CRITIQUES & CONTENTIONS

Alfred Marshall, W. Stanley Jevons, and the Mathematization of Economics

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IF ANY SINGLE CHARACTERISTIC DIFFERENTIATES current, neoclassical economics from that of the classical period of Adam Smith and David Ricardo, it is the use of mathematics. By the early 1950s economists such as Paul Samuelson, Kenneth Arrow, and Gérard Debreu had rendered the general equilibrium theory of prices and distribution into a highly sophisticated mathematical science. Contemporary economists generally trace the origins of neoclassical economics to the Marginal Revolution of the 1870s, but maintain that the transformation into a mathematical discipline only took hold in the interwar years.[1] It is argued here that the mathematization of economic theory was already well under way in late Victorian England.

I

The Marginal Revolution in economics is commonly said to have begun with the publication of three books: William Stanley Jevons's *Theory of Political Economy* (1871), Carl Menger's *Grundsätze der Volkswirtschaftslehre* (1871), and Léon Walras's *Eléments d'économie politique pure* (1874). Each of these professors of political economy—Jevons at Owens College in Manchester, Menger at the University of Vienna, and Walras at the University of Lausanne—had independently arrived at the principle of diminishing marginal utility and proclaimed this insight on exchange value to be the cornerstone of a significantly new science of economics. For this reason, the so-called Marginal Revolution has traditionally been characterized as a recasting of the theory of value. Whereas

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I wish to acknowledge helpful suggestions from Timothy Alborn, Thomas Archibald, Samuel Hollander, Trevor Levere, Gail Maxwell, and Mary Morgan.

[1] When looking to the 1920s and 1930s as the time of transition, scholars have relied far too heavily upon a survey by George Stigler of selected American periodicals from 1892 to 1963. But he and others have mistakenly assumed that the periodical, rather than the book, was representative of seminal work in economics before World War I. Moreover, it is misleading to generalize from the American context until the middle of this century. Recollections by leading theorists, such as Paul Samuelson, of entering the field in the 1930s and finding the courses still entirely literary tell us something about the situation in the United States, but imply little or nothing about the earlier decades in England. See George J. Stigler, *Essays in the History of Economics* (Chicago: Univ. Chicago Press, 1965), p. 48; and Paul A. Samuelson, "Economics in My Time," in *Lives of the Laureates,* ed. William Breit and Roger W. Spencer (Cambridge, Mass.: MIT Press, 1986), pp. 59–76.

the classical theory of Smith, Ricardo, and John Stuart Mill focused upon costs (labor) as the determinant of exchange value, the neoclassical theory emphasized utility and the role of the consumer. Or so the story was told in countless texts on the history of economic thought.[2]

Since the 1970s, however, scholars have come to question this account. The impetus for their reinterpretation derived more, it appears, from careful historical research than from collateral debates on the structure of scientific revolutions, though these also left their mark. Studies of various pre-1870 economists established quite conclusively that the principle of diminishing marginal utility had been discovered countless times throughout the last century and recognized as a partial explanation of market prices. Other studies challenged the received view that, as Joseph Schumpeter once put it, "Jevons, Menger, and Walras taught essentially the same doctrine."[3] William Jaffé, for one, has argued that each of these economists, in relative isolation from one another, built upon quite distinct intellectual traditions and drew different implications from the principle of marginal utility. Erich Streissler has also done much to separate Menger from Walras and Jevons, not only because Menger eschewed mathematics, but also because marginalism per se was not a major component of his theory. Indeed, the difficulties involved in treating Jevons, Menger, and Walras as a united front have led some, such as Mark Blaug, to deny categorically that a revolution happened at the time.[4]

If one looks only at England, however, a fairly compelling case can be made for a Jevonian revolution. As Terence Hutchison has argued, "in the space of a few years in the late 1860s and early 1870s the classical structure of 'theory' underwent a remarkably sudden and rapid collapse of credibility and confidence, considering how long and authoritative had been its dominance in Britain."[5] And in many important respects, Jevon's *Theory* marked the beginning of a new school of thought. Precursors notwithstanding, Jevons was the first in Britain to expound at length the psychological dimensions of the principle of diminishing marginal utility, and thereby to provide a richer foundation for the analysis of demand and prices. But for all his talk of the novelty of the utility calculus, he never denied the importance of costs in a long-run account of exchange value. More significantly, he was the first to argue systematically that the true science

[2] See, e.g., Joseph A. Schumpeter, *A History of Economic Analysis* (New York: Oxford Univ. Press, 1954); and Henry William Spiegel, *The Growth of Economic Thought* (Durham, N.C.: Duke Univ. Press, 1971).

[3] Schumpeter, *History,* p. 952. The main locus for these debates is a special issue of *History of Political Economy,* 1972, *4*:267–624, reprinted as R. D. C. Black, A. W. Coats, and C. D. W. Goodwin, eds., *The Marginal Revolution in Economics: Interpretation and Evaluation* (Durham, N.C.: Duke Univ. Press, 1972). On scientific revolutions see Martin Bronfenbrenner, "The 'Structure of Revolutions' in Economic Thought," *Hist. Polit. Econ.,* 1971, *3*:136–151; and A. W. Coats, "Is There a 'Structure of Scientific Revolutions' in Economics?" *Kyklos,* 1969, *22*:289–296. On pre-1870 economists see, e.g., Marian Bowley, "The Predecessors of Jevons—The Revolution That Wasn't," *Manchester School,* 1972, *40*:9–29; and George Stigler, "The Adoption of the Marginal Utility Theory," *Hist. Polit. Econ.,* 1972, *4*:571–586.

[4] See William Jaffé, "Jevons, Menger, and Walras De-Homogenized," *Economic Inquiry,* 1976, *14*:511–524; Erich Streissler, "To What Extent Was the Austrian School Marginalist?" *Hist. Polit. Econ.,* 1972, *4*:426–441; and Mark Blaug, "Was There a Marginal Revolution?" *ibid.,* pp. 269–280.

