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Author(s): H. Peyton Young

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Progressive Taxation and Equal Sacrifice

By H. PEYTON YOUNG*

Fairness is the dominant theme in almost every political debate about income tax policy.[1] Yet when it comes to actually assessing the treatment of different income groups, there is little or no agreement on how, or even whether, fairness can be meaningfully measured. The difficulty is that most criteria of vertical equity are based on the notion of equal sacrifice. While this idea was influential around the turn of the last century, it is now considered quite unfashionable, if not downright disreputable, since it relies heavily on interpersonal utility comparisons (Paul Samuelson, 1947; Richard Musgrave, 1959; Anthony Atkinson and Joseph Stiglitz, 1980). In spite of its dubious theoretical foundations, however, we propose to examine whether equal sacrifice may explain why observed tax rates have the particular structure that they do. In other words, is it a valid empirical principle?

Equal sacrifice is an elaboration of the notion that a rich person should pay more in taxes than a poor person because the former feels a given monetary loss to a lesser degree.[2] The case for it was put most succinctly by

John Stuart Mill:

> As a government ought to make no distinction of persons or classes in the strength of their claims on it, whatever sacrifices it requires from them should be made to bear as nearly as possible with the same pressure upon all ...Equality of taxation, therefore, as a maxim of politics, means equality of sacrifice. [1848, p. 804]

This passage spawned a large and illustrious literature on sacrifice theory around the turn of the century (Henry Sidgwick, 1883; Arnold Jacob Cohen Stuart, 1889; Gustav Cassell, 1901; F. Y. Edgeworth, 1897, 1919; Arthur Pigou, 1928). Below we shall briefly review the various interpretations that have been given to the term "equal sacrifice." The point that bears emphasizing here is that Mill was suggesting the concept as a *political* principle. Equal sacrifice is a natural corollary of egalitarianism. If we consider Mill's statement in this light, then it is reasonable to ask whether equal sacrifice is discernible in the way that legislators actually do distribute the tax burden. Specifically, we shall ask whether different income groups give up approximately the same amount (alternatively, the same proportion) of their utility in paying taxes. The credibility of the answer will depend, of course, on whether the estimated form of the utility function accords well with estimates of utility derived from other sources, such as the finance literature. We shall find that it does.[3]

*H. Peyton Young is Professor of Public Policy, School of Public Affairs, University of Maryland, College Park, MD 20742.

[1] For example, the title of the recent U.S. tax reform proposal was *Tax Reform for Fairness, Simplicity, and Economic Growth* (U.S. Department of the Treasury, 1984).

[2] Equal sacrifice does not necessarily imply (as some early authors erroneously assumed) that the rich should pay *proportionally* more of their incomes in tax (Samuelson, 1947, p. 247).

[3] An early attempt to estimate the marginal utility of income from tax data is due to Koichi Mera (1969). Irving Fisher (1927) suggested the reverse procedure: estimate the marginal utility of income from consumption data, and then substitute this into an equal sacrifice formula to determine the "just" rate of income tax progression. For related work, see Gabrielle Preinreich (1948), Otto Eckstein (1961), Robert H. Haveman (1965), and Burton Weisbrod (1968).

The data on which we test this hypothesis consist of federal income tax schedules in the United States over the period 1957–1987. During this period the income tax underwent a half-dozen substantial reforms.[4] The top bracket dropped from 91 percent in 1957 to 38.5 percent in 1987 to 33 percent today. In spite of these dramatic shifts, the distribution of the tax burden is explained quite well by the equal sacrifice model in most years. The post-1986 tax reform schedule is, however, a notable exception, as we shall presently see. In every case where a good fit is obtained, the estimated utility function exhibits constant proportional risk aversion with a coefficient between 1.5 and 1.7. These values are in good agreement with recent estimates based on cross-sectional studies of household demand for risky assets (Irwin Friend and Marshall E. Blume, 1975.) Similar results are obtained for recent nominal tax schedules in West Germany, Japan, and Italy.

The United Kingdom, like the United States, deviates significantly from the equal sacrifice model however. A possible explanation for this is that in both cases the schedules resulted from tax reforms in which great importance was attached to reducing the *number* of distinct tax brackets (so-called "tax simplification"). A small number of distinct brackets, with sizable jumps between the brackets, results in a choppy pattern for the average tax rate that does not fit the equal sacrifice model nearly as well as a gradually rising series of brackets.

Although we cannot draw definitive conclusions from such a limited set of data, the results suggest that equal sacrifice provides a reasonably accurate model of how the U.S. federal tax burden has been distributed among most taxpayers, at least until recently. It is also significant (or at least a remarkable coincidence) that the estimated elasticity of the utility of income is in agreement with estimates based on the demand

for risky assets. At the lower and upper ends of the distribution, however, the equal sacrifice model does not fit the data well. One explanation is that at lower incomes the need to raise revenue forces an initial rate that is higher than equal sacrifice requires, whereas at higher incomes the need to preserve economic incentives holds the marginal rates below what equal sacrifice requires. These results appear to generalize to other industrialized countries, but more work on this aspect remains to be done.

I. Concepts of Equal Sacrifice

Any empirical test of the equal sacrifice hypothesis is complicated by the existence of several competing versions of the concept (Musgrave, 1959). The idea originally advanced by Mill was that everyone should suffer the same absolute loss of utility. That is, if $U(x)$ represents the utility corresponding to income level x, then the tax t as a function of x should satisfy

$$(1) \qquad U(x) - U(x - t) = s,$$

where s is the constant level of sacrifice for all income classes, x. This implies that the tax schedule takes the form

$$(2) \quad t = x - U^{-1}[U(x) - s] \quad \text{for all } x > 0.$$

Ideally, individuals should be differentiated according to their particular utility functions. But this is impossible in practice, and, even if it were possible, would be based on false premises because it requires making fine-tuned interpersonal utility comparisons. A more plausible point of view is to consider $U(x)$ as a social norm—the utility function of a "representative" member of society (Lionel Robbins, 1938; Musgrave, 1959). In this sense no interpersonal utility comparisons are being made; rather, individuals are being treated as if they were all alike. This is a typical assumption in many types of economic models, including most treatments of optimal taxation.

