $S_m' = w/f_m'' < 0$ and $S_g' = w/(pf_g'') < 0$.

Full employment of the labour supply N, assumed fixed, requires

$$N = h_m(w) + h_g(w/p) \tag{7}$$

This defines a relationship $p = \theta(w)$ with elasticity

$$w\theta'(w)/\theta(w) = (ph_m' + h_g')/h_g' \tag{8}$$

exceeding unity. When $p < \theta(w)$, producers are unwilling to hire all the available labour and there must be an excess supply. Conversely, when $p > \theta(w)$ there must be an excess demand for labour unless prevented by a Keynesian deficiency of demand for goods. The latter possibility is excluded until Section II, supposing meanwhile that all markets but the labour market are always cleared.[3] Any rationing of labour is supposed for simplicity to fall wholly on goods production. Thus, production of money is invariably given

[3] Since the excess demand or supply of labour is only "notional" and not "effective", it need not be offset by excess supply or demand in another market. Walras' law holds only over the equilibrated markets, one of which can be left implicit: the bond-market is chosen for this, as usual.

by $S_m(w)$, but production of goods is given by $S_g(w/p)$ only if $p \leqslant \theta(w)$ and by $S_g(\theta(w)/w)$ if $p > \theta(w)$.[4]

If consideration is restricted to full-employment situations in which the labour market is also cleared, so that $p = \theta(w)$ always, the full-employment supply functions

$$x = S_m(\theta^{-1}(p)) \equiv x(p), \qquad x'(p) < 0 \tag{9}$$

$$y = S_g(\theta^{-1}(p)/p) \equiv y(p), \qquad y'(p) > 0 \tag{10}$$

apply. $x(p)$ and $y(p)$ give a parametric representation of the economy's transformation frontier, which has slope

$$dx/dy = x'(p)/y'(p) = -f'_m/f'_g = -p \tag{11}$$

and therefore the usual convex-outward shape. As p rises from zero to infinity, y rises from zero to $y_{\max} \equiv f_g(N)$, x falls from $f_m(N)$ to zero, and w/p falls from infinity to $f'_g(N)$. From (11), full-employment real income, $y(p) + x(p)/p$, falls with p, so that

$$y(p) + x(p)/p > y_{\max} = y(\infty) > y(p) \tag{12}$$

for any $p \geq 0$.

The demand for goods, y^d, has both consumption and investment components. Adopting a standard formulation which neglects distributional effects, suppose that

$$y^d = \xi(v, r, M/p); \qquad 0 < \xi_1 < 1, \quad \xi_3 > 0 > \xi_2 \tag{13}$$

ξ_i is the partial derivative of ξ with respect to the ith argument. v is real disposable income. M is the existing stock of money, so that M/p is real balances, surely net wealth with a commodity money.[5] r is the rate of interest on (one-period) private bonds used to finance investment: the nominal and (expected) real interest rates coincide given assumption (iii) above. On the assumption that all markets but perhaps the labour market are cleared, we have

$$y^d = y; \qquad v = y + x/p; \qquad M^d = M \tag{14}$$

where y and x are given by the relevant supply functions, and M^d is the demand for money to hold.

[4] Attention will be restricted throughout to $w > w_{\min} > 0$, where $h_m(w_{\min}) = N$, so that money production does not absorb the whole labour force. Note that $\theta(w)/w = 0$ for $w = w_{\min}$ and increases towards $1/f'_g(N)$ as $w \to \infty$.

[5] It is tempting to argue that the relevant price deflator for real income and real balances is an output-weighted average of the price of goods, p, and the price of newly-produced money, unity. But *in the absence of non-monetary use of the money commodity*, the number of monetary units produced or held is entirely without significance to the producer or holder: all that matters is purchasing power over goods. In particular money is *not* held in order to buy more money in the future, only more goods or bonds. See Section IV.

The demand for money function is taken—again adopting a standard formulation—as

$$M^d = p\psi(v, r); \qquad \psi_1 > 0 > \psi_2 \tag{15}$$

The use of v as a transaction variable follows Johnson (1967, p. 321). It could, indeed, be argued that money-producing firms might manage with somewhat smaller money balances by making some disbursements directly from the flow of output, but this is ruled out if money requires governmental certification before circulation, as will be assumed.

The elasticities of the functions ξ and ψ with respect to the ith argument will be denoted ε_i and η_i respectively. The interest elasticities ε_2 and η_2 are negative and all others positive. It is convenient to assume that the transactions elasticity of demand for money never exceeds unity, as seems plausible, so that

$$\eta_1 \equiv v\psi_1/\psi \leq 1 \quad \text{for all} \quad v, \qquad r > 0 \tag{16}$$

The general specification is completed by assuming that the money stock moves over time, t, according to

$$\mathrm{d}M/\mathrm{d}t = x - \delta M, \qquad \delta > 0 \tag{17}$$

where δ is the physical depreciation rate of the money commodity. For simplicity, the dampening effect upon the demand for goods of this depreciation, which has real value $\delta M/p$, is incorporated into the real-balance effect and assumed insufficiently strong to reverse its direction.