[5] T. W. Hutchison, *On Revolutions and Progress in Economic Knowledge* (Cambridge: Cambridge Univ. Press, 1978), p. 58. A "Jevonian revolution" is also explicitly acknowledged by Maurice Dobb in *Theories of Value and Distribution since Adam Smith: Ideology and Economic Theory* (Cambridge: Cambridge Univ. Press, 1973).

of economics called for mathematics, and he persisted in promoting this cause throughout his career.[6] In short, the new direction set by Jevons was methodological. Although others, notably William Whewell and Thomas Robert Malthus, had recognized long before him the possibility of a mathematical economics, Jevons discovered the means to anchor the calculus, by focusing on utility maximization and by bringing to bear numerous mechanical analogies.[7]

By the time Jevons died in 1882, a number of economic theorists had rallied around his call "to fling aside, once and for ever, the mazy and preposterous assumptions of the Ricardian School."[8] The two most important converts were Philip Henry Wicksteed and Francis Ysidro Edgeworth, who subsequently devised much of the mathematical apparatus—indifference and contract curves, Lagrangian multipliers, and production coefficients—that students of microeconomics still imbibe. Wicksteed, perhaps more than anyone, followed Jevons most closely with his insistence "that certain fundamental relations and conceptions in the theory of political economy are essentially mathematical." Edgeworth, who had been a neighbor and close friend of Jevons for several years, also took this dictum to heart in his *Mathematical Psychics* (1881), of which John Creedy has remarked that "there is probably no other apology in the whole of economic literature which compares with Edgeworth's passionate plea for the application of mathematics." In later papers, Edgeworth seemed less committed to a belief in the necessity of mathematics, though his use of the calculus continued unabated.[9]

Other economists active in the 1880s and 1890s, notably Herbert Somerton Foxwell at Cambridge and University College, London, and the two leading Cambridge philosopher-economists, Henry Sidgwick and John Neville Keynes, expressed considerable appreciation for the new approach. In 1886, Foxwell wrote to Léon Walras: "I went to see Dr. Sidgwick myself, in order to be quite sure about his relation to Jevons. He authorizes me to say that he is 'quite content to be described in general terms as a follower of Jevons.' So am I." In

[6] See the many excellent papers by R. D. Collison Black, e.g., "W. S. Jevons and the Foundations of Modern Economics," *Hist. Polit. Econ.,* 1972, *4*:364–378.

[7] Although political economists had long sought to emulate Newtonian physics, they did not readily take to mathematics. Antoine Augustin Cournot had issued his *Recherches sur les principes mathematiques de la théorie des richesses* in 1838, but it was virtually ignored until the late 1870s. Both Jean-Baptiste Say and John Stuart Mill had discredited the applicability of mathematics, although Mill, following Auguste Comte, advocated its study as a way of training the mind. Other economists of the Millian persuasion—John Elliott Cairnes at University College, London, Henry Fawcett at Cambridge, and Bonamy Price and Thorold Rogers at Oxford—made a point of upholding the literary tradition well into the 1870s.

[8] William Stanley Jevons, *The Theory of Political Economy,* 5th ed. (London: Macmillan, 1957), p. xliv. One of the first to promote Jevons's cause was George Darwin, second wrangler and son of the famous naturalist. On his contribution, and on the general reception of the *Theory,* see Margaret Schabas, "Some Reactions To Jevons' Mathematical Program: The Case of Cairnes and Mill," *Hist. Polit. Econ.,* 1985, *17*:337–353.

[9] Philip Henry Wicksteed, *The Common Sense of Political Economy and Selected Papers and Reviews on Economic Theory,* ed. Lionel Robbins, 2 vols. (London: Routledge, 1935), Vol. II, p. 811; John Creedy, *Edgeworth and the Development of Neoclassical Economics* (Oxford/New York: Basil Blackwell, 1986), p. 51; and Francis Ysidro Edgeworth, *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences* (New York: Augustus M. Kelly, 1967). Edgeworth's 1889 presidential address to Section F of the British Association, "On the Application of Mathematics to Political Economy," notes some of the limitations of mathematical applications but points to certain results, such as Cournot's study of monopoly, that would be difficult to derive without its aid: Edgeworth, *Papers Relating to Political Economy,* 3 vols. (1925; rpt. New York: Burt Franklin, 1970), Vol. II, pp. 273–291.

Foxwell's estimation, "there is no doubt Mill is dead in this country. The mathematical method for pure theory, the historical for practical questions—this is what we are all coming to agree in."[10]

Neville Keynes's celebrated *The Scope and Method of Political Economy* (1891) consolidated this perspective. While granting the value of the historical and literary tradition, he maintained that "it would be difficult to exaggerate the gain that has resulted from the application of mathematical ideas to the central problems of economic theory."[11] In America, Jevons's innovations were readily praised by Simon Newcomb and subsequently developed by Irving Fisher, whose doctoral dissertation at Yale (under the supervision of J. Willard Gibbs) yielded the widely appreciated *Mathematical Investigations in the Theory of Value and Prices* (1892). Fisher also wrote the first textbook on the subject, *A Brief Introduction to the Infinitesimal Calculus, Designed Especially to Aid in Reading Mathematical Economics and Statistics* (1897).

All of this lends considerable weight to the view that mathematical economics took hold and was widely accepted, in the English-speaking world at least, by the end of the last century. But an explicit recognition of this transformation has thus far escaped entrenchment in the secondary literature. Historians have generally been led to diminish the significance of Jevons's campaign to mathematize the subject by the main exception to its success: the contribution of Alfred Marshall (1842–1924). Doyen of economists, Marshall purportedly turned back to the classical theorists and resisted the urge to mathematize the subject.[12] And yet, when one delves into Marshall's relationship to Jevons, considering his teaching and early writings, it appears that Marshall's role in the rise of mathematical economics was of greater significance than has commonly been supposed.

