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The Unwieldy Time-Dimension of Space

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"Coming events cast their shadows before."—THOMAS CAMPBELL

ABSTRACT: This paper introduces the concept of "time-indivisibility," and suggests that it may interfere with optimal allocation of durable resources, and especially permanent resources. Space on the earth's surface is taken as a representative permanent resource. The limitations of leasing and lending as time-dividers are briefly sketched. A simple technique is advanced for analyzing on an annual basis the effects of time-indivisibility, and it is demonstrated that permanent goods do not tend to be allocated in keeping with the equimarginal ideal. The technique is further developed to analyze the effects on allocation of depreciation and appreciation, the latter tending to aggravate and the former to meliorate the diseconomies inherent in time-indivisibility.

I

Time-indivisibility

IT IS A BASIC MAXIM of economic theory, and rather plain common sense as well, that indivisibility of resources tends to interfere with their ideal allocation. If Q is inseparable from P, those wanting Q alone must settle for less of it, and more of P, than they would most prefer; and *vice versa.* Accordingly, economists have tended to frown on rigid attachments such as tie-in sales, appurtenances, all-or-nothing deals, full-line forcing, bundles of rights, exclusive patronage contracts, and other artificial limitations on free choice and the free flow of individual resources to their most productive uses.

This paper concerns a particular aspect of indivisibility: time-indivisibility. By "time-indivisibility" the writer means a tieing of the present to

the future such that he who would use a durable good cannot easily buy its services from week to week or year to year as he needs them, the way he buys his labor and raw materials, but must buy in one piece a claim to its services over its full span of years.

Time-indivisibility affects the allocation of durable goods in the measure of their durability. It hardly affects how the market allocates goods that endure only, say, two or three years. Its influence over durable goods like buildings is more perceptible. It most affects the allocation of permanent goods, of which the most evident example is space on the earth's surface, or land, which in this study we will let represent the whole genus of permanent goods, of which it constitutes a major part. He who buys title to land can divide it down to a very small piece in two dimensions, and dispense if he will with the third, both aerial and subterranean. But the fourth dimension runs from now to doomsday. That is longer than he is likely to need it, and costs more money, certainly, than many hard-pressed entrepreneurs can conveniently raise at one time.

To put it in conventional symbolic terms, the buyer of title to durable goods must pay not just for the immediate annual income, *a,* but for that plus a stream of future incomes (which he discounts at a rate *i*):

$$a_0 + \frac{a_1}{1+i} + \frac{a_2}{(1+i)^2} + \cdots + \frac{a_n}{(1+i)^n} \qquad (1)$$

When he buys impermanent products, the anticipated annual incomes decline with time and in a few years the series ends. But when he buys land, the incomes are not in general expected to decline, and the series is infinite. Where *n* equals infinity and *a* and *i* are constant in perpetuity, the series may be simplified, as is well known, to the form:

$$a + \frac{a}{i}, \qquad (2)$$

the latter term being much the larger, and generally much larger than a corresponding expression for a declining and finite series. This extended time-dimension constitutes a costly appurtenance to the current services of land.

Let us see how time-indivisibility might interfere with optimal allocation of permanent resources. Moving a step closer to reality, what one buys, of course, is not a specified guaranteed annual income, *a,* but the use of land, call it *U,* a potentiality from which the user may take various incomes depending on how he uses it, and how external circumstances develop. Different persons attach different values to *U,* depending in general on how productive they would make the land. In a perfect market, *U* would tend

to gravitate to holdings in which its marginal productivity would be a maximum.

Now time-indivisibility would constitute no problem if the highest bidder for U could also bid the most, or even the same amount as others, for the subsequent terms. But this is not in general the fact. Even assuming for brevity's sake that all individuals' valuations of U remain constant in perpetuity, there is another element, i, which varies among persons—a crucial postulate, which will be defended—and affects what they are willing to pay for land titles. Formally i represents the rate of interest at which the individual discounts future values. More generally it may be taken, again for brevity's sake, to subsume the individual's inclination to speculate and his financial power to indulge it. It is this ability to carry interest burdens, as much as the ability to make land productive, that determines who will bid highest for land titles.

Suppose, for example, the marginal productivity of certain lands would be \$20 a year in the enterprise of individual A, and twice that, \$40 a year, in the enterprise of individual B, both in perpetuity. But suppose A has funds of his own to invest, whose best alternative yield he reckons at 2 per cent, while B can borrow only at 6 per cent. B, then, can bid no higher than \$40 plus $\frac{\$40}{.06}$, or \$707. B could not meet the top bid of A, which is \$20 plus $\frac{\$20}{.02}$, or \$1020.