Mill proposed using the Bernoullian utility function, which was the standard of his day. In this case a fixed percentage decrease in

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[4] In each of the periods 1954–1963, 1964, 1965–1976, 1977–1978, 1979–1981, U.S. individual income tax schedules remained fairly stable. From 1982–1987 major tax reforms were instituted through a series of transition schedules.

income represents the same loss of utility at every income level. Therefore, everyone sacrifices the same amount of utility if each person pays the same percent of income in tax.[5] It is noteworthy that this solution, which is considered by many to be the simplest and fairest, can be justified on equal sacrifice grounds.

Subsequent to Mill, the equal sacrifice doctrine was elaborated in several directions. Cohen Stuart (1889) proposed that everyone should suffer the same *relative* loss in utility. If r is the rate of loss in utility, then for all $x > 0$,

$$(3) \qquad U(x - t)/U(x) = 1 - r.$$

This criterion is known as "equal proportionate sacrifice."

For present purposes there is no need to distinguish the case of equal absolute from equal proportionate sacrifice, because equal proportionate sacrifice is nothing but equal absolute sacrifice relative to a different utility function. Namely, if we take the logarithm of both sides of (3), then we see that equal proportional sacrifice with respect to $U(x)$ amounts to equal absolute sacrifice with respect to $\ln U(x)$.[6]

II. A Test for Equal Sacrifice

To make any progress on testing equal sacrifice, it would appear that we must first specify the form of the utility function. Actually, this is not so. Instead, we shall postulate that equal sacrifice holds for some (unknown) utility function, and then show that important information about the utility

function can be derived directly from the tax data. The equal sacrifice hypothesis will be plausible if: (i) the estimated utility function is reasonably consistent with utility theory; and (ii) the equal sacrifice schedule derived from this utility function fits the empirical tax data.

In the modern theory of risk bearing, two parameters play a key role in defining the utility function: the coefficient of absolute risk aversion $R(x) = -U''(x)/U'(x)$ and the coefficient of proportional risk aversion $C(x) = -xU''(x)/U'(x)$ (John Pratt, 1964, Kenneth J. Arrow, 1971.) It is now generally accepted that the coefficient of absolute risk aversion is decreasing, while the coefficient of proportional risk aversion is more or less constant. Constant proportional risk aversion implies that people hold a constant *proportion* of their wealth in any one class of risky assets as their wealth varies. Empirical studies of household wealth have tended to support this hypothesis, and the coefficient C has been estimated to be greater than 1, and probably in the general neighborhood of 2 (Friend and Blume, 1975). This implies that the utility function is of the form:

$$(4) \qquad U(x) = -A(x)^{1-C} + B,$$

$$A > 0, \qquad C > 1.$$

Turning now to the test for equal sacrifice, the first step is to examine the behavior of C as a function of x. It turns out that C can be estimated directly from the tax data. To see this, consider an empirically given tax schedule $t = f(x)$, where t is the amount of tax paid by persons at income level x. Assume that there exists a utility function $U(x)$ such that the loss of utility at all levels of income x is approximately constant:

$$U(x) - U(x - t) = s.$$

Dividing both sides by t, we obtain

$$(5) \qquad [U(x) - U(x - t)]/t = s/t.$$

By the Mean Value Theorem, the left-hand side of (5) is equal to the derivative of U at some intermediate value, w, between x and

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[5] Mill defined taxable income to be income net of subsistence requirements as well as savings.

[6] A third variation of the equal sacrifice theme is to minimize aggregate sacrifice. This means that taxes should be distributed so as to minimize the total loss of utility summed over all individuals. The solution (assuming that the utility of income is increasing and strictly concave) is to equalize everyone's after-tax income (Edgeworth, 1897). This welfare maximization approach can be made much more appealing by employing a more realistic utility function—one that incorporates, for example, the tradeoff between income and leisure (James A. Mirrlees, 1971; J. K. Seade, 1977).

$x - t$. Of course, w cannot be known precisely unless we know $U(x)$, which is what we are trying to estimate. Nevertheless, it turns out that w can be estimated quite accurately without knowing U. To see this, assume that the coefficient C is more or less constant in some neighborhood that includes x and $x - t$. Then, in this neighborhood, $U(x) = -Ax^{1-C} + B$, and without loss of generality we may take $A = 1$ and $B = 0$. Thus $U'(x) = (C-1)x^{-C}$ and the defining equation for w is

$$U'(w) = (C-1)w^{-C}$$
$$= [U(x) - U(x-t)]/t,$$
$$= [(x-t)^{1-C} - x^{1-C}]/t.$$

After some algebraic manipulation, we find that

$$w/x = \left(\frac{(C-1)(t/x)}{(1-t/x)^{1-C} - 1} \right)^{1/C}.$$

Take a typical value of t/x, say $t/x = 0.2$, and substitute in various plausible values for C. As the following table shows, the resulting value of w/x (and hence of w, given a specific value of x) is quite insensitive to the choice of C.

C	w/x
3	0.893
2.5	0.894
2.0	0.894
1.5	0.895
1.1	0.896

The upshot is that we may safely choose any value of C in this range in order to estimate w. The value $C = 2$ seems like a good choice and leads to the particularly simple formula $w = \sqrt{x(x-t)}$. From this and equation (5) we therefore have

$$U'\left(\sqrt{x(x-t)}\right) = s/t.$$

Without loss of generality we may take $s = 1$.