These misinterpretations occur mainly because Marshall's own retrospective account cannot be taken as an accurate representation of his earlier beliefs, a problem compounded by his failure to publish many of his ideas until his later years. A study of Marshall's "memorials" has led various scholars to charge him with inconsistency, a poor memory, a fear of being found wrong, an "inadequate acknowledgment of priority," and a tendency "of being less than frank." He is also reputed to have had a "talent for generating antagonism among his contemporaries," such that their appraisals of him must also be taken with a grain of salt.[13] An assessment of Marshall's part in the story is thus no easy matter. My

[10] Foxwell also referred to Wicksteed as "an enthusiastic follower of Jevons" and noted that Wicksteed's impression of the profession in Holland was that "all the leading [Dutch] economists [are] Jevonians": *Correspondence of Léon Walras and Related Papers,* ed. William Jaffé, 3 vols. (Amsterdam: North-Holland, 1965), Vol. II, pp. 160–162.

[11] John Neville Keynes, *The Scope and Method of Political Economy* (London: Macmillan, 1891), p. 251.

[12] See the oft-cited G. F. Shove, "The Place of Marshall's *Principles* in the Development of Economic Theory," *Economic Journal,* 1942, *52*:294–329.

[13] On the issue of inconsistency see H. M. Robertson, "Alfred Marshall's Aims and Methods Illustrated from His Treatment of Distribution," *Hist. Polit. Econ.,* 1970, *2*:1–65. For the next two accusations see Richard S. Howey, *The Rise of the Marginal Utility School, 1870–1889* (Lawrence: Univ. Kansas Press, 1960), pp. 85–87n; and Schumpeter, *History* (cit. n. 2), p. 839. Maynard Keynes has perhaps the best account of Marshall's character flaws; see "Alfred Marshall," in John Maynard Keynes, *Essays in Biography* (London: Rupert Hart-Davis, 1951), pp. 174–176. For Marshall's tendency "of being less than frank" see *The Early Economic Writings of Alfred Marshall, 1867–1890,* ed. John K. Whitaker, 2 vols. (London: Macmillan, 1975), Vol. I, p. 103; and for the last characteristic see John Maloney, *Marshall, Orthodoxy, and the Professionalisation of Economics* (Cambridge: Cambridge Univ. Press, 1985), p. 65.

task here will be primarily to clarify and evaluate this muddled record and to bring to light the direct lineage between Jevons and Marshall on the issue of mathematical economics.

II

In treating the relationship between Jevons and Marshall, one must bear in mind that Marshall, though only seven years younger than Jevons, did not publish his first book (*The Economics of Industry,* coauthored with his wife, née Mary Paley), and an elementary textbook at that, until 1879, just three years before Jevons drowned. Although Jevons had copies of two short pamphlets by Marshall (distributed privately in 1879) that suggested the greatness of things to come, he could not have gathered from Marshall's written work that late Victorian economics would be seen as the "Marshallian Age."[14] Jevons recognized Marshall as a promising teacher of the subject and had considered appointing him as his substitute at University College in 1875; he was also quite impressed by the 1879 essays.[15] During the years in which they corresponded (1875–1879), Jevons evidently regarded Marshall more as a possible recruit to his campaign than as a rival.

Like Jevons, Marshall turned to economics having already studied mathematics and the natural sciences, and he was similarly attracted to the ethical dimensions of the subject. It is all the more surprising, then, that they did not collaborate. Following his success as second wrangler in 1865, Marshall was appointed a fellow of St. John's College, Cambridge, and in 1868, lecturer of political economy, a position he held until 1877. Jevons may have known of Marshall's appointment in 1868, but they did not correspond until January 1875 and did not meet until the end of that year.[16] In his capacity as external examiner for the moral sciences tripos at Cambridge in 1874 and 1875, Jevons had been very impressed by the performance of Marshall's students (including Mary Paley). He duly noted their facility with graphical techniques in his first letter to Marshall, with the express hope that the latter might publish these findings.

Jevons also sent Marshall a copy of his "Progress of the Mathematical Theory of Political Economy" (1874), most likely with the idea of gathering support. But Marshall, rather than enlist, merely conceded to Jevons that "the substantive difference between us is less than I once supposed." From that point on, their few exchanges were cordial but reserved. In 1877 Jevons wrote a favorable letter

[14] See Schumpeter, *History* (cit. n. 2), p. 830. The two pamphlets, available as London School of Economics Reprints, are entitled "Pure Theory of Foreign Trade" and "Pure Theory of Domestic Values." Despite the immense popularity of *The Economics of Industry,* Marshall was in later years to deplore its existence and even went to the trouble in 1892 of reissuing, though as sole author, a quite different version with the same title. Prior to 1879 Marshall had published only four essays, perhaps the most important of which was "On Mr. Mill's Theory of Value," *Fortnightly Review,* April 1876, rpt. in *Memorials of Alfred Marshall,* ed. A. C. Pigou (London: Macmillan, 1925; rpt., New York: Augustus M. Kelly, 1966), pp. 119–133. It defended Mill against the recent criticisms of Cairnes. On Mary Paley's interesting career see "Mary Paley Marshall," in Keynes, *Essays in Biography* (cit. n. 13), pp. 324–347.

[15] See *Papers and Correspondence of William Stanley Jevons,* ed. R. D. Collison Black, 7 vols. (London: Macmillan, 1972–1981), Vol. IV, pp. 204–205. Jevons chose Foxwell as his substitute instead.

[16] This seems to have been their only encounter, unless their paths crossed at British Association meetings. Jevons had attempted unsuccessfully to meet Marshall on his first visit to Cambridge.

of recommendation for Marshall when the latter applied for a position at Bristol, and in 1879, by way of thanking Jevons for mentioning two of his unpublished papers in the preface to the second edition of the *Theory,* Marshall acknowledged Jevons as "the chief author . . . of abstract quantitative reasoning."[17] Neither economist, however, wrote to the other as openly or as frequently as both did to their mutual friends, Foxwell and Edgeworth. In fact, the content of the Jevons-Marshall correspondence and its paucity suggest that Marshall wished to remain as independent of Jevons as possible (the same is true of Marshall's correspondence with Léon Walras). As Maynard Keynes later remarked, Marshall was "extraordinarily reluctant to admit that he owed anything to Jevons."[18]