II. *The properties of the model*

The "short-term" equilibrium situation, with p and r clearing the goods and money markets for temporarily fixed values of M and w, is considered first. The movement of this equilibrium is then analysed, with w responding to the excess demand or supply for labour and M obeying (17). This movement continues unless or until a "long-term" equilibrium, with M and w unchanging, is reached. Perpetual full employment, with the labour market, as well as the goods and money markets, cleared for each value of M, is then viewed as a special case, and some less formal consideration is finally given to the stability of the short-term equilibrium and to non-clearing of the goods market.

For fixed values of M and w, the short-term equilibrium is defined by the goods- and money-market clearing conditions

$$\xi(S_m(w)/p + S_g(w/\sigma(p, w)), r, M/p) = S_g(w/\sigma(p, w)) \tag{18}$$

$$\psi(S_m(w)/p + S_g(w/\sigma(p, w)), r) = M/p \tag{19}$$

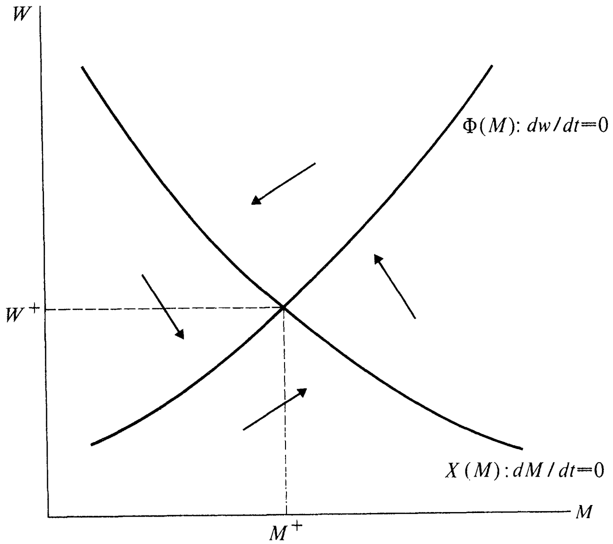
where $\sigma(p, w) = p$ for $p \leq \theta(w)$ and $\sigma(p, w) = \theta(w)$ for $p > \theta(w)$, when labour rationing arises. It is shown in Appendix A that the loci in (p, r) space (i.e. with p on the horizontal axis) defined by (18) and (19) for fixed w and M must have negative and positive slope, $\partial r/\partial p$, respectively, and thus can intersect only once. The short-term equilibrium (p^*, r^*) is thus unique if it exists. Assuming the latter for each relevant w and M, we have a function $p^* = p^*(M, w)$ which obviously increases with M, but not necessarily with w. Excess demand for labour in the short-term equilibrium is (see (7)) $h_g(w/p^*(M, w)) - h_g(w/\theta(w))$ which is positive or negative as $p^*(M, w) \gtrless \theta(w)$.

Assume now that

$$dw/dt = \lambda(h_g(w/p^*(M, w)) - h_g(w/\theta(w))), \lambda > 0 \tag{20}$$

Together with (17), (20) forms a pair of simultaneous differential equations in M and w. It is shown in Appendix B that there is an increasing function, $\phi(M)$, such that $dw/dt \gtrless 0$ as $w \lesseqgtr \phi(M)$. Moreover, (17) with $x = S_m(w)$ defines a decreasing function, $\chi(M)$, such that $dM/dt \gtrless 0$ as $w \lesseqgtr \chi(M)$. It follows that any long-term equilibrium (M^+, w^+), defined by $dM/dt = dw/dt = 0$, must be unique. That such an equilibrium exists and is locally stable, but perhaps approached cyclically, is shown in Appendix C. The implied phase diagram is shown in Fig. 1.[6]

The perpetual-full-employment case arises when the adjustment speed, λ, is increased without limit so that w jumps vertically to the $\phi(M)$ locus and



[6] Figure 1 suggests that stability may be global, but it is difficult to prove that limit cycles cannot occur.

then moves along it, keeping the labour market cleared, as M changes more slowly according to

$$dM/dt = S_m(\phi(M)) - \delta M \tag{21}$$

As Fig. 1 indicates, the resulting convergence of M to M^+ must be globally stable.

The long-term equilibrium value, M^+, for the money stock is such that depreciation δM^+ is just made good by new production $S_m(w^+)$. For a given supply function $S_m(\cdot)$, M^+ and w^+ are thus necessarily inversely related. Moreover, the higher is M^+ the lower must full-employment goods production be to permit higher money production, and hence the *higher* the real wage, $w^+/p^*(M^+, w^+)$. It is shown in Appendix B that an autonomous increase in the demand for goods, or decrease in the demand for money, raises the locus $\phi(M)$ bodily in Figure 1 and thus lowers M^+.

The intuitive basis for convergence of M to M^+ is obvious and familiar. If M is low, so that w also tends to be low, money production is attractive and exceeds replacement needs. Growth of M tends to raise p and w and discourages money production while increasing replacement needs. With perpetual full employment, M and w rise uninterruptedly to their long-term equilibrium values, or decline uninterruptedly if initially above them. With w responding only sluggishly to excess demand or supply, convergence to M^+ and w^+ may involve the alternation of intervals of unemployment and falling wages with intervals of excess demand for labour and rising wages.