Taking logarithms we obtain

$$(6) \qquad \ln U'\left(\sqrt{x(x-t)}\right) = -\ln t.$$

Recall now that we are attempting to estimate the coefficient of proportional risk aversion $-zU''(z)/U'(z)$. This is the rate of change of $-\ln U'(z)$ with respect to $\ln z$, which is $d(-\ln U'(z)/d(\ln z) = -[U''(z)/U'(z)]/[1/z]$. Let $X = \ln z$ and let $Y = -\ln U'(z)$. If we regress Y against X, then the slope of the regression line will be an estimate of C.

Let $z = \sqrt{x(x-t)}$. Then

$$X = \ln z = \ln\sqrt{x(x-t)}$$

and, by (6), $Y = -\ln U'(\sqrt{x(x-t)}) = \ln t$. Thus, we wish to regress $Y = \ln t$ against $X = \ln\sqrt{x(x-t)} = (1/2)\ln x(x-t)$ for various levels of tax t and pre-tax income x. The higher the R^2 is, the more plausible is the hypothesis that C is independent of x, and that the tax equalizes sacrifice relative to an isoelastic utility function.

III. Empirical Results

We illustrate the approach for United States tax data in 1957. Table 1 shows tax paid as a function of Adjusted Gross Income, which is the closest approximation we have to the effective tax schedule.[7]

The first step is to estimate the coefficient of proportional risk aversion as described in the preceding section. Let $Y = \ln t$ and $X = (1/2)\ln x(x-t)$ for the various values of x and t in the table. If the equal sacrifice hypothesis is correct relative to an isoelastic utility function, then we would expect to see an approximately linear relationship between X and Y, and the slope of the regression line will be an estimate of the coefficient C. Figure 1a shows that this hypothesis is

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[7]It would be far better, of course, to make this estimation relative to total personal income rather than Adjusted Gross Income. Unfortunately, these data are not available.

TABLE 1—FEDERAL TAX PAID AS A FUNCTION OF ADJUSTED GROSS INCOME (AGI)
UNITED STATES, 1957[a]

Income Class	Percent of Total Returns in the Class	Average Income	Tax Paid	Percent
Under $600	6.5	328	0	0
600– 1,000	5.0	798	13	1.6
1,000– 1,500	7.0	1,241	48	3.9
1,500– 2,000	6.2	1,752	90	5.1
2,000– 2,500	6.5	2,252	136	6.0
2,500– 3,000	6.4	2,748	188	6.8
3,000– 3,500	6.5	3,246	247	7.6
3,500– 4,000	6.6	3,751	309	8.2
4,000– 4,500	6.7	4,250	370	8.7
4,500– 5,000	6.5	4,748	431	9.1
5,000– 6,000	11.0	5,474	525	9.6
6,000– 7,000	7.9	6,472	690	10.7
7,000– 8,000	5.4	7,466	870	11.7
8,000– 9,000	3.5	8,467	1,065	12.6
9,000– 10,000	2.2	9,458	1,257	13.3
10,000– 15,000	3.7	11,744	1,740	14.8
15,000– 20,000	0.9	17,112	3,013	17.6
20,000– 25,000	0.4	22,256	4,468	20.1
25,000– 50,000	0.6	33,373	8,472	25.4
50,000–1000,000	0.2	65,652	23,262	35.4
Above 100,000	0.3	–	–	–

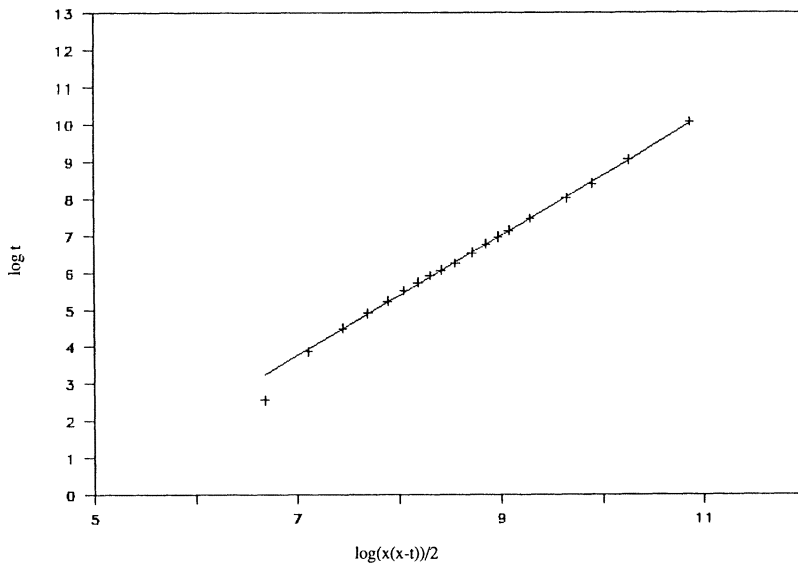[a]Source: Statistics of Income, U.S. Internal Revenue Service, 1957.