Marshall's first publication was a less-than-glowing review of Jevons's *Theory of Political Economy* for the *Academy* (April 1872). We know from the extant manuscripts that, when Marshall reviewed the book, he was hard at work on a mathematical treatment of economic theory. One might expect him to have welcomed a fellow practitioner of mathematical economics, as Jevons did on many occasions. But, though Marshall grants that Jevons's book is strewn with "suggestive remarks and careful analyses," he emphasizes the various inaccuracies and in places absurdities of Jevons's mathematical manipulations. In a retrospective note of the late 1890s, Marshall attributed his begrudging review to Jevons's less-than-adroit mathematics. Whereas Cournot and Johann Heinrich von Thünen "handled their mathematics gracefully," Jevons seemed "like David in Saul's armour." Moreover, the substance of Jevons's *Theory* failed to match its revolutionary claims. The difference between Jevons and David Ricardo, he maintains, is primarily one of form: "we continually meet old friends in new dresses." Jevons's repudiation of Ricardo and Mill was even more unforgivable: "He [Jevons] seemed perversely to twist his own doctrines so as to make them appear more inconsistent with Mill's and Ricardo's than they really were. But the genius which enabled Ricardo—it was not so with Mill—to tread his way safely through the most slippery paths of mathematical reasoning, . . . had made him one of my heroes; and my youthful loyalty to him boiled over when I read Jevons' *Theory.*"[19]

It has often been suggested that Marshall's unsympathetic treatment of Jevons was due in part to his disappointment at having been anticipated. But while Marshall was not totally indifferent to issues of priority, it seems implausible, given his appreciation for Cournot and von Thünen, that Jevons would have irritated him so excessively on this account.[20] Rather, Marshall initially took

[17] Jevons, *Correspondence* (cit. n. 15), Vol. IV, p. 100 (first quotation), and Vol. V, p. 66.

[18] "William Stanley Jevons," in Keynes, *Essays in Biography* (cit. n. 13), p. 287. From the extant correspondence, which consists of five letters in all, it is apparent that there must have been at least two additional letters. Of the three exchanges, in 1875, 1877, and 1879, not one of the letters exceeds a printed page. On Marshall's low regard for Walras see Marshall, *Writings* (cit. n. 13), Vol. I, pp. 103–107.

[19] Alfred Marshall, "Mr. Jevons' Theory of Political Economy," rpt. in Pigou, *Memorials* (cit. n. 14), pp. 93–99, on pp. 93, 99, 95; and Marshall, "Comment on the Above Review in an Undated MS. . . . ," *ibid.,* pp. 99–100. Similar uncharitable comments appear in his later work; see Alfred Marshall, *Principles of Economics,* 8th ed. (London: Macmillan, 1920; rpt. 1949), pp. 673–675. As far as Marshall was concerned, the fact that Ricardo did not use mathematics was of little consequence: "There is a class of economic problems which cannot be safely treated by anyone of less genius than Ricardo without the aid of some apparatus, either of mathematics or of diagrams": *ibid.,* p. 688.

[20] In 1908, Marshall recollected the following: "My main position as to the theory of value and distribution was practically completed in the years 1867 to 1870, when I translated Mill's version of

umbrage at Jevons's pretensions to complete novelty. He firmly believed that "the older economists had not recognized the mathematical conceptions that were latent in their own [work]" and that Ricardo was "feeling his way towards the distinction between marginal and total utility." Jevons, in Marshall's estimation, had done much "mischief by implying that the older economists were more at fault than they really were."[21]

Following the publication of Jevons's journal and selected correspondence in 1889, Marshall may have come to realize that Jevons had indeed broken new ground in the early 1860s without the least awareness of his Continental precursors. Marshall later expressed some regret for his harsh treatment of Jevons: "I have since learnt to estimate him better . . . and I reverence him now as among the very greatest of economists."[22] But it is his earlier, less favorable reactions that are most frequently recollected.

While there can be no question that Jevons had developed the ideas presented in his 1871 work independently of Marshall, what of Marshall's claim to complete independence from Jevons? Marshall maintained that he had only needed Adam Smith, Ricardo, Mill, Cournot, and von Thünen in order to develop the central points of his main work, *Principles of Economics* (1890), and that all of this had taken place before 1872, when he reviewed Jevons's *Theory*. It has never been doubted that it would have been possible for a mind such as Marshall's to have used only those sources. After all, Jevons, though he failed to draw out the full implications of the new approach, had even fewer works to guide him. But scholars have suggested that Marshall's own account—written some thirty or more years after the fact and with very few supporting documents—is not wholly convincing.[23] We know that Marshall read Jevons carefully in order to write his review, that he used Jevons's *Theory* for his lectures to women at Cambridge in the years 1873–1875 and perhaps on other occasions, and that he wrote the bulk of the mathematical appendix to the *Principles* between 1870 and 1874. Moreover, in one of his autobiographical fragments he states that his "apprenticeship to economic studies [occurred] between 1867 and 1875."[24] It seems quite likely, then, that Marshall had learned something from Jevons. In a letter to Jevons (1879) Foxwell wrote: "Marshall always spoke in the highest terms of your book

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Ricardo's or Smith's doctrines into mathematics; and that, when Jevons's book appeared, I knew at once how far I agreed with him and how far I did not": Marshall to John Bates Clark, 24 Mar. 1908, in Pigou, *Memorials,* p. 416. In his early work Marshall also shared with Jevons an appreciation for drawing analogies to classical mechanics, though he shifted over time toward a preference for biological motifs. See H. Scott Gordon, "Alfred Marshall and the Development of Economics as a Science," in *Foundations of Scientific Method: The Nineteenth Century,* ed. Ronald Giere and R. S. Westfall (Bloomington: Indiana Univ. Press, 1973), pp. 234–258.

[21] Alfred Marshall, "Fragments," in Pigou, *Memorials,* p. 359; and Marshall, *Principles* (cit. n. 19), pp. 670, 76.

[22] Marshall, "Jevons' *Theory*" (cit. n. 19), p. 99. Marshall also wrote: "There are few writers of modern times who have approached as near to the brilliant originality of Ricardo as Jevons has done": Marshall, *Principles,* p. 673.

[23] See Marshall, *Writings* (cit. n. 13), Vol. I, p. 103; and Howey, *Marginal Utility School* (cit. n. 13), pp. 79–80.