This is not to deny that B could realize an income from year to year on his investment, a higher income than A could realize. And when his need terminated, if we wish to assume that it would, he could sell the title probably undepreciated, and often as not appreciated, in the same markets open to A, perhaps to A himself. But this in no way disposes of the problem of time-indivisibility. For the problem is not that B would have to throw money away on a "tie-in" that is entirely worthless to him. The problem is rather that he would have to carry the burden of interest on this investment over each year he held the title. And this he is less well equipped than A to do, for exactly the same reason that he cannot meet A's price for $\frac{U}{i}$:—B must reckon a higher interest rate.

Operationally this is probably the most convenient way to pose the problem, one which we will pursue from here on. The individual's annual marginal cost of holding land is interest on the price of the title. As different individuals have access to funds at different rates of interest, the marginal cost of holding the same lands varies from person to person (and, of course,

from firm to firm). As each individual tends to equate marginal cost with marginal productivity on his own holdings, the marginal productivity of land comes to vary from holding to holding, contrary to the equimarginal ideal. Time indivisibility deters many land-hungry individuals from buying at all. Others take some land, but less than if they could buy it a month or a year at a time. Because they cannot divide it temporally, they divide it spatially, lopping off much needed acres to fit their lands to the Procrustean beds of their finances. On the other hand, financially powerful individuals and firms may extend their holdings without much concern over productivity, current or future. These are the land-surfeited, holding too much to let them earn a very good income from each unit.[1]

Here, then, is a matter of some practical consequence. It is a problem with which many economists have grappled when investigating markets for land. Here, too, is a matter of important theoretical implications. But thus far the investigators have had to proceed without benefit of the sharper focus which theory might afford. To be sure several economists, working from particulars of the land markets toward generalizations of theory, have advanced promising suggestions.[2] But little of their work has found its way into that accumulating body of thought which the profession passes on as "the" basic principles of economics. This paper suggests that time-indivisibility might warrant such consideration. And it advances a simple technique for integrating it into the structure of economic theory.

II

Leasing and Lending: Imperfect Time-dividers

THE CRITICAL READER may already have wondered whether our economy does not already provide adequate time-dividing facilities in the institutions of leasing and lending. Both, without question, are means for dividing up the time-dimension of durable goods, and theorists who maintain, with Ely and Wehrwein, that "Rent acts as the 'sorter' and 'arranger' of land uses"[3] doubtless do so on the assumption that the time problem can be so disposed

[1] An important corollary problem is obviously that persons like B must, in order to finance that much of *U* that they do take, sink more of their limited funds than they would most prefer into claims to remote future values. This, of course, cuts into alternative investments they might make in, say, machinery and equipment. And persons like A sink less of their funds into claims to remote future values than they would most prefer. In the present brief paper we will not pursue this corollary problem.

[2] Some of these are the following: Philip Cornick, "Land Prices in a Commodity Price System," *Journal of Land and Public Utility Economics* 10 (1934), pp. 217–31; Clyde R. Chambers, "Farm Land Income and Farm Land Value," *American Economic Review* 14 (1924), pp. 673–98; and David Weeks, "A Suggested Approach to the Farm Land Valuation Problem," Berkeley, n.d. (typewritten ms. in the Giannini Library, Berkeley, California).

[3] R. T. Ely and G. S. Wehrwein, *Land Economics*, New York, Macmillan, 1940, p. 139.

of. It will be well to digress for a bit, therefore, to outline the reasons why leasing and lending appear to be so imperfect and to entail costs so heavy that the pressure of economic loss from time-indivisibility must mount high before individuals can find any net advantage in relieving it by resort to either.

From a remote enough vantage, leasing and lending seem to promise complete solutions. By leasing land, a user can buy its services a year or less at a time, leaving to some absentee landlord the burden of financing title. Or, by recourse to lending institutions, a land user can borrow the wherewithal to finance title himself. Those who take leasing as a perfect substitute for owner-operation may then write of land gravitating to the individual in whose care it yields (and who will, therefore, pay) the highest rent. Others who put their faith in lending may mention the "normal rate" or "the going rate of return" on investments, which by implication any investor can get at will and at which any would-be land user can borrow.[4]

To inspect leasing and lending at all closely, however, is to see that both are very imperfect arrangements. They are by no means entirely unworkable, and one must interpret "time-indivisibility" not as an absolute but as short for "imperfect time-divisibility." But each involves costs so high that the market must tolerate considerable deviation from ideal equimarginal allocation of land among different owner-operated, self-financed holdings, before resorting to either. That is, affluent A may extend his lands until their marginal productivity becomes a good deal lower than it is on the narrow holding of impecunious B without its becoming feasible for A either to lease land to B, or lend him the money to buy it.