Return now to the short-term situation with M and w both held fixed. Imagine that p temporarily differs from the market-clearing level $p^*(M, w)$. If $p < p^*(M, w)$, there must be excess demand for goods, while if $p > p^*(M, w)$ there must be excess supply.[7] This suggests that $p^*(M, w)$ is a stable short-term equilibrium for the price level, with p raised by excess demand if initially below equilibrium and lowered by excess supply if initially above.

With $p > p^*(M, w)$, so that goods are in excess supply, a fundamental distinction opens up between the production of goods and money. The production of money cannot be limited by demand so long as money retains general acceptability in exchange. But the production of goods must be demand constrained whenever there is an excess supply of goods. The notional excess supply of goods ceases to be an effective one, and output is cut back to meet the limited demand, which is in turn altered by induced

[7] Equations (18) and (19) continue to apply at any p where the r determined by (19) is less than the r determined by (18), implying that goods output is supply constrained and that excess demand for goods is present. This is true (see Appendix A) whenever $p < p^*(M, w)$. If $p > p^*(M, w)$ there cannot be excess demand in the goods market, since the r determined by (19) is above the goods-market clearing level that would be determined by (18) if output were supply constrained.

multiplier repercussions. y and r are now jointly determined for given p as the underemployment equilibrium of the ISLM system

$$\xi(S_m(w)/p+y, r, M/p)=y \tag{22}$$

$$\psi(S_m(w)/p+y, r)=M/p \tag{23}$$

This has a unique solution for given M, w, p since the IS curve defined by (22) has negative slope $\partial r/\partial y$, while the LM curve defined by (23) has positive slope.

Money production will not be altered by the presence of an excess supply of goods and labour, so long as w remains unchanged. But even if w remains rigid, a reduction in p will increase the real value of money production, $S_m(w)/p$, as well as increasing real balances, M/p. The increase in real balances exerts the usual Pigou effect through (22) and Keynes effect through (23), both of which serve to increase y. The increase in the real value of money production has an expansionary multiplier effect on y in (22) but a *deflationary* interest-rate-raising effect in (23). However, the latter only partially offsets the Keynes effect,[8] so that a reduction in p must increase y while simultaneously reducing the desired supply of goods. On the other hand, a reduction in w at fixed M, p could lower y by raising the interest rate enough to offset the direct multiplier effect of increased money production. Conceivably unemployment might be *increased* by such a wage cut, reduced employment in goods production as w falls more than offsetting increased employment in money production induced by the lower w. Nevertheless, a sufficient cut in w would clearly produce full employment by absorbing, if necessary, the entire labour force in money production.[9] Of course, this conclusion is subject, as Keynes (1936, pp. 230–1) emphasised, to the proviso that limited natural resources (such as gold reefs) do not set an upper limit to money production. Such a limit is excluded here by the assumption that the marginal product f'_m always remains positive. A related possibility, to be considered in Section IV, is that rapid movement of resources into or out of money production may be inhibited by adjustment costs.

[8] The price elasticity of M/p is -1. The price elasticity of ψ in (23) is (for fixed r, y) $-\eta_1 S_m/(S_m+py)$, which lies between 0 and -1 given (16). Thus r must fall as p is reduced with y held fixed.

[9] A curious perpetual-full-employment case arises if the assumed orders of magnitude for the adjustment speeds of wages and prices are reversed, so that w and r adjust to keep the labour and money markets cleared for any M and p. A generalisation, encompassing both this case and that of the text as limits, supposes that both w and p simultaneously change in proportion to the excess demands for, respectively, labour and goods, while r keeps the money market cleared and M remains temporarily fixed. The equilibrium $w=\phi(M), p=\theta(w)$ can then be shown globally stable for *all* choices of the adjustment speeds governing wage and price movements.

III. *Comparison with a fiat-money system*

The aim is to contrast an economy with commodity money against another, alike in all respects except for using a fiat money. From a *policy* viewpoint, comparison of the likely money stocks under these alternative monetary regimes is a crucial consideration in the choice between them. To its exponents, much of the appeal of a commodity money comes from its automaticity and impersonality, promising a degree of predictability for money creation in contrast to the potential vagaries of a fiat system, ever at the caprice of its managers and the political or social pressures besetting them. But from an *analytical* viewpoint, the determinants of money stock in a fiat system must remain somewhat imponderable. Moreover, admitting differences in the *size* of money stock obscures the narrower comparisons of economic activity and prices under alternative *means* of creating money. For these reasons, the same money stocks are assumed for both alternatives. Fiat money is supposed issued by open-market purchases of private securities,[10] and to not count as net wealth.[11]

Most discussions of the question presume that adopting a commodity money tends, at a given M, to boost economic activity or the price level, as well as stabilising them against exogeneous disturbance.[12] We will discover that these consequences may not follow. The ostensible direct multiplier effects of commodity-money production on the demand for goods may be offset by less obvious indirect effects working through the demand for money. These questions are considered first for a rigid-wage short-term equilibrium involving unemployment and then for the case of perpetual full employment.