[24] Marshall to John Bates Clark, in Pigou, *Memorials* (cit. n. 14), pp. 416–417 (on the *Principles*); and Marshall, "Fragments," *ibid.,* p. 358. Mary Paley Marshall maintained that Jevons's *Principles* was used in the Cambridge lectures, but Whitaker has noted, quite soundly, that she must have meant Jevons's *Theory,* since the other work was not published until 1905. See Whitaker, in Marshall, *Writings,* Vol. I, p. 11.

from its first appearance, which might prove either that he was prepared to agree with it, or that he had learnt a good deal from it, or both."[25]

Nevertheless, the onus is still on those who question Marshall's assertions to point to portions of his work that reveal the direct influence of Jevons's work. In this regard, Marshall's silence during the period in question works very much in his favor. According to John K. Whitaker, one of the few documents we have to go by is Marshall's review of Jevons, which he claims "is consistent with the view that Marshall became prone in later life to exaggerate his precocity as a theorist." Whitaker has claimed, from his study of the Marshall manuscripts written between 1872 and 1879, that "even without the evidence of the explicit reference to Jevons, the deference to his ideas and terminology is sufficiently marked to indicate his influence. Marshall's stance is much more explicitly 'hedonic' here than in the remainder of his early work, a fact which helps confirm the suspicion that Jevons's utilitarian calculus had more influence on Marshall's development than he was willing to admit."[26]

Another historian, Richard S. Howey, has maintained that a comparison of Marshall's 1872 review of Jevons with his 1881 review of Edgeworth, which was both favorable and cognizant of Edgeworth's debt to Jevons, strongly suggests that Marshall had grown accustomed to the powers of marginal utility analysis in the years after 1871 rather than before. All of this reinforces Whitaker's position, namely, that is must have taken at least a decade for Marshall to refine and clarify what were, in 1871, "little more than half-formed thoughts."[27] But neither historian has made a totally convincing case, one that would allow us to discredit completely Marshall's own claims to the contrary. Indeed, in a statement written around 1900, Marshall is adamant that he was not indebted to Jevons for the basic principles of marginal analysis: "On many aspects of economics I have learnt more from Jevons than from anyone else. But my obligations which I had to acknowledge in the Preface to my *Principles* were to Cournot and von Thünen and not to Jevons."[28]

Presumably Marshall had learned much from Jevons's empirical work, his *Money and the Mechanism of Exchange* (1875), *The State in Relation to Labour* (1882), and *Investigations in Currency and Finance* (1884), since he assigned them as reading for the new tripos in economics at Cambridge (1903). But can one detect further links, particularly in the form of mathematical techniques?

[25] Foxwell to W. S. Jevons, 12 Nov. 1879, in Jevons, *Correspondence* (cit. n. 15), Vol. V, p. 78. Foxwell also mentions that Henry Sidgwick "thinks that Marshall's theory of Distribution is founded on your [Jevons's] theory of Final Utility." Although Foxwell was a colleague of Marshall's, he was also prone to playing the ambassador and may have embellished things in order to improve relations between Jevons and Marshall.

[26] Whitaker, in Marshall, *Writings* (cit. n. 13), Vol. II, p. 286.

[27] See Howey, *Marginal Utility School* (cit. n. 13), pp. 79–80; and Whitaker, in Marshall, *Writings,* Vol. I, p. 44. Although Marshall used Jevons's phrase "final utility" in his 1879 papers, he made a point, in his *Principles,* of switching to "marginal utility." In the preface to the first edition he maintained that he had known of the latter phrase all along from the work of von Thünen, that is, from 1869 or 1870. But it was pointed out to him that von Thünen never used the phrase in German, and Marshall corrected this error in the second edition. However, in a letter to J. B. Clark, dated 1900, Marshall forgot his error and restated his illegitimate debt to von Thünen, another instance, perhaps, of his refusal to acknowledge his links to Jevons. See Howey, *Marginal Utility School,* pp. 83–84, for this interesting detective work.

[28] Marshall, "Comment on Above Review" (cit. n. 19), p. 100. Marshall also once claimed in a letter that "I make it a point of honour to acknowledge my obligations—whenever I contract them, and when they are not obvious": quoted by Whitaker, in Marshall, *Writings* (cit. n. 13), Vol. I, p. 49.

According to Whitaker, "there is from the first an awkwardness and hesitancy about Marshall's efforts at mathematical economics that argues against him ever having breathed wholly freely on the pinnacles of abstraction. Both Jevons and Edgeworth seem to have dwelt more comfortably in the realm of abstract logic, despite their inferiority to Marshall in mathematical training." An overview of Marshall's work before the *Principles* suggests that he only gradually shed his predilections for geometry in favor of algebraic representations. In early correspondence with Edgeworth (1880), Marshall waxed enthusiastic about what "seems to be a very close agreement between us as to the promise of mathematics in the sciences that relate to man's actions," but also noted that "I intend never to use analysis when I can use geometry."[29] Ten years later, his mathematical appendix to the *Principles* amply demonstrated the value of a judicious application of linear algebra and the differential and integral calculus. Quite possibly he came to appreciate the powers of algebraic formalizations in part by studying Jevons's work over the course of many years.

III

It may never be possible, given the extant evidence, to resolve this controversy over the degree of Marshall's indebtedness to Jevons. But to a large extent, it does not matter. For if, as Howey and Whitaker maintain, Marshall was influenced by Jevons in the 1870s and 1880s more than he admitted or even realized, then he can be viewed, together with Edgeworth, Wicksteed, Foxwell, Sidgwick, and Neville Keynes, as reinforcing the shift begun by Jevons. But even if we take Marshall completely at his word and accept his claim to have worked out his theoretical insights quite independently, then, by the same token, we may conclude that the primary catalyst for Marshall's discoveries was mathematics itself. He made this point on several occasions over a number of years, speaking, for example, of himself as a young man "accustomed to think in Mathematics more readily than in English, and bewildered on his sudden entry into the strange land of economics." He wrote John Bates Clark in 1900: "My acquaintance with economics commenced with reading Mill, while I was still earning my living by teaching Mathematics at Cambridge; and translating his doctrines into differential equations as far as they would go; and, as a rule, rejecting those which would not go."[30] This method led him to discard Mill's wage-fund