Consider leasing. Sentimental preference for ownership over tenancy seems to be the rule, and as all economic value originates in human preference, this one is not to be despised. But the more tangible drawbacks are more serious. Leasing, as compared with owner-operation, intrudes into management an operationally superfluous human relationship, involving extra costs that detract from the net produce of the land, and discords and conflicts of interest that tend to preclude full development. The mere cost of collecting rent, for example, often consumes an appreciable part of it, especially where rent is low. Where collection difficulties warrant it, landlords often resort to sharecropping, or some urban equivalent, arrangements destructive of much incentive. More important, short-term lessees have no interest in the future of the assets they work and manage, and will

[4] J. G. Sutherland and C. E. Bishop, *Increasing Production and Incomes*, N.C.A.E.S. Tech. Bul. No. 117, 1955, p. 6; Theodore Morgan, *Introduction to Economics*, 2d ed., Englewood Cliffs, N. J., Prentice-Hall, 1956, p. 356.

hardly give them an owner's care. They cannot rationally attach long-term improvements to land they may lose at any lease-end, nor can they sacrifice any present values to conserve future values either in the soil or in improvements the landlord provides. They cannot plan ahead, but only from lease to lease, although efficient land use requires long-range planning. In all their plans they must build in flexibility, at some cost, for the contingency of eviction. All this is well enough established, especially among students of land tenure, that we need not dwell on it.[5]

To obviate part of these extra costs, landlords sometimes grant long-term leases, but not as a general rule. There appear to be compelling reasons for this. Should economic rent rise above contract rent, the landlord would receive none of the increase; while should it fall below, the tenant might very likely fail to pay. Too, the landlord who abdicates his power to evict cannot protect himself against tenant practices exploitive of the soil or of the landlord's improvements; nor is he free to seize the opportunity to sell at the most opportune moment.[6] For these and for other reasons, long-term leases are rare, and short ones, with all their disadvantages, the general rule.

Leasing, therefore, is a poor substitute for landownership. As a time-divider it leaves much to be desired. Most of the telling points of those who advocate tenancy as a social norm seem to be variations on the theme that it is a time-divider.[7] But other than that, there is little to commend it. There are good reasons why land users strongly prefer to couple occupation with ownership.

Next, consider lending. This lacks the major drawbacks of leasing and, like leasing, contributes something toward solving the problem of time-indivisibility. It might go far toward solving it satisfactorily were the

[5] For example, see the following: Interregional Land Tenure Research Committee, *Agricultural Land Tenure Research*, Chicago, Farm Foundation, 1955, pp. 11–14. J. A. Baker, "Tenure Status and Land Use Patterns in the Corn Belt," Bureau of Agricultural Economics Land Economics Report No. 5, 1939 (mimeographed); Hoyle Southern, "Land Tenure and Soil Conservation," in H. Hoffsomer, ed., *The Social and Economic Significance of Land Tenure in the Southwestern States*, pp. 200–24, Chapel Hill, University of North Carolina Press, 1950, pp. 216–7; Peter Nelson, "Land Tenure and Agricultural Conservation, *Current Farm Economics*, Vol. 2, No. 2, Stillwater, Oklahoma Agricultural Experiment Station, April, 1938; Rainer Schikele, *et al.*, *Economic Phases of Erosion Control in Northern Iowa and Southern Minnesota*, Iowa Agricultural Experiment Station Bulletin No. 333, June, 1935.

[6] A persistent tendency grossly to overestimate the possibilities of future resale at advanced prices has been noted by the British Uthwatt Committee. (Expert Committee on Compensation and Betterment, *Final Report*, [The Uthwart Report], [Cmd. 6386], (1942) paragraphs 23, 24. Cited in G. L. S. Shackle, *Expectations in Economics*, Cambridge, the University Press, 1949, pp. 91–3.) The authors write of the "floating value" of land in and around urban areas, which they estimate in the aggregate to exceed by many times the values that the city can support.