The goods and money-market clearing conditions for the alternative monetary regimes can be written in general form as

$$\xi(y^f, r^f, 0) - y^f = 0 = \xi(y^c + x^c/p^c, r^c, M/p^c) - y^c \tag{24}$$

$$\psi(y^f, r^f) - M/p^f = 0 = \psi(y^c + x^c/p^c, r^c) - M/p^c \tag{25}$$

with y^f, p^f, r^f the levels of goods output, price level and interest rate in the fiat-money case, and y^c, p^c, r^c the corresponding variables for the commodity-money case, whose money output is denoted x^c. As well as the

[10] A more expansionary alternative would be to issue new money as an income transfer, thus replicating the disposable-income effect of commodity money without its resource-absorption or demand-for-money consequences. The effects of a commodity money would be exactly reproduced if fiat money was issued in exchange for resources used in "pyramid building" or "leaf raking".

[11] The lack of a wealth effect may reflect a fallacy of composition. Issuers of private securities held by the monetary authority may not feel relieved of debt, but collectively they are. If the money stock is to be maintained, the authority has no option but to receive interest and redemption payments in the form of further securities, ad. inf.

[12] See, for example, Friedman (1951).

value of M, all functions are to be assumed identical between the two cases—the sameness of the function ξ relying on the non-use of capital in money production, so that no investment demand is eliminated when this production is suppressed. The fiat-money case simply omits the income from money production, and also the real balance effect, since fiat money is not net wealth. The open-market operations by which fiat money is issued are assumed not to require the public to hold extra transactions balances.

Suppose first that w, as well as M, is held fixed at the same level in the two cases. Then $x^c = S_m(w)$ and, in the absence of labour rationing, $y^f = S_g(w/p^f)$ and $y^c = S_g(w/p^c)$. Labour rationing occurs in the commodity-money case if $p^c > \theta(w)$, defined by (7), and in the fiat-money case if $p^f > \theta^f(w) = wf_g'(N) > \theta(w)$. With labour rationing, $y^f = S_g(w/\theta^f(w))$ or $y^c = S_g(w/\theta(w))$. On these assumptions, both alternatives have a unique equilibrium if one exists, as will be supposed. Moreover, an autonomous increase in the demand for goods must increase both p^f and p^c, while an autonomous increase in the demand for money must lower them. (See Appendix A, whose arguments specialise to cover the fiat-money case also.) Suppose now that both alternatives are in underemployment equilibrium (i.e. $p^f < \theta^f(w)$ and $p^c < \theta(w)$).

It is easily seen that r^f must be less than r^c. Suppose it were not. Then $p^f > p^c$ would be needed for money-market equilibrium (for if $p^f \leq p^c$ then $y^c + x^c/p^c > y^f$ certainly holds, so that (in shorthand notation) $\psi^c > \psi^f$ while $M/p^c < M/p^f$.) But if $r^f \geq r^c$ and $p^f > p^c$ then goods-market equilibrium cannot hold in both cases. For we must have $y^f - \xi^f > y^c - \xi^c$, because in the fiat case (a) the higher production y^f increases ξ^f by a smaller amount since $\xi_1 < 1$, (b) r^f is no lower, (c) income from money production and the real-balance effects are absent.

But it is also easily seen that p^f may be higher or lower than p^c. For a given p^f, consider the trial solution $p^c = p^f$, so that $y^c = y^f$. To clear the money market, we must have $r^c > r^f$, but with $r^c > r^f$ we can have $\xi^c \gtrless \xi^f$, depending on whether the demand-reducing effect of a higher r in the commodity-money case either fails to offset or more than offsets the demand-increasing influences of the real-balance effect and positive income from money production. If $\xi^c > \xi^f$ we must have $\xi^c > y^c$, since $\xi^f = y^f = y^c$ by the assumptions that $p^c = p^f$ and p^f an equilibrium. Since there is excess demand for goods when the money market is cleared at the trial value of p^c, the true equilibrium value must exceed this trial value: hence, $p^c > p^f$. Conversely, if $\xi^c < \xi^f$ at the trial value then $p^c < p^f$.[13] The latter outcome is favoured by a high interest elasticity of demand for goods, ε_2, or a low interest elasticity of demand for money, η_2.

When the equilibrium p^c is below the equilibrium p^f, commodity money is

[13] These conclusions follow from the slope properties established in Appendix A.

the more deflationary alternative. It has a lower price level, a lower output of goods, and lower employment in goods production. It may, however, still have higher total employment because of the positive employment in money production, but this is by no means necessary: the only general restriction is that this employment be positive.[14]

If a log-linear approximation is taken to the functions, ξ, ψ and S_g (but with ξ log-linear in $1+M/p$ rather than M/p to avoid the difficulty otherwise arising when $M/P=0$) we have

$$A \log (p^f/p^c) = (\eta_1(\varepsilon_2/\eta_2) - \varepsilon_1) \log B - \varepsilon_3 \log E \tag{26}$$