FIGURE 1a. SLOPE ESTIMATE OF C, U.S. EFFECTIVE TAX SCHEDULE, 1957,
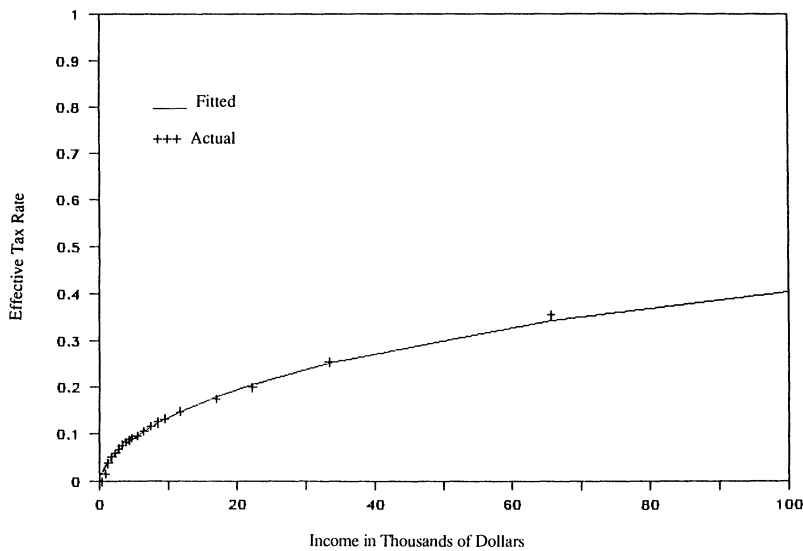$C = 1.61$, S.E. $= 0.008$, $R^2 = 99.9$

FIGURE 1b. EQUAL SACRIFICE TAX FITTED TO U.S. EFFECTIVE SCHEDULE,
1957. ES TAX $= x - (x^{-0.610} + 0.000337)^{-1/0.610}$. INCOME RANGE:
$1,000 $\leq x \leq$ $100,000. COEFFICIENT OF VARIATION OF ES TAX/ACTUAL
TAX $= 3.8$ PERCENT

strongly confirmed for adjusted gross in-
comes above $1,000, which represents 88.2
percent of all returns in 1957.[8] The esti-
mated value of C is 1.61, the standard error
of the estimate is 0.008, and the R^2 is 99.9
percent. This finding does not confirm the
hypothesis of equal sacrifice itself; it merely
gives us confidence in the estimated value of
C and the isoelasticity of the utility function
assuming that equal sacrifice holds. To a
good approximation, therefore, the utility
function may be written as $U(x) = -x^{-0.61}$.

Next, we plot the differences $U(x) -$
$U(x - t)$ to estimate the level of sacrifice, s.
The value of s has no absolute significance,

of course, since it depends on the scaling of
the utility function. Nevertheless, it is a nec-
essary parameter for computing the equal
sacrifice tax, given that the utility function
has been specified. Again, treating incomes
below $1,000 as outliers, the estimated mean
level of sacrifice is $s = 3.37 \times 10^{-4}$ and the
standard deviation is 0.1×10^{-4}, which is
about 3 percent of the mean.

The final step is to use the estimated val-
ues of C and of s to compute a fitted tax
schedule $t = x - (x^{1-C} + s)^{1/(1-C)}$. This is
shown in Figure 1b. For incomes between
$1,000 and $100,000, the ratio of the equal
sacrifice tax to the actual tax has a coeffi-
cient of variation of ± 3.8 percent. To a very
good approximation, therefore, the effective
tax rate in 1957 is consistent with equal
absolute sacrifice relative to the isoelastic
utility function $U(x) = -x^{-0.61}$.[9]

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[8]The data point corresponding to the income class
$600–$1,000 is treated as an outlier (see Figure 1a). If
this point is included in the estimation, then the esti-
mated value of C is 1.67 and the standard error is 0.033.
As noted earlier, there are good reasons to expect a
departure from the equal sacrifice model at the lower
end of the income distribution. Hence we have esti-
mated C after excluding the lower tail, where linearity
does not appear to be confirmed.

[9]It should be noted that these results are also consis-
tent with equal *proportionate* sacrifice relative to the
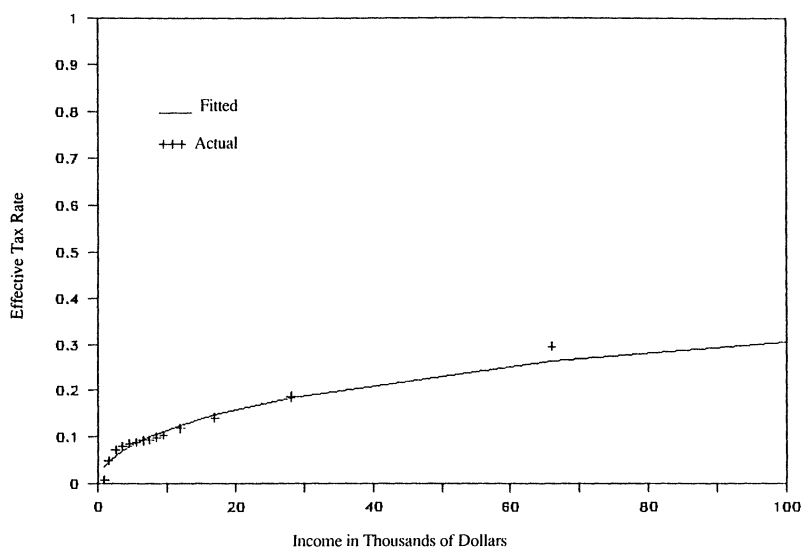utility function $U(x) = \exp[-Ax^{-0.61} + B]$. While this

FIGURE 2. EQUAL SACRIFICE TAX FITTED TO U.S. EFFECTIVE SCHEDULE, 1967.
ES TAX = $(x^{-0.519} + 0.000566)^{-1/0.519}$. INCOME RANGE: $\$3,000 \leq x \leq \$100,000$.
COEFFICIENT OF VARIATION OF ES TAX/ACTUAL TAX = 7.2 PERCENT

Similar results are obtained for the tax years 1967 and 1977, though the fit is somewhat less good. (See Figures 2 and 3). The estimation for 1987 could not be done because the relevant data are not yet available from the Internal Revenue Service. The nominal tax schedules are available, however, and this is the estimation we turn to next.