[7] The most widely cited example seems to be T. W. Schultz, "Capital Rationing, Uncertainty, and Farm-Tenancy Reform," *Journal of Political Economy*, 48 (1940), pp. 309–24.

money markets able to establish one universal market interest rate, that "going rate of return" of ancient legend, at which all lenders might lend and any borrower freely borrow any sum for any term. But, of course, the markets achieve no such thing. The economic worlds of borrowers and lenders remain well insulated by the high cost of transferring funds. Keynes called the barrier "lenders' risk," and he incorporated the concept most explicitly in his theories as a floor under interest rates.[8] Hart and others have added "credit rationing" as a barrier to lending; and, from the borrowers' side, "linkage of risks" and the need to maintain "financial respectability" as deterrents to borrowing.[9] If these barriers affect the aggregates, how much more so the components.

Like resistors in an electrical circuit, these "lending costs," if one may lump them in that phrase, repel part of the flow. And the funds that do trickle through the resistance lose much of their "charge," their economic potential. Between affluent A, who deposits surplus funds in the bank for 2 per cent, and impecunious B, who borrows from it at 6 per cent, there remains a wide barrier. B, borrowing limited amounts at high rates on short terms without guarantee of renewal, and only for specified conservative projects approved by the banker and the several regulating, insuring, and endorsing agencies who peer over the banker's shoulder, does not by borrowing acquire the full financial potential of A. When it comes to bidding for land titles, A and B live, although they may be neighbors, in two different economies.

Further, among B and C, D, E, and F, each of whom borrows at a different interest rate in different amounts for different periods and under different limitations, there is a series of barriers. And between them and those many others who cannot borrow at all on any terms, the insulation is impenetrable.

Were differences in interest rates and other conditions of loans based solely on lenders' well-founded judgments of the relative probabilities of individual borrowers' producing from loan-financed land the incomes they anticipate, there would still be reasons for denying that lending adequately solved the problem of time-indivisibility. But there is no need to pursue the implications of conditions contrary to fact. The conventional basis of lending is set forth clearly by Schikele:

> The principle of allocation is collateral security, not marginal productivity. . . . These two principles tend to work at cross purposes: with increas-

[8] J. M. Keynes, *The General Theory of Employment, Interest, and Money*, New York, Harcourt, Brace, 1936, pp. 144, 208.

[9] A. G. Hart, *Money, Debt, and Economic Activity*, New York, Prentice-Hall, 1948, pp. 202–3.

ing collateral security, the marginal productivity of capital tends to decline, and vice versa. Instead of allocating capital to where it is scarce, our credit system allocates it to places where it is ample.[10]

Liquid assets do not seem to lap along the horizontal contour of an isoproductivity shoreline, enveloping headlands and promontories, penetrating bays and inlets, seeking a universal level in the impartial way envisaged in some of the more hydraulic passages of economic literature. Liquid assets often flow uphill; they tend to gravitate towards established nuclei. Not only does the market not allocate funds in the most economical way, but it allocates them in an especially pernicious way, such as to aggravate and, perhaps more than any other factor, to initiate and perpetuate the concentration of economic power.[11]

Lenders do not, of course, completely ignore marginal productivity. If they did the economic machine might not carry us through the month. But certainly they do not accord it the steady primacy that would be necessary to let economists take comfort in the thought that lenders effectively determine who can afford to take and hold title to much of our land.

III

How Time-indivisibility Affects Allocation

IT IS OF SOME THEORETICAL INTEREST and practical consequence, therefore, to set out precisely the factors that determine the allocation of space and other durable goods among different holdings in a world, such as ours, where interest rates vary widely among individuals, and leasing so dulls the edge of husbandry as not to become a feasible alternative to owner-operation until the marginal productivities of land on different owner-operated holdings have come to diverge widely.

To begin simply, suppose that land prices are expected to remain constant; that leasing is too costly to consider at all; that there are no taxes of any kind; and that the costs of transferring land are negligible. Then each individual will extend his landholdings until the last unit just yields him his interest rate—that is, until its annual marginal product divided by its price equals his interest rate. Algebraically, let R be the revenue product, L the unit of land, P the price per unit of land, and i the individual's rate. Then each individual expands his holdings until:

$$\frac{\frac{dR}{dL}}{P} = i. \qquad (3)$$

[10] Rainer Schikele, "Farm Tenure Under the Strain of War," *Journal of Farm Economics, 25* (Proceedings) (1943), pp. 235–44, p. 240.

[11] This does not necessarily imply that it is possible or desirable to abolish collateral requirements. Policy proposals, of which credit reform is only one of many, are beyond the scope of this paper.