[29] Whitaker, in Marshall, *Writings,* Vol. I, p. 5; and Marshall to Edgeworth, 8 Feb. 1880, quoted in Creedy, *Edgeworth* (cit. n. 9), p. 51; Marshall may have been restricted by the more traditional Cambridge curriculum. See Harvey W. Becher, "William Whewell and Cambridge Mathematics," *Historical Studies in the Physical Sciences,* 1980, *2*:1–48. Jevons had the good fortune to have studied mathematics for four years under Augustus De Morgan at University College, London, and went on to make important contributions to symbolic logic in response to the work of George Boole and De Morgan. Both Jevons and Marshall were impressed with Fleeming Jenkin's explorations into economic geometry. See Jenkin, "The Graphical Representation of the Laws of Supply and Demand, and Their Application to Labour" (1870), rpt. in *Papers: Literary, Scientific, &c. by the Late Fleeming Jenkin,* 2 vols., ed. Sidney Colvin and J. A. Ewing (London: Longmans, Green, 1887), Vol. II, pp. 76–106. Keynes has suggested that an important source of inspiration was W. K. Clifford, who was friendly with Jevons, Edgeworth, and Marshall, but I have been unable to substantiate this point. Keynes, *Essays in Biography* (cit. n. 13), p. 159n.

[30] Marshall, "Fragments," in Pigou, *Memorials* (cit. n. 14), p. 359; and Marshall to J. B. Clark, 2 July 1900, *ibid.*, p. 412. A perusal of Marshall's early papers suggests that he derived a number of insights by playing around with demand and supply curves, rather than with the differential calculus. See, e.g., Marshall, *Writings,* Vol. I, pp. 117–164.

theory and thereby to begin to see that the determination of wages and profits is analogous to rents. The heuristic value of mathematics, he recognized, was due in part to the nature of the phenomena themselves:

> Our observations of nature, in the moral as in the physical world, relate not so much to aggregate quantities, as to increments of quantities, and that in particular the demand for a thing is a continuous function, of which the "marginal" increment is, in stable equilibrium, balanced against the corresponding increment of its cost of production. It is not easy to get a clear full view of continuity in this aspect without the aid either of mathematical symbols or of diagrams.[31]

In this regard, Jevons could not have wished for a better flag bearer.

Jevons and Marshall were also very much at one with respect to the epistemological standing of mathematical economics. Indeed, one of the most unambiguous compliments in Marshall's review of Jevons's *Theory* was directed at the author's "singularly good" arguments for applying mathematics to economics. The opening sections of the *Theory* discussed these matters at length. While Jevons insisted that mathematics was the best means of deriving sound propositions, he always granted the possibility of translating from mathematics back into ordinary language.[32] Moreover, mathematics per se would not impart greater certainty or exactitude to economic doctrines. It primarily served to clarify one's assumptions, to ensure that a determinate solution existed, but only empirical methods, particularly the use of statistics, could ascertain the degree of correspondence between theory and reality. This was precisely Marshall's point of view, and, whether he liked it or not, he was beholden to Jevons (and to a lesser extent Edgeworth) for having worked much of this out.

Marshall, however, is more commonly known for his hostility toward mathematical economics. But this, I submit, is an artifact of his peculiar regard for Jevons, of his delays in publishing, and of the wider publicity given to his later correspondence. True, his *Principles of Economics* has virtually no algebraic formulas in the body of the text, owing to his wish to appeal to the same general audience as had read Mill's *Principles of Political Economy* (1848). Nevertheless, his many footnotes display to the careful reader the value of a geometrical treatment, and he refers throughout to the relevant sections in the mathematical appendix. Marshall's peculiar claim that his diagrams were not mathematics, even though he knew full well that they had algebraic equivalents, allowed them to grace the same pages as his text. But although he insists, in the preface to the *Principles,* that "the argument in the text is never dependent on [the diagrams]," he is hard pressed to explicate a number of key points, such as derived demand or consumer's surplus, without them.[33]

[31] Marshall, *Principles* (cit. n. 19), pp. viii–ix (preface to first edition); see also Marshall to Clément Colson, 1907, printed in *Econometrica,* 1933, *1*:221–222.

[32] "The symbols of mathematical books are not different in nature from language; they form a perfected system of language, adapted to the notions and relations which we need to express. They do not constitute the mode of reasoning they embody; they merely facilitate its exhibition and comprehension. If, then, in Economics, we have to deal with quantities and complicated relations of quantities, we must reason mathematically; we do not render the science less mathematical by avoiding the symbols of algebra—we merely refuse to employ, in a very imperfect science, much needing every kind of assistance, that apparatus of appropriate signs which is found indispensable in other sciences." Jevons, *Theory* (cit. n. 8), p. 5.

[33] Marshall, *Principles* (cit. n. 19), p. ix. In a draft of a letter to his publisher, 1878, Marshall

Also puzzling is Marshall's assumption that his geometry was readily accessible to the layman. We learn from Maynard Keynes that Marshall "always used [diagrams] freely in his lectures," but that this tended to have "choked off—more or less deliberately—the less serious students."[34] Surely he must have realized, with attrition averaging close to fifty percent year after year, that his curves were just as inscrutable to the average Cambridge student as any algebraic formulas. Although Marshall elsewhere remarked that the mathematics should be kept "in the background," this does not square with either his written work or his university lectures. As many have noted, the mathematics required for Marshall's analysis lie just beneath the surface. Even Marshall admitted to his readers that "there are many problems of pure theory, which no one who has once learnt to use diagrams will willingly handle in any other way."[35]

Moreover, one must explain why, if Marshall was so antagonistic toward mathematics, he took the trouble to include the appendixes and spent the good part of several months insuring that they were printed correctly. The answer is readily apparent in his first letter to Frederick Macmillan (1887). "Mathematics," he wrote, "cannot now be avoided in some branches of economics." And in the first edition of the *Principles,* he made a point of praising mathematics for having "compelled a more careful analysis of all the leading conceptions of economics, and especially of demand."[36] In subsequent editions he omitted this remark, though by the eighth edition (1920) he recognized a different virtue, the furthering of consensus within the profession:

> The new analysis is endeavouring gradually and tentatively to bring over into economics, as far as the widely different nature of the material will allow, those methods of the science of small increments (commonly called the differential calculus) to which man owes directly or indirectly the greater part of the control that he has obtained in recent times over physical nature. . . . There is a remarkable harmony and agreement on essentials among those who are working constructively by the new method.