Here $A = \gamma(1-\varepsilon_1+\eta_1(\varepsilon_2/\eta_2)+\varepsilon_2/(\gamma\eta_2))$, $B = 1+S_m(w)/(p^c S_g(w/p^c))$ and $E = 1+M/p^c$. The ε_i and η_j are the elasticities defined in Section I (all positive but ε_2 and η_2) and $\gamma > 0$ is the constant elasticity of supply of goods, so that $\log (y^f/y^c) = \gamma \log (p^f/p^c)$. In the fiat case $\varepsilon_1 = \xi_1 < 1$, so $\varepsilon_1 < 1$ is required, ensuring $A > 0$. Since $\log B$ and $\log E$ are both positive, a sufficient condition for $p^f < p^c$ is that $\eta_1 \varepsilon_2/\eta_2 < \varepsilon_1$, as must be the case if ε_2 is sufficiently small or η_2 sufficiently large. Suppose, however, that p^c, and hence B and E, are held fixed (by altering the implicit constant terms of $\log \xi$ and $\log \psi$) as the elasticities ε_2 and η_2 are varied. Then if ε_2 is made sufficiently large, or η_2 sufficiently small, $p^f > p^c$ must hold and the commodity-money alternative is the more deflationary.

This log-linear approximation permits comparison of, not only the levels of prices and output, but also their sensitivity to exogeneous disturbance.[15] Let the functions ξ and ψ now become $k_1\xi$ and $k_2\psi$, with k_1 and k_2 multiplicative shift parameters, both initially unity. From (26) we then have, for $i=1, 2$

$$A\,\partial \log (p^f/p^c)/\partial \log k_i = [(\eta_1(\varepsilon_2/\eta_2) - \varepsilon_1)\, \partial \log B/\partial \log p^c - \varepsilon_3\, \partial \log E/\partial \log p^c]\, \partial \log p^c/\partial \log k_i \tag{27}$$

with $\partial \log B/\partial \log p^c = (1+\gamma)(1/B-1) < 0$ and $\partial \log E/\partial \log p^c = 1/E - 1 < 0$. Now (27) is proportional to the difference between the elasticities of p^f and p^c with respect to k_i. As observed above, these elasticities are both positive for $i=1$ and both negative for $i=2$. Thus, multiplying (27) by -1 when $i=2$, and working with the absolute values of the elasticities $\partial \log p^f/\partial \log k_i$ and $\partial \log p^c/\partial \log k_i$, permits identical arguments for $i=1$ and $i=2$. The

[14] Aggregate income must be perceived as higher in the commodity-money case even though goods output and perhaps employment may be smaller. For $y^c + x^c/p^c > y^f$ must hold, even when $p^c < p^f$, if the money market is to clear with $r^c > r^f$. If fiat money was issued as an income transfer and depreciated at the same rate as commodity money then the demand for goods in the fiat case would become $\xi(y^f + x^c/p^f, r^f, M/p^f)$, with fiat money being issued at the same rate, x^c, as commodity money and now counting as net wealth. It is easily seen that $p^c < p^f$ *must* hold now and that $y^f + x^c/p^f > y^c + x^c/p^c$ may hold.

[15] Compare Johnson (1967, pp. 321–3) where an extreme quantity-theory case is considered.

elasticity for p^f must be the larger one if $\eta_1\varepsilon_2/\eta_2 < \varepsilon_1$, but the smaller if (holding p^c constant) ε_2/η_2 is sufficiently great. If the elasticity for p^f is the smaller, then the fiat-money regime is the more able to stabilise prices, production and employment against the exogeneous demand shifts. Note that the fiat regime may be the more stabilising even though it is the more deflationary.

Turn now to the perpetual-full-employment case in which p, w, r always adjust to keep the goods, labour and money markets cleared for the given M. We now have in (24) and (25) $y^f = y_{\max} \equiv f_g(N)$ and y^c, x^c given by $y(p), x(p)$, the full-employment supply functions of (9) and (10). By (B3) and (B4) of Appendix B (which specialise to cover the fiat case) each monetary regime has a unique equilibrium, with the equilibrium price level raised by an autonomous increase in demand for goods and lowered by an autonomous increase in demand for money. It can be shown, by an argument similar to that for the under-employment case, that $p^f \gtreqless p^c$ may hold and that $r^f < r^c$. By (12) we must now have $y^c + x^c/p^c > y^f > y^c$, implying that real wages are lower in the fiat money case.

Carrying on with the same log-linear approximations to ξ and ψ we have

$$\log(p^f/p^c) = (\varepsilon_1\eta_2/\varepsilon_2 - \eta_1)\log F - (\eta_2/\varepsilon_2)(\log G + \varepsilon_3 \log E) \qquad (28)$$

where $F = y^f/(y^c + x^c/p^c)$ and $G = y^f/y^c$, so that $F < 1 < G$, while $E = 1 + M/p > 1$, as before. If $\varepsilon_1\eta_2/\varepsilon_2 > \eta_1$ then (28) must be negative, but if η_2 is sufficiently small or ε_2 sufficiently large (for fixed p^c and hence fixed E, F, G) then (28) is positive and the fiat-money alternative is the more inflationary.