For the annual marginal cost of holding land is its price times the individual's interest rate (*Pi*); and the individual expands his use of this, as of all resources, until its marginal revenue product equals its marginal cost:

$$\frac{dR}{dL} = Pi \tag{3a}$$

Figure 1 illustrates the point. *MRP*, or $\frac{dR}{dL}$, represents the annual mar-

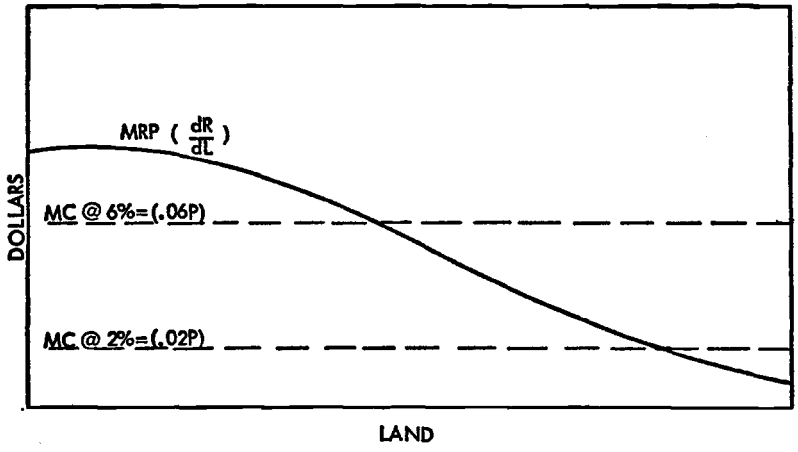

Figure 1

Marginal revenue productivity of land added to fixed complements; and annual marginal cost of land, at 2 per cent and 6 per cent

ginal revenue product of land added to a fixed complement of men and capital. The horizontal broken curves represent the annual cost per unit (*Pi*) at 2 per cent and 6 per cent. Messrs. 2 Per Cent and 6 Per Cent pay the same price for title to each unit of land but very different prices for the annual use of land, due to the difference of their interest rates. Mr. 2 Per Cent, therefore, will combine a good deal of land with each unit of labor and equipment, lavishing it down to low marginal returns. Mr. 6 Per Cent will economize on land much more stringently, adding it only until the last unit yields him $6 for each $100 of land price.

Figure 1 has wider bearing than its simplifying and limiting assumptions might suggest. In practice Mr. 2 Per Cent's marginal revenue productivity curve will not look just like Mr. 6 Per Cent's. But of course the point is that no matter what the entire curves look like, the marginal returns to land will differ on their two holdings. Again, of course, neither

is limited to a fixed complement of men and equipment, but either man can increase them to delay the advent of decreasing returns to additional land. But whatever complement of men and equipment an individual finally musters, he will ultimately encounter some kind of diminishing returns, and he will then increase his landholdings until the last unit just yields him his interest rate. And a third elementary point: the knowing marginalist need hardly be reminded that Mr. 2 Per Cent tends to spread complementary labor and equipment equally thin over his entire holdings. He tends to work each unit of his lands, and not just the last acquired, in such a way that its marginal productivity is $2 per $100.

Some important results follow immediately. Current doctrine evidently misses the mark. The rent of land, or, if you prefer, its marginal productivity, does not "sort and arrange" the pattern of land use. This is not to say that it should not. Economists of past and present have demonstrated convincingly that would be ideal. But it does not. Many different land economies exist side by side. One extreme is prodigal of land as of culls from a local mill; the other is precious of land as of rare treasure from Cathay. Actual land use deviates enormously from the equimarginal ideal. Even assuming, as so far we have done, that no one holds land for the rise of price, its marginal revenue productivities on adjacent holdings may diverge as much as the interest rates reckoned by different individuals.

These deviations are not just exotics, to be explained *ad hoc* in each situation and expected to work out ideally "in the long run" or under "normal" conditions. No word has been breathed of "speculators," scape-goats of many a mono-diabolistic interpretation. Nor is Mr. 2 Per Cent sacrificing present values to conserve more valuable future ones—we assumed yields constant in perpetuity and discovered that Mr. 2 Per Cent would use land less productively in perpetuity. Here, rather, is a permanent underlying tendency, a tendency as permanent as the indivisibility of U, the current use of land, from U/i, a claim to future use of land in perpetuity. As long as individuals differ in power to finance claims on the future (which is likely as long as individuals differ at all), there is no reason to expect ideal allocation of U alone. Mr. 2 Per Cent holds land for an annual cost much lower than its social cost, its best alternative use. For Mr. 6 Per Cent, who could earn $6 from land that yields only $2 to Mr. 2 Per Cent, has no way of making Mr. 2 Per Cent feel this social cost as his individual cost, either directly or as foregone gain. To Mr. 2 Per Cent the annual cost is $2 per $100, and that is all. Mr. 6 Per Cent's high productivity is no concern of his.