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describes a potential "book on the method of diagrams distinct from but allied to the methods of analytical mathematics applied to economic theory including Foreign trade curves": Marshall, *Writings* (cit. n. 13), Vol. I, p. 63. Marshall may also have avoided mathematics out of fear of offending his mentor, Benjamin Jowett: see Marshall, "Reminiscences," in Pigou, *Memorials* (cit. n. 14), p. 66. According to Jowett, "Political Economy is human & concrete & should always be set forth in the best literary form: the language of symbols may be relegated to notes & appendices": Jowett to Marshall, 25 Dec. 1884, quoted in Marshall, *Writings,* Vol. I, p. 28.

[34] Keynes, *Essays in Biography* (cit. n. 13), pp. 160n, 164n, 196. E. A. Benians also mentions the high attrition in Marshall's course: see Benians, "Reminiscences," in Pigou, *Memorials,* p. 78.

[35] Marshall, *Principles* (cit. n. 19), p. ix. On the math beneath Marshall's analysis see A. C. Pigou, "In Memoriam: Alfred Marshall," in Pigou, *Memorials,* p. 86; "Marshall," in Keynes, *Essays in Biography,* pp. 159–160; and, most recently, Philip Mirowski, "Physics and the 'Marginalist Revolution,' " *Cambridge Journal of Economics,* 1984, *8*:362–379. For keeping math in the background see Marshall to Walras, 19 Sept. 1889, in Walras, *Correspondence* (cit. n. 10), Vol. II, p. 355.

[36] Marshall to F. Macmillan, 12 Apr. 1887, as quoted in C. W. Guillebaud, "The Marshall-Macmillan Correspondence over the Net Book System," *Econ. J.,* 1965, *75*:518–538, on p. 519; and Alfred Marshall, *Principles of Economics,* 1st ed. (London: Macmillan, 1890), p. 85. On Marshall's oversight of the printing see "Marshall," in Keynes, *Essays in Biography,* p. 180. Marshall also wanted to keep the price of the book low and helped Macmillan to experiment with a new pricing system: see Guillebaud, "Marshall-Macmillan Correspondence." Marshall again owns up to the need for mathematics in the *Principles* (cit. n. 19), p. 688; and in his and his wife's *Economics of Industry* a problem is passed over because "an exact treatment . . . requires the aid of mathematics": quoted in J. K. Whitaker, "The Marshallian System in 1881: Distribution and Growth," *Econ. J.,* 1974, *84*:1–17.

CHAPTER IV

THE ELASTICITY OF WANTS

III, IV, 1. Definition of *elasticity of demand.*

§ 1. We have seen that the only universal law as to a person's desire for a commodity is that it diminishes, other things being equal, with every increase in his supply of that commodity. But this diminution may be slow or rapid. If it is slow the price that he will give for the commodity will not fall much in consequence of a considerable increase in his supply of it; and a small fall in price will cause a comparatively large increase in his purchases. But if it is rapid, a small fall in price will cause only a very small increase in his purchases. In the former case his willingness to purchase the thing stretches itself out a great deal under the action of a small inducement: the elasticity of his wants, we may say, is great. In the latter case the extra inducement given by the fall in price causes hardly any extension of his desire to purchase: the elasticity of his demand is small. If a fall in price from say 16*d.* to 15*d.* per lb. of tea would much increase his purchases, then a rise in price from 15*d.* to 16*d.* would much diminish them. That is, when the demand is elastic for a fall in price, it is elastic also for a rise.

And as with the demand of one person so with that of a whole market. And we may say generally:—The *elasticity* (or *responsiveness*) *of demand* in a market is great or small according as the amount demanded increases much or little for a given fall in price, and diminishes much or little for a given rise in price.[1]

[1] We may say that the elasticity of demand is one, if a small fall in price will cause an equal proportionate increase in the amount demanded; or as we may say roughly, if a fall of one per cent. in price will increase the sales by one per cent.: that it is two or a half, if a fall of one per cent. in price makes an increase of two or one half per cent. respectively in the amount demanded; and so on. (This statement is rough; because 98 does not bear exactly the same proportion to 100 that 100 does to 102.) The elasticity of demand can be best traced in the demand curve with the aid of the following rule. Let a straight line touching the curve at any point *P* meet *Ox* in *T* and *Oy* in *t*, then *the measure of elasticity at the point P is the ratio of PT to Pt.*

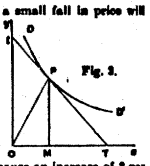


Fig. 2.

If *PT* were twice *Pt*, a fall of 1 per cent. in price would cause an increase of 2 per cent., in the amount demanded; the elasticity of demand would be two. If *PT* were one-third of *Pt*, a fall of 1 per cent. in price would cause an increase of ⅓ per cent. in the amount demanded; the elasticity of demand would be one-third; and so on. Another way of looking at the same result is this:—the elasticity at the point *P* is measured by the ratio of *PT* to *Pt*, that is of *MT* to *MO* (*PM* being drawn perpen-

Figure 1. *A page of text from Alfred Marshall,* Principles of Economics, *8th ed. (London: Macmillan, 1920). Note the relegation of mathematics to the footnotes.*

On many occasions Marshall encouraged his students to study mathematics. In his words, "a training in mathematics is helpful by giving command over a marvellously terse and exact language for expressing clearly some general relations and some short processes of economic reasoning; which can indeed be expressed in ordinary language, but not with equal sharpness of outline."[37] In an inaugural address to the Cambridge Economic Club (founded in 1897), Marshall acknowledged the powers of mathematics and told his audience to carry on: "This work of the new methods is far from finished: much remains for your generation to do." He also instructed his students to carry out translations into mathematics, much as he had done in his own early years, and attached considerable importance to his diagrammatic exercises in the classroom, as Keynes emphasized. In 1902 he noted that all of his most-valued students were "mathematical casuals," that is, candidates for the mathematical tripos at Cambridge.[38] Certainly his most prominent pupils—Keynes, Arthur C. Pigou, and Arthur Bowley—were mathematically astute.