It is quite conceivable that the schedule of published rates (the nominal schedule) is also consistent with equal sacrifice. The hypothesis here is that the public is at least as sensitive to the apparent distribution of tax rates as they are to the effective rates. In other words, the appearance of equity may be as important as equity in fact. Certainly, the question seems worthy of investigation. Furthermore, the nominal schedules have a distinct advantage over the effective schedules in that they are not subject to measurement error.

For purposes of this analysis we chose U.S. Schedule X, which applies to individuals. A good fit is obtained for the years 1957, 1967, and 1977, except at the very lower and upper ends of the income scale (Figure 4 is illustrative.)[10] The estimated elasticities are somewhat higher than for the corresponding effective schedules, as is to be expected, since the higher the elasticity of marginal utility, the greater the progressivity of the schedule (see Table 2).

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is a less standard representation of utility than the isoelastic functions, it is not wholly unreasonable. In fact, the coefficient of proportional risk aversion for $U(x) = \exp[-Ax^{-p} + B]$ is $1 + p - Ap/x^p$, which is slightly increasing but very close to the coefficient for $-Ax^{-p} + B$ when x is large relative to A. In other words, the two functions are scarcely distinguishable with respect to risk aversion, and hence the model cannot really say whether equal absolute or equal proportionate sacrifice is more credible. Given the general preference for the isoelastic representation of utility in the literature, we shall interpret our results as implying equal absolute sacrifice relative to an isoelastic utility function.

[10] The coefficient of variation for the equal sacrifice tax divided by the actual tax is 8.8 percent for 1957, 5.6 percent for 1967, and 6.9 percent for 1977.
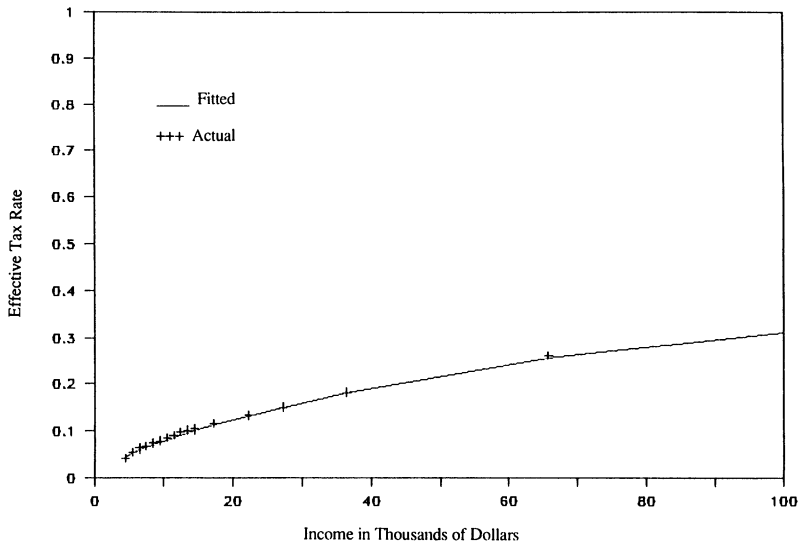
FIGURE 3. EQUAL SACRIFICE TAX FITTED TO U.S. EFFECTIVE SCHEDULE, 1977.
ES TAX $= x - (x^{-0.718} + 0.00008)^{-1/0.718}$. INCOME RANGE:
$4,000 $\leq x \leq$ $100,000. COEFFICIENT OF VARIATION OF ES TAX/ACTUAL
TAX $=$ 4.0 PERCENT

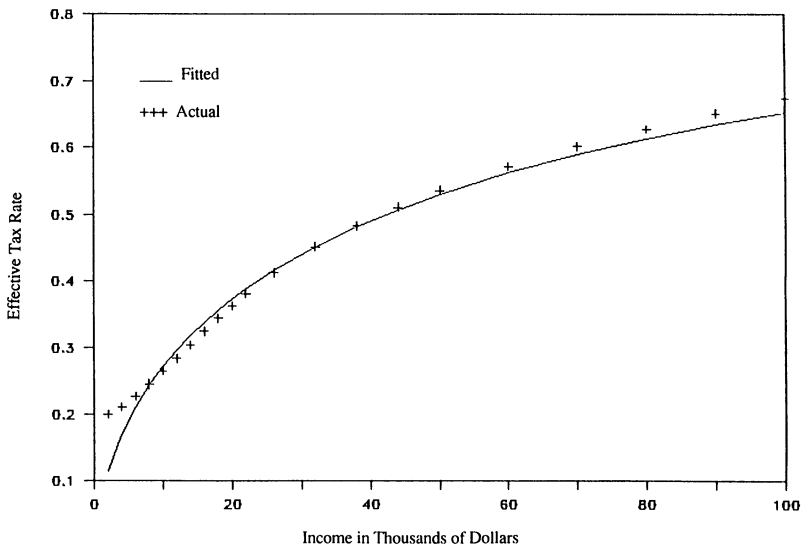FIGURE 4. EQUAL SACRIFICE TAX FITTED TO U.S. NOMINAL SCHEDULE, 1957.
ES TAX $= x - (x^{-0.631} + 0.000664)^{-1/0.631}$. INCOME RANGE:
$3,000 $\leq x \leq$ $100,000.
COEFFICIENT OF VARIATION OF ES TAX/ACTUAL TAX $=$ 5.2 PERCENT

TABLE 2—ESTIMATED ELASTICITY OF MARGINAL
UTILITY, U.S. NOMINAL AND EFFECTIVE
TAX SCHEDULES, 1957–1987.