IV

Appreciation and Depreciation

WE INFERRED those results simply from incorporating time into the analysis. We postulated no changes through time. But when land buyers expect changes—and in this world of ebb and flow they usually do—practice may depart even further from the equimarginal ideal. Let us cease assuming, now, that buyers expect land prices to hold steady, and explore the changes which the prospect of change brings. This is perhaps the most interesting and rewarding aspect of the study.

Suppose, now, buyers expect the value of U to rise, or i to fall, and hence P, the price of land, to rise. "Values may be created by the mere expectancy of some new use . . ."[12] likewise of lower capitalization rates, and part of this hope materializes immediately in higher prices for land titles. But not all the hope so manifests itself, because some time lies between now and its realization. Part of it appears instead as expectations of annual increments to land prices. Higher present prices are only the embryo of expectation, which takes on more and more substance until the moment of birth.

Now the individual faces a more complex decision. We can formulate it simply, however, by compressing its elements to an annual basis. This device is handy for him in practice as for us in theory, which augurs well for its realism, since no economizing technique, however valid, is likely to find much use unless easily rendered in the common tongue.

The individual who expects land prices to rise still higher will now consider that increase an additional gain, on top of the annual marginal product, to be had by holding land. To the individual, the annual price increment becomes part of the annual "yield" of land.[13] He may regard this aggressively as an increase in the price he can demand of someone else; or defensively as an increase in what someone else might demand of him: he responds much the same to either feeling. Each individual now adds land to his holdings until the last unit thus "yields" him his interest rate: until the marginal product plus the price increment, each divided by

[12] H. M. Lewis, W. D. Heydecker, and R. A. O'Hara, *Land Values, Distribution Within the New York Region and Relation to Various Factors in Urban Growth*, Engineering Series, Monograph No. 3, Regional Plan of New York and Its Environs, New York, 1927, p. 31.

[13] We probably understate the case a good deal by assuming that all parties have equal-valued expectations of annual price increments. Financially less powerful individuals, it seems plausible, would not always be able to attach as much weight to given price increments as would the more powerful, especially if they must commit land to heavy improvements whose presence tends to preclude quick resale. The possibility of resale in a remote and doubtful future may for some of them have very little meaning. Unfortunately we cannot here pursue this thread.

the price, equals his interest rate. Symbolically, until:

$$\frac{\frac{dR}{dL}}{P} + \frac{\triangle P}{P} = i \qquad (4)$$

This equation epitomizes the whole study. Let us christen it the "dynamic equilibrium equation." It is an instrument of many uses and implications, which we can only begin to explore here.

It does not follow that all individuals will extend their holdings, an obvious impossibility. The annual increment that buyers anticipate encourages every one to buy more land; but higher land prices, the other manifestation of their higher expectations, discourage it. But it does not follow, either, that these two new forces will just offset one another in the reckoning of each individual, so that no one will change his holdings. For while the annual increment encourages each in the same measure (assuming, for simplicity, that each anticipates the same increment), the higher price of land titles discourages each buyer in a different measure, in proportion to his interest rate. The net result is that high-interest individuals must contract their already skimpy holdings, some no doubt below the limit of viability, while low-interest individuals may extend theirs.

Those who pay or impute low interest rates buy more land because the expected annual price increment exceeds the increase of their interest burden. Those who reckon high interest rates must contract because the increase of their interest burden exceeds the expected annual price increment. There is of course some intermediate interest rate, call it *j*, those reckoning which need neither expand nor contract. *j* equals the expected annual price increment divided by the price increment that has already materialized. Symbolically,

$$j = \frac{\triangle P}{P_1 - P_0} \qquad (5)$$

where $\triangle P$ is the expected annual increment to land price; P_o is the price that would prevail if buyers expected *a* and *i* to hold steady; and P_1 is the present price. This value for *j* derives from observing that the added incentive to hold land is $\triangle P$, and the added cost is $(P_1 - P_0)\ i$. When the added incentive just equals the added cost, the basic equation still holds true, with no change in the marginal revenue productivity of land $\frac{(dR)}{(dL)}$. This is when:

$$\triangle P = (P_1 - P_0)\ i, \text{ or} \qquad (5a)$$

$$i = \frac{\triangle P}{P_1 - P_0} \equiv j \qquad (5)$$

j is the dividing line between lower interest rates that allow those reckoning them to expand, and higher rates that force those reckoning them to contract.