The sensitivity of employment to exogeneous shocks is no longer an issue, and goods output is obviously less variable in the fiat-money case where it is entirely unaffected by demand-side shocks. But there remains a question about price-level sensitivity in the alternative regimes.[16] For the log-linear approximation, with the shifts k_1, k_2 defined as before, we have for $i = 1, 2$

$$\partial \log(p^f/p^c)/\partial \log k_i = [(\varepsilon_1\eta_2/\varepsilon_2 - \eta_1)\,\partial \log F/\partial \log p^c$$
$$- (\eta_2/\varepsilon_2)(\partial \log G/\partial \log p^c + \varepsilon_3\,\partial \log E/\partial \log p^c)]\,\partial \log p^c/\partial \log k_i \qquad (29)$$

Here, using (11), $\partial \log F/\partial \log p^c = x^c/(p^c y^c + x^c) > 0$, $\partial \log G/\partial \log p^c = -p^c(dy^c/dp^c)/y^c < 0$, and $\partial \log E/\partial \log p^c = 1/E - 1 < 0$, as before. The argument proceeds essentially as in the underemployment case. If the term in square brackets in (29) is positive, as it must be when $\varepsilon_1\eta_2/\varepsilon_2 > \eta_1$, then the fiat-money case has the higher elasticity of price-level response to exogeneous demand shifts. But if η_2/ε_2 is sufficiently small, this expression must be

[16] It might be thought unfair to contrast the sensitivity of p, when the whole labour force is devoted to goods production in the one case and only part in the other. But the remarks of footnote 5 apply with equal force here.

positive and the fiat-money case then has the lower response elasticity and thus the more stable price level for a given pattern of random shifts.

It might be reasonable to expect on empirical grounds that a commodity money gives greater stability of prices and economic activity. But this is not a logical necessity, failing to be true in our model if the interest elasticity of demand for money is sufficiently low, or the interest elasticity of demand for goods is sufficiently high.

The actual transition between a commodity and fiat money raises delicate institutional questions. It might, however, be worth noting that banning or taxing prohibitively the production of a commodity money, while the existing stock continues in use as the sole money, might increase economic activity and prices in the short run rather than cause a depression. Conversely, the initiation of a commodity-reserve currency, with all new money issued in exchange for specified commodities delivered to the monetary authority, might prove deflationary in the short term unless the money stock was first jumped upwards.

IV. *A partial-equilibrium approach to commodity money*

This section restricts attention to the perpetual-full-employment case, so that the full-employment supply functions $x(p)$ and $y(p)$ of (9) and (10) apply.

An economy producing commodity money fails to satisfy the usual static neutrality property: doubling the money stock will not merely double the full-employment p and w while leaving all real variables unchanged. For a change in p will alter the short-term equilibrium rate of money production, $x(p)$, and hence goods output, the real wage and the interest rate (see Burstein (1963, pp. 97–100)). Suppose, however, that attention is restricted to values of M sufficiently large to ensure that the short-term equilibrium p is high enough to make the value of money production a negligible fraction of the value of goods output. That is, suppose that p always remains above p_0 defined by

$$x(p_0)/p_0y(p_0)=k \tag{30}$$

where k is a sufficiently small fraction (say 0.01, although somewhat larger values might be tolerable). From (30) and (12) we have

$$(1+k)y(p)>y(p)+x(p)/p>y_{\max}=y(\infty)>y(p) \tag{31}$$

for any finite $p>p_0$, so that real income, $y(p)+x(p)/p$, must lie very close to $y_{\max}$ so long as p exceeds p_0.

A near-neutrality property will now hold. Even though they alter the rate of money production, variations in M will have only negligible effects on other real variables, which will remain approximately constant while p and w approximately double. Thus the demand for real balances will also remain approximately constant at $\psi = \psi(y + x/p, r) \simeq \psi(y_{max}, r)$. A partial-equilibrium approach to the money-producing industry now becomes possible with

$$M = \psi p, \qquad \psi > 0 \tag{32}$$

representing the demand function for the cumulated stock of the industry's output, whose price level in terms of goods is $1/p$. ψ is regarded as a parameter. This is the approach latent in much nineteenth-century discussion (for example Marshall (1871), Mill (1965, Book III), Walras (1874, Lessons 29–32 of 1954 edition), Havrilesky (1972).

Using (17) and (32) gives

$$\psi \, \mathrm{d}p/\mathrm{d}t = x(p) - \delta\psi p \tag{33}$$

which clearly converges monotonely over time to the unique equilibrium value p^+, defined by $x(p^+)/p^+ = \delta\psi$. This corresponds to the long-term equilibrium price level, p^+, of Section II. We must, naturally, assume that both p^+ and the initial p exceed p_0 defined by (30). Otherwise the approximation underlying (33) will not remain valid at all points of the solution trajectory.

The formulation is now so simple that it can be extended without difficulty in other directions. This is illustrated by adding non-monetary uses for the money commodity and a Marshallian adjustment process reflecting a slow movement of resources into and out of money production.

Non-monetary uses will differ in the extent to which they preserve the commodity intact, or in a form easily and cheaply adaptable to monetary use. In reality there is a spectrum, but here only two polar cases are admitted: a flow demand $q(p)$ for a use yielding utility by destruction of the commodity; and a stock demand $R(p)$ for a use yielding utility by holding the commodity without impairment (e.g. using it for ornament). These non-monetary uses become cheaper compared to other goods as p rises, so that $q'(p), R'(p) > 0$ may be assumed. The stock demand differs from the demand for money in being a demand for nominal rather than real balances.