	Nominal	Effective
1957	1.63	1.61
1967	1.53	1.52
1977	1.79	1.72
1987	1.37	n/a

By contrast, the 1987 nominal schedule does not fit the equal sacrifice model very well (see Figure 5). The reason is that the tax is nearly flat-rate for incomes up to $16,800 and hence progressivity is almost nil in this range.[11] Above $16,800 progressivity is modest but fairly steady as marginal rates rise from 15 percent to 38.5 percent. This example clearly demonstrates that the equal sacrifice model does not explain all tax schedules. But it also shows that the model is not tautological: There exist perfectly reasonable tax schedules that do *not* support an equal sacrifice interpretation, at least not relative to an isoelastic utility function.

A possible explanation for the departure from equal sacrifice in 1987 (as compared with prior years) is the emphasis placed in the 1986 Tax Reform Act on "simplifying" the tax structure. One of the supposed simplifications was to reduce the number of brackets. But no schedule composed of just two or three marginal rates will fit the equal sacrifice model well, because equal sacrifice relative to any smooth utility function implies a continuously varying marginal rate.[12] The converse is not true: just because a tax schedule exhibits a continuously increasing marginal rate, does not imply that it is "almost" an equal sacrifice tax. Indeed, an equal sacrifice tax relative to an isoelastic

utility function (with $1 < C < 2$) has a very special shape: the effective tax rate t/x is concave, increases continuously from zero and is asymptotic to 100 percent as income goes to infinity. Actually, more is true: once two points on the schedule are chosen—that is, once the tax is specified for two distinct incomes—then the equal sacrifice schedule and the corresponding utility function are fully determined. So, it would be highly coincidental if an arbitrarily chosen tax rate schedule (even one with many brackets) were to meet these requirements. For example, a tax rate schedule in which the effective rate t/x is first concave, then convex, then concave (as in the 1987 U.S. Schedule) does not fit the equal sacrifice model that we have described.

While earlier U.S. tax schedules are generally consistent with the equal sacrifice hypothesis over most of the income distribution, they do not fit the model at the lower end. The reason is that the tax brackets are not graduated finely enough for low incomes. Indeed, the equal sacrifice model (with $C > 1$) requires that the marginal tax rate decrease continuously to zero as income approaches zero. Any schedule based on a finite number of brackets obviously violates this condition. Given that the initial brackets in the U.S. schedules varied between 11 percent and 20 percent during the period 1957–1987, it can hardly be expected that the fit would be good at the lower end of the income scale. The reason for such large initial rates is a matter of fiscal arithmetic: In order to capture enough revenue, one must tax where the income is, and the lion's share of taxable income lies in the income brackets just above the minimum exemption level. So it is almost necessary for the initial marginal rate to be large, or at least to rise very steeply. This constraint may override considerations of fairness at the bottom of the income scale.

At the upper end of the income distribution we find departures from the equal sacrifice model for quite a different reason. Marginal tax rates must be truncated well below 100 percent in order to provide adequate incentives for people to work and invest. This is inconsistent with equal sacrifice

[11]Income up to $1,800 is taxed at 11 percent, then at 15 percent up to a total of $16,800. The fit is even worse for 1988, where the initial rate is 15 percent on the first $17,850, 28 percent up to $43,150, and 33 percent up to $89,560 (Internal Revenue Service, 1987, 1988.)

[12]From $U(x) - U(x - t) = s$ it follows by differentiation that $dt/dx = 1 - U'(x)/U'(x - t)$. Hence, if U' is continuous, then so is the marginal rate dt/dx.
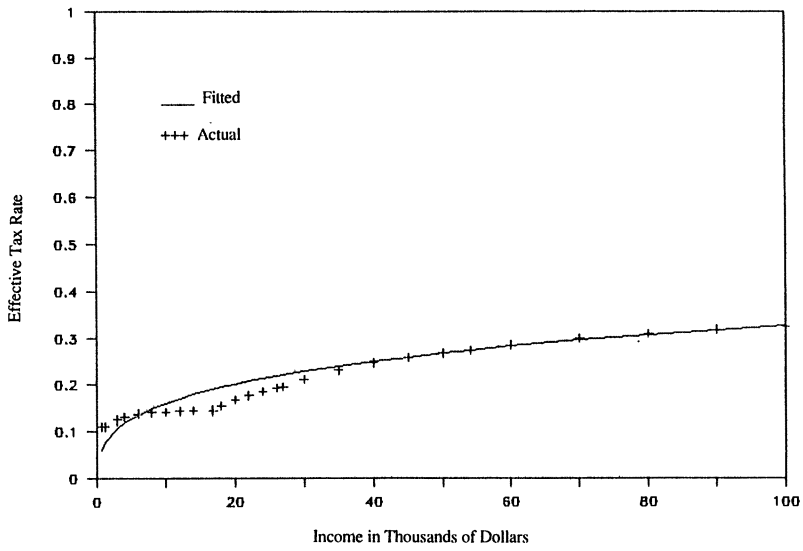
FIGURE 5. EQUAL SACRIFICE TAX FITTED TO U.S. NOMINAL SCHEDULE, 1987.
ES TAX $= x - (x^{-0.373} + 0.00218)^{-1/0.373}$. INCOME RANGE:
$\$3,000 \le x \le \$100,000$.
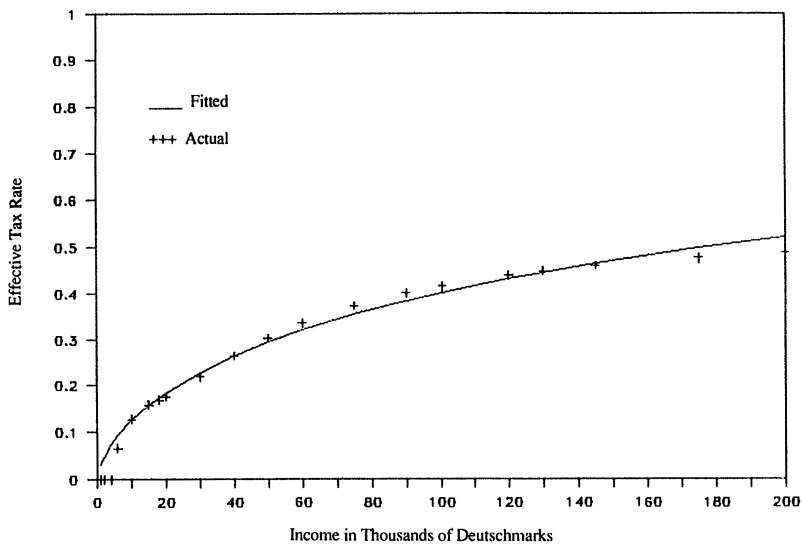COEFFICIENT OF VARIATION OF ES TAX/ACTUAL TAX $= 10.3$ PERCENT