Returning to Messrs. 2 Per Cent and 6 Per Cent for a numerical example, suppose they expect annual price increments of 2 per cent, and suppose land prices have doubled. Mr. 6 Per Cent's marginal revenue product, formerly 6 per cent of the price of land, is now become only 3 per cent of it, too little to justify his holding so much. To this 3 per cent he adds the expected 2 per cent price increment, but still lacks something of justifying his investment. He must therefore contract until his marginal productivity equals 4 per cent of the present price, or 8 per cent of the original price.

Now Mr. 2 Per Cent, on the other hand, finds the annual increment of 2 per cent adequate to warrant his holding land yielding him no marginal revenue product whatever. He will expand until the marginal revenue productivity of land to him equals zero. This may take the form of spreading his men and equipment very thin over a wide area, or of holding land completely out of use. One need not search far for examples of both practices.

The only persons whom these new expectations leave unmoved are the Messrs. 4 Per Cent, whose interest rate is the *j* of this example ($4/$100 equals 4 per cent). For them the expected price increment of 2 per cent, or $4, just offsets the extra interest on the title. All those with lower interest rates expand; all with greater ones contract.

In result, the marginal productivities of land on different holdings come to differ in greater proportion than individuals' interest rates. As between Messrs. 2 Per Cent, 4 Per Cent and 6 Per Cent, their interest rates are as 1:2:3, but the marginal productivities of land on their different holdings are as 0:2:4.

On the other hand, when buyers expect land to depreciate, marginal productivities of land in different holdings tend to converge with one another. For then interest, the cost that varies most markedly among individuals, ceases to be the only annual cost of holding land. It is joined by depreciation, a cost which interest rates do not determine and which tends, therefore, to afflict all landholders the same. Interest cost loses absolute as well as relative weight. For not only may "Values . . . be created by the mere expectancy of some new use . . . (but) they may depreciate as a result of failure of expectations."[14] When buyers foresee lower land prices, present prices become somewhat lower, too; and price, of course, is the base on which interest cost is reckoned.

[14] Lewis, *et al., loc cit.*

In terms of the dynamic equilibrium equation (4), let buyers expect land prices (P) to fall, and the annual price increment ($\triangle P$) becomes negative. To gather all costs together on the right side, multiply both sides by P, and subtract $\triangle P$ from both. The equation becomes:

$$\frac{dR}{dL} = Pi - \triangle P \qquad (4a)$$

To obviate any confusion in algebraic signs, take the absolute value of $\triangle P$, which here is negative, and write the equation:

$$\frac{dR}{dL} = Pi + |\triangle P| \qquad (4b)$$

(This form applies only to depreciating assets). Each individual now adds land to his holdings until its marginal revenue productivity $\frac{(dR)}{(dL)}$ equals interest cost plus depreciation. Depreciation has shouldered its way next to interest to stand as a major annual cost and share its previously exclusive control over marginal revenue productivity. Interest loses influence also because the prospect of depreciation has eroded off part of its base, P.

Reverting to the numerical example of Messrs. 2 Per Cent, 4 Per Cent, and 6 Per Cent, suppose they expect land price to depreciate by $2 this year, and suppose its price has already dropped to $50. A few simple computations establish that they will adjust their holdings until their respective marginal productivities stand in the proportion 1½:2:2½, closer than their interest rates.

It is an exceptional period, to be sure, when people expect land prices to depreciate. This second form of the dynamic equilibrium equation has its uses, however. Almost all durable assets other than land do depreciate (and/or obsolesce and/or turn over, which for brevity may be included with depreciation). The equation applies as well to them—let us follow the classical terminology and call them "capital"—as to depreciating lands.

The equation, so applied, establishes what intuition suggests, that time-indivisibility little perverts the allocation of mortal capital. Most capital loses value rapidly with time, much more so than in the numerical example just cited, so its annual depreciation exceeds its annual interest cost by a good deal. Indeed many, probably most, economists have come to doubt that businessmen let interest rates influence their decisions to invest in any but the most durable goods.[15]

[15] M. Moonitz, "The Risk of Obsolescence and the Importance of the Rate of Interest," *Journal of Political Economy*, 51 (1943), pp. 349–55; F. A. Lutz, "The Interest Rate and

It is, rather, immortal land, and especially appreciating land, whose allocation time-indivisibility seriously perverts from the equimarginal ideal. With the dynamic equilibrium equation one may contrast the allocation of appreciating land with depreciating capital, simply by adjusting P (up for land, down for capital), and changing the sign of $\triangle P$. Depreciation, added to an interest cost which its prospect has reduced, so dilutes it that large differences among individual's interest rates affect allocation only a little; appreciation, subtracted from an interest cost which its prospect has magnified, so fortifies it that small differences among individuals' interest rates affect allocation a great deal.