The price level of goods, p, is now only a sectoral consumption price index. In calculating real income, the price of money, unity, should be included in the overall price index just like the price of any other commodity, but only with respect to *non*-monetary use. However the difference between p and such an overall price index will be negligible if both the value of money production and the income equivalent of the utility derived from the non-monetary-stock use always remain negligible compared to the value of goods production, as will be assumed.

The evolution of the money stock now satisfies

$$\mathrm{d}M/\mathrm{d}t = x - q(p) - R'(p)\,\mathrm{d}p/\mathrm{d}t - \delta M \tag{34}$$

instead of (17). Eliminating M by use of (32) gives

$$\mathrm{d}p/\mathrm{d}t = (x - q(p) - \delta\psi p)/(\psi + R'(p)) \tag{35}$$

Assume now that a change in p exerts its long-run effect on money production, x, only with a geometric distributed lag, so that

$$\mathrm{d}x/\mathrm{d}t = \mu(x(p) - x), \qquad \mu > 0 \tag{36}$$

The phase diagram of the system (35) and (36) governing p and x is exactly like Fig. 1, but with p replacing w and x replacing M. The unique long-term equilibrium (x^+, p^+) must be locally stable, but may be approached cyclically.[17] If the lag in the response of money production is eliminated, so that $x = x(p)$ always, then p is governed by

$$\mathrm{d}p/\mathrm{d}t = (x(p) - q(p) - \delta\psi p)/(\psi + R'(p)) \tag{37}$$

and clearly approaches the long-term equilibrium monotonely.

In the long-term equilibrium, the production of money must replace depreciation of the money stock and also satisfy the non-monetary flow demand forthcoming at the equilibrium price level. The non-monetary stock demand has no influence on the equilibrium price level, since the stock must remain constant in equilibrium. However, the speed of convergence to long-term equilibrium is increased by the existence of both types of non-monetary demand. A fall in p now increases M, not merely by increasing production of the money commodity, but also by reducing the amount of this production destroyed in the non-monetary flow use, and by switching already-existing stock from non-monetary to monetary use. A rise in p acts obversely.

V. *Concluding reflections*

From a very general viewpoint, a commodity-money system involves a specific control or feedback rule relating money creation to other endogenous variables. As a control rule it is costly to implement because of the resources absorbed in money production.[18] The initial selection of which

[17] Linearising (35) and (36) about (x^+, p^+) gives equations equivalent to (C1) and (C2) of Appendix C if we take $m = (p - p^+)$, $\omega = -(x - x^+)$ and $\lambda = \mu$. We have $a = (q' + \delta\psi)/(\psi + R')$, $b = 1/(\psi + R')$, $c = -x'$, $d = 1$, all positive. Thus the argument of Appendix C also applies to the present case.

[18] The share of national income generated in money production may overstate social cost. Rents to non-transferable money-producing resources (e.g. gold reefs) should be excluded. The cost will also be less if some or all of the money can be expected to revert ultimately to non-monetary use.

commodity is to serve as money offers some choice of the applicable control rule and associated resource cost, and further emendation of these is attainable by such expedients as taxing or subsidising production or non-monetary use of the chosen commodity. It is not a simple matter to conceptualise choice of the optimal commodity-money arrangement from the set of all such possible arrangements. For instance, taxing away all new production of the chosen commodity eliminates any resource cost but may also eliminate some degree of built-in stabilization (see Section III). Inevitably, an ideal fiat money, with its wider choice of feedback rules (including Friedman-like choice of no feedback) and its negligible resource cost, will always be superior to any commodity-money arrangement (compare Friedman (1951)). This does not mean, of course, that any fiat-money arrangement is better than every commodity-money one.

The preceding analysis could be generalized to allow all functions and parameters to vary autonomously over time. Long-term constancy of the price level would no longer be assured by commodity money and additional stabilizing measures might be called for—such as Irving Fisher's compensated dollar (Fisher 1913) which involved systematic adjustment of the commodity content of the currency unit. The case of a money commodity requiring, like gold, an important exhaustible resource could be incorporated by including cumulated production of the commodity as an argument in the supply function for new production—a kind of "forgetting by doing." All these changes would complicate the arguments, without calling for any fundamental revision of the analytical framework, but they would have to be carefully weighed in any discussion of policy. More radical revision would be necessary, however, to rectify some of the shortcomings of the theoretical approach, especially with regard to expectation formation, income distribution and capital accumulation.

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APPENDIX A: SHORT-TERM EQUILIBRIUM

From (18) the slope of the locus of goods-market-clearing points (p, r) is for $p<\theta(w)$

$$\partial r/\partial p=(\xi_1 S_m-(1-\xi_1)wS'_g+\xi_3 M)/\xi_2 p^2 \tag{A1}$$

This is negative since $\xi_1, \xi_3, (1-\xi_1)>0>S'_g, \xi_2$. If $p>\theta(w)$ then (A1) holds with $S'_g=0$ and the slope remains negative.