FIGURE 6. EQUAL SACRIFICE TAX FITTED TO WEST GERMAN NOMINAL
SCHEDULE, 1984. ES TAX $= x - (x^{-0.633} + 0.000260)^{-1/0.633}$. INCOME RANGE:
DM $10,000 \le x \le$ DM $200,000$. COEFFICIENT OF VARIATION OF ES
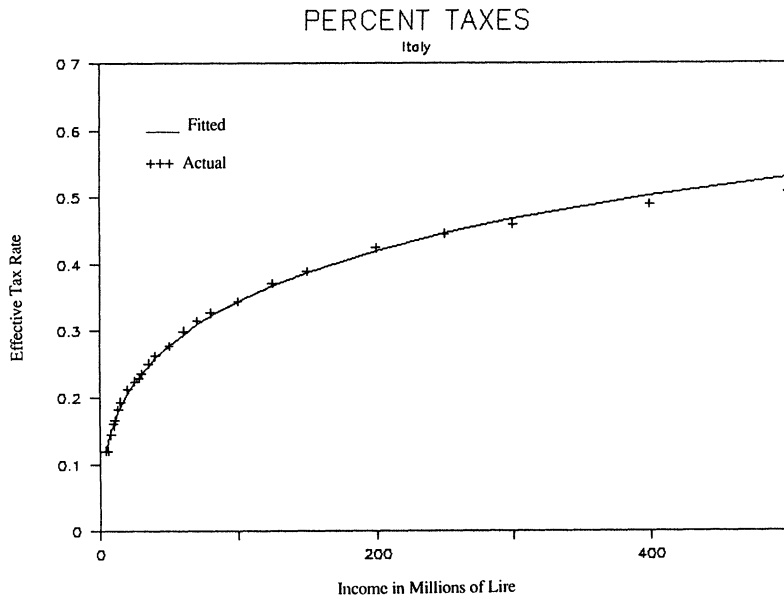TAX/ACTUAL TAX $= 3.2$ PERCENT

PERCENT TAXES
Italy

FIGURE 7. EQUAL SACRIFICE TAX FITTED TO ITALIAN NOMINAL SCHEDULE, 1987. ES TAX $= x - (x^{-0.403} + 0.00179)^{-1/0.403}$. INCOME RANGE: $4 \leq x \leq 500$ MILLION LIRE. COEFFICIENT OF VARIATION OF ES TAX/ACTUAL TAX $=$ 3.9 PERCENT
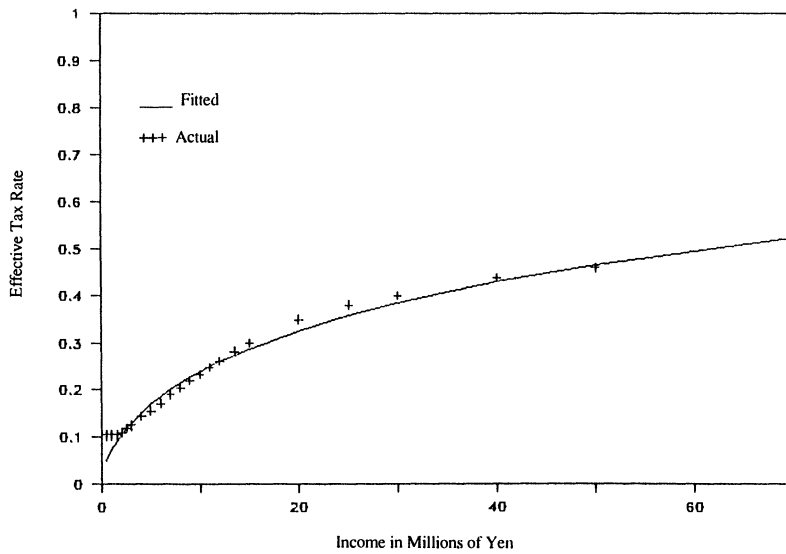