V. *Conclusion*

Ideas come crowding to mind for qualifying, as well as applying the dynamic equilibrium equation. Like other abstractions it sacrifices something of reality for clarity and generality; to apply it one must reconstitute it with ingredients from the particular environment, which would pollute such crystalline simplicity and symmetry as it may have with the murkier qualities of the world we live in. This would be all to the good, but is beyond the scope of a few pages. Here we must leave it, with a brief appraisal.

The equation as developed thus far is limited by several assumptions: no taxes; prohibitive leasing costs; negligible transfer costs; identical and certain expectations; permanence of land; and others unspecified. This adds up to rather a lavish use of the *ceteris paribus* prerogative, and in application these must all be relaxed. In doing so one would discover, not that the equation dissolved, but that it proved a useful instrument affording insights, respectively, into tax policy, land tenure, real estate markets, forecasting and uncertainty, and conservation, as well as subjects such as business cycles and imperfect competition for which its present form may be adequately adapted. Thus the equation might serve as a vehicle for re-integrating land economics into the broadening mainstream of developing economic theory from which, since Ricardo, it has tended more and more to flow apart.

To be sure, even with the assumptions relaxed, the equation cannot

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Investment in a Dynamic Economy," *American Economic Review*, 35 (1945), pp. 811–30; H. D. Henderson, "The Significance of the Rate of Interest," *Oxford Economic Papers*, No. 1, October, 1938, p. 9; H. C. Wallich, "The Changing Significance of the Interest Rate," *American Economic Review*, 36 (1946), pp. 761–87; J. E. Meade and P. W. S. Andrews, "Summary of Replies to Questions on Effect of Interest Rates," *Oxford Economic Papers*, No. 1, October, 1938, pp. 14–31; J. F. Ebersole, "The Influence of Interest Rates upon Entrepreneurial Decisions in Business," *Harvard Business Review*, 17 (1938), pp. 35–39. A recent contrary view is expressed in H. W. White, "Interest Inelasticity of Investment Demand," *American Economic Review*, 46 (1956). pp. 565–87.

presume to substitute for mature understanding of all that lies behind it. In compressing all factors to an annual basis it submerges some of the finer points. For example, it subsumes all differences in the terms of lending, in prospects for refinancing, all budgetary restraints of any sort, in the single symbol *i*. And it subsumes all prospects for future improved incomes, or resale at whatever date, in the single symbol $\triangle P$. Such shorthand notation can convey its full meaning only to those knowing something of the complexities it represents—and only to them if they are willing to translate their knowledge into its terms.

But brevity is also a virtue. The equation brings data quickly to focus in a workable form which it is hoped the layman may understand and the specialist find operable. Simplicity lends it a kind of realism that more comprehensive alternative formulations may lack, since probably many landholders actually think in its terms. And its compactness has let us in a few pages reduce to some order a complex tangle of relationships and reach an important substantive conclusion.

Let us review the conclusion. Time-indivisibility deflects from the equimarginal ideal the allocation of durable goods in proportion to their durability: capital, by and large, only a little; stable-priced land more; and appreciating land very seriously. In the bidding for appreciating lands marginal productivity recedes to a secondary role, and yields place to financial power as the prime allocating agent, the "sorter and arranger" of land use. Nothing could be more wrong than the often expressed idea that "Careful maintenance and administration of land are likely to be stimulated by the prospect of a rise in its value; . . ."[16] That prospect even leads financially powerful firms and individuals to hold lands yielding them no income at all.

This conclusion suggests a different emphasis on land in the thinking of economists, many of whom have tended to assess its importance by its imputed income. This income, shrouded as it is in "entrepreneurial withdrawals" and wreathed in "profits," has not made a very tangible object for study. But the importance of land to economic theory, and to social welfare, may lie not so much in this unknown figure as in the latent income unrealized from lands that the "sorter and arranger" withholds from their most productive uses; and from the unemployed and underemployed labor and capital these lands might complement. It may lie, too, in the distribution of land: the conclusion suggests that land may be, contrary to a widespread impression, near the focus of the concentration of economic power.

[16] Shorey Peterson, *Economics*, Rev. Ed., New York, Henry Holt, 1954, p. 674.

In another era men reproached the "dead hand of the past" for keeping land from best use. They had good cause. But is not the unborn hand of the future more to blame? Coming events cast their shadows before to become substance in the prices of land titles. These are anachronisms too, anachronisms out of tomorrow. They ill suffice the needs of the day.

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