From (19) the corresponding slope for the money-market-clearing locus is for $p<\theta(w)$

$$\partial r/\partial p=(\eta_1(S_m+wS'_g)/(S_m+pS_g)-1)/\eta_2(p/r) \tag{A2}$$

This is positive since $\eta_2<0<\eta_1\leq 1$ and the term multiplying η_1 must be less than unity. For $p>\theta(w)$, (A2) holds with $S'_g=0$ and the slope remains positive. It follows from these slope properties that an autonomous increase in the demand for goods, which raises the goods-market locus bodily, must raise the equilibrium price level, p^*. An autonomous increase in the demand for money, by bodily raising the money-market locus, must lower p^*.

APPENDIX B: THE LOCUS $w = \phi(M)$

If $w = \theta^{-1}(p)$ and

$$\xi(y(p)+x(p)/p, r, M/p) = y(p) \tag{B1}$$

$$\psi(y(p)+x(p)/p, r) = M/p \tag{B2}$$

all hold (with $x(p)$, $y(p)$ the full-employment supply functions (9) and (10)) then the labour, goods and money markets are all cleared. For a given M, the solution of (B1) and (B2) must be unique if it exists, as will be assumed. For the (p, r) locus satisfying (B1) has slope (using (11))

$$\partial r/\partial p = (y'(p) + (\xi_1 x(p) + \xi_3 M)/p^2)/\xi_2 \tag{B3}$$

which is negative, while the locus satisfying (B2) has slope

$$\partial r/\partial p = (\eta_1 x/(x+py) - 1)/\eta_2(p/r) \tag{B4}$$

which is positive given (16). Moreover, as M increases, the solution p must also increase. For the (p, r) loci defined by (B1) and (B2) both shift bodily to the right, since at fixed r (again using (11))

$$\partial p/\partial M = \begin{Bmatrix} p\xi_3/(\xi_1 x + \xi_3 M + p^2 y') > 0 \text{ for (B1)} \\ 1/\psi(1-\eta_1 x/(x+py)) > 0 \text{ for (B2)} \end{Bmatrix} \tag{B5}$$

Let $p = \alpha(M)$ with $\alpha'(M) > 0$ denote this solution p. Then $w = \theta^{-1}(\alpha(M))$ defines the locus $w = \phi(M)$ of labour-market clearing points in Fig. 1. Since $\theta' > 0$ from (8), $\phi' > 0$ must hold as claimed in the text.

Now let w be increased above $\phi(M)$ and p correspondingly increased above $\alpha(M)$ to preserve the condition $p = \theta(w)$ built into (B1) and (B2). When p is raised above $\alpha(M)$, the value of r satisfying (B1) must exceed the value of r satisfying (B2) because of (B3) and (B4). Thus, if r is determined by (B2) (and also by (19) at the given p, w) there must be an excess supply of goods. From (A1) and (A2) of Appendix A, p must therefore be above the value $p^*(M, w)$ clearing the goods market for the given M, w. If p is lowered to $p^*(w, M)$ with r adjusted to preserve (19) for the given M, w, an excess supply of labour must emerge since $p < \theta(w)$. Thus $w > \phi(M)$ involves an excess supply of labour and hence $dw/dt < 0$ from (20). Conversely, $w < \phi(M)$ implies $dw/dt > 0$.

An autonomous increase in the demand for goods shifts the locus of (B1) upward in (p, r) space without affecting the locus of (B2). Hence, by (B3) and (B4), it increases $\alpha(M)$, and therefore $\phi(M) = \theta^{-1}(\alpha(M))$, for each M. An autonomous *decrease* in the demand for money has a similar effect by shifting the locus of (B2) downward while leaving the locus of (B1) unchanged.

APPENDIX C: EXISTENCE AND LOCAL STABILITY OF THE LONG-TERM EQUILIBRIUM

Consider the locus $\chi(M) \equiv S_m^{-1}(\delta M)$ of Fig. 1. The properties imposed on $f_m(\cdot)$ ensure that $\chi(0) = \infty$ and that $\chi(M) = w_{\min}$ (so that the entire labour force is absorbed in money production) at some finite positive M. Consider now the locus $\phi(M)$. We must have $\phi(0) \geq w_{\min}$ since there would be full employment at $w_{\min}$ even if $M = 0$ implied zero demand for goods. With $\chi(M)$ and $\phi(M)$ continuous and, respectively, decreasing and increasing, there must be an intersection in Fig. 1 at finite positive M, w, such that $w > w_{\min}$.

Linearising equations (17) and (20) (with $x = S_m(w)$ in (17)) at the long-term equilibrium (M^+, w^+) we have

$$dm/dt = -am - bw \tag{C1}$$

$$d\omega/dt = \lambda cm - \lambda\, dw \tag{C2}$$

where $m = M - M^+$, $\omega = w - w^+$, $a = \delta > 0$ and $b = -S'_m(w^+) > 0$. That $c, d > 0$ can be inferred from Fig. 1. Also, $\lambda > 0$, where λ is the wage-adjustment coefficient of (20). The two roots of the characteristic equation of (C1) and (C2) can be written as

$$[-(\lambda\, d + a) \pm \sqrt{((\lambda\, d - a)^2 - 4\lambda bc)}]/2 \qquad \text{(C3)}$$

They must have negative real parts, and will be complex if λ is sufficiently near to a/d and real if λ is either sufficiently large or sufficiently near zero.

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