FIGURE 8. EQUAL SACRIFICE TAX FITTED TO JAPANESE NOMINAL SCHEDULE, 1987. ES TAX $= x - (x^{-0.587} + 0.0448)^{-1/0.587}$. INCOME RANGE: $1.5 \leq x \leq 70$ MILLION YEN. COEFFICIENT OF VARIATION OF ES TAX/ACTUAL TAX $=$ 5.4 PERCENT
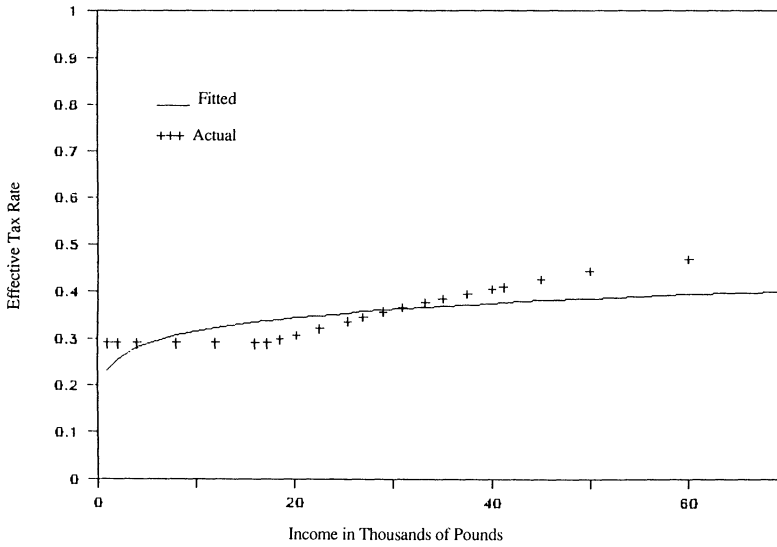
FIGURE 9. EQUAL SACRIFICE TAX FITTED TO U.K. NOMINAL SCHEDULE, 1987.
ES TAX $= x - (x^{-0.163} + 0.0143)^{-1/0.163}$. INCOME RANGE: $1 \le x \le 70$
THOUSAND POUNDS. COEFFICIENT OF VARIATION OF ES TAX $= 10.4$ PERCENT

relative to an isoelastic utility function, however, which requires that tax rates gradually approach 100 percent. For high incomes, therefore, the departure from equal sacrifice may be due to efficiency considerations, whereas for low incomes it is probably due to revenue requirements. The middle- to upper-middle income range is where considerations of vertical equity can be given somewhat freer rein.

IV. Data from Other Countries

It is natural to ask whether the preceding results are in some way peculiar to the United States. To investigate this possibility in a preliminary way, we chose four major industrialized countries—West Germany, Italy, Japan, and the United Kingdom—and analyzed the most recent nominal tax schedules available to us. The results are illustrated in Figures 6–9. The equal sacrifice model gives an excellent fit for West Germany, Italy, and Japan, and the estimated coefficients are 1.63, 1.40, and 1.59, respectively. It is rather remarkable that these values all lie within such a narrow range, and that they are so similar

to the U.S. results. Italy is particularly noteworthy because the nominal schedule fits the equal sacrifice model very closely even at the low end of the income scale. All three countries exhibit a much more finely graduated rate structure than the current U.S. schedule does. The United Kingdom, however, is similar to the United States in that the 1987 schedule does not fit the equal sacrifice model at all well. The reason is that it employs a flat-rate tax on taxable income up to £17,200. and a mildly progressive series of rates thereafter. As in the case of the recent U.S. tax reform, this appears to be the result of a political compromise in which the drive toward a flat-rate tax had to be modified by demands for progressive treatment of the well-to-do.

V. Conclusion

In this paper we have described a general method for testing whether a tax schedule exhibits equal sacrifice relative to an isoelastic utility function. The technique can be applied to any schedule, whether nominal or effective, and even to particular portions of a

given schedule. The method estimates the coefficient of risk aversion in the utility function, the level of sacrifice at each level of income, and the tax function that would be implied by equal sacrifice. The latter may then be compared with the actual schedule to see how far the actual tax deviates from equal sacrifice on different portions of the income distribution.

Over much of the postwar period, the observed tax rates in the United States conformed to the equal sacrifice model quite closely, and the estimated curvature of the utility function was fairly stable. Of course, obtaining a reasonably good fit does not prove causation. Nor do we have any direct evidence that legislators actually invoked equal sacrifice arguments in proposing the rate structures that we observe. It seems likely, however, that intuitive notions of "relative sacrifice" and "ability to pay" are one factor in the way that legislators evaluate the fairness of tax proposals. And it does not seem too far-fetched to suppose that the aggregate of these intuitions, as expressed in a majority vote, might come close to an equal sacrifice tax relative to an "average" utility function.

Several questions remain to be explored. First, it would be interesting to know whether these results hold up when we analyze the effective rather than the nominal schedules in other industrialized countries. Even for the United States, it would be far better to carry out the analysis relative to full income, rather than Adjusted Gross Income, as we were forced to do because of lack of data.

Second, it may well be that some other theory can explain why tax schedules merely look like equal sacrifice schedules. Such a theory would need to explain why effective rates t/x tend to be concave, and more specifically why they tend to fit functions of the form

$$(7) \qquad t = x - \left[x^{-p} + s \right]^{-1/p},$$

$$s > 0, \qquad 0 < p < 1.$$

Third, it may be that equal sacrifice has some other explanation or justification than the traditional utilitarian one. One idea along

these lines is the following. If we hypothesize that taxes are distributed according to *some* measure of ability to pay (not necessarily a utilitarian one), and if we suppose further that the criterion applies not only to the distribution of the whole tax, but to every tax increase (or decrease), then the measure of ability to pay must be equal sacrifice relative to a social utility of income (Young, 1988).[13] This argument suggests that the equal sacrifice "look" might result from legislators trying to balance equity in incremental changes that they make to the tax distribution with equity in the overall result.

Resolving these issues is beyond the scope of the present paper. The evidence suggests, however, that equal sacrifice may play a significant role in the way that people think about taxation, and that it needs to be taken more seriously as the 'maxim of politics' that Mill claimed it to be.

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[13]We assume here that taxes are positive, continuous, strictly increasing in income, strictly increasing as a function of the total tax burden, and that marginal rates are less than 100 percent.

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