All the same, one must take into account Marshall's frequent caveats on the

[37] Marshall, *Principles* (cit. n. 19), pp. xiv–xv, 644.

[38] Quoting from Maloney, *Marshall* (cit. n. 13), p. 59; on the value of translations see A. W. Coats, "Alfred Marshall and the Early Development of the London School of Economics: Some Unpublished Letters," *Economica,* 1967, *36*:408–417, on pp. 410–411. On the use of diagrams see Keynes, *Essays in Biography* (cit. n. 13), pp. 190, 252. For the Cambridge address see Alfred Marshall, "The Old Generation of Economists and the New," in Pigou, *Memorials* (cit. n. 14), p. 301.

misuse or hazards of mathematics, particularly in his later correspondence. According to Maynard Keynes, Marshall "arrived very early at the point of view that the barebones of economic theory are not worth much in themselves."[39] There is reason to believe that he delayed publishing his major treatise (apart, of course, from long bouts of poor health) because it took a substantial amount of research to find the "flesh" for his "theoretical skeleton." It was only after having spent several years in search of the empirical foundations of his theoretical model that Marshall began to realize that economics is seldom as straightforward and simple as a mathematical treatment might lead one to believe. He then, in an oft-cited letter of 1906, expressed certain fears about the potential abuses of applied mathematics. To Arthur Bowley he wrote that it would be better to "burn the mathematics" than to allow economic theory to stray away from real-life examples.[40]

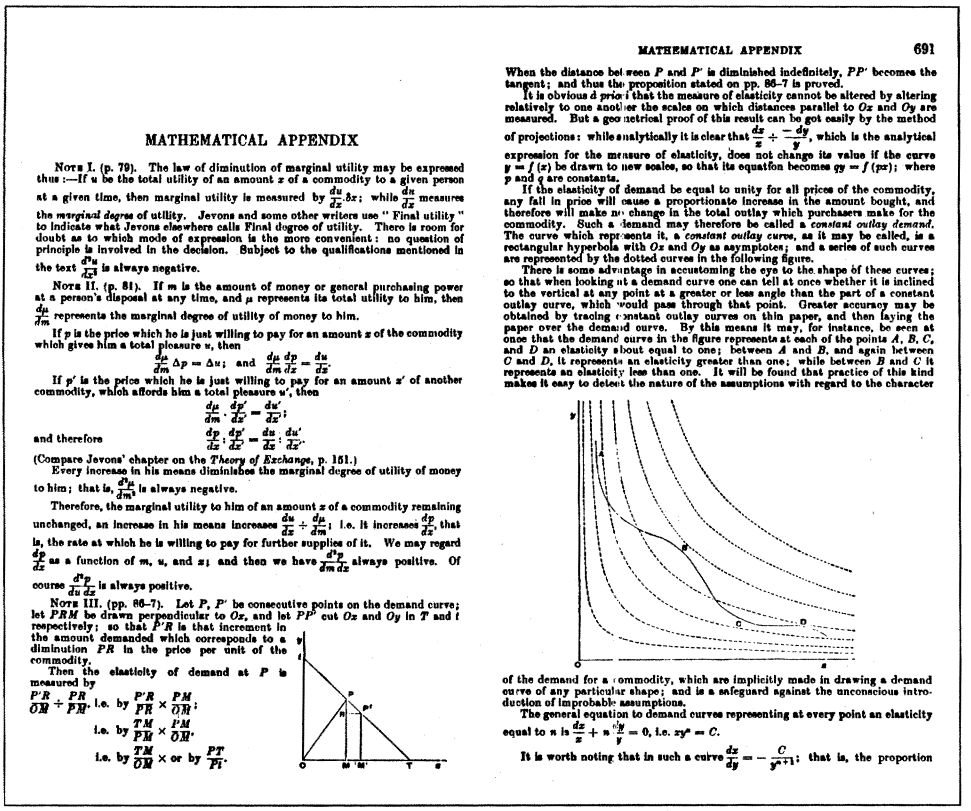
Considerable weight has been placed on these remarks by subsequent economists and historians, leading many to discount Marshall's role in the rise of mathematical economics. John Maloney, in his otherwise excellent study of Marshall and the Cambridge school, draws this conclusion: "From Marshall's comments on 200-odd pupils, recorded between 1895 and 1902, it is clear that he regarded the mathematicians so highly not because he had any great opinion of mathematical economics as such—his correspondence with Bowley makes it clear that he did not—but because he saw their analytical sophistication as symptomatic of a generalised intelligence." But rarely is it mentioned that Marshall prefaced his remarks to Bowley with the following qualification: "I have very indistinct memories of what I used to think on the subject [of mathematical economics]. I never read mathematics now: in fact I have forgotten even how to integrate a good many things." In almost every other instance where Marshall criticizes mathematics, one finds remarks restoring its value within certain guidelines. As Pigou, his successor at Cambridge, once observed: "So far as [Marshall] was against mathematical elaborations in economics, it was only because he feared that realism might suffer. Convince him that any particular line of mathematical attack would indirectly help realism, and he would have been its enthusiastic friend."[41]

If Marshall was primarily bent on empirical veracity, his misgivings came too late. When he built the Cambridge school in the last few decades of the nineteenth century, we know from various sources that he was, as Foxwell put it, "thoroughly mathematical." However much Marshall might have tried in his

[39] Keynes, *Essays in Biography,* p. 170. This view may also explain his preference for diagrams, since he and others realized the possibility, in the not-too-distant future, of statistics, particularly the method of least squares, lending empirical credibility to the theoretical analysis of markets. On the development of econometrics see Mary S. Morgan, *The History of Econometric Ideas* (Cambridge: Cambridge Univ. Press, forthcoming).

[40] Marshall to A. Bowley, 27 Feb. 1906, in Pigou, *Memorials* (cit. n. 14), p. 427. The passage in full is as follows: "But I know I had a growing feeling in the later years of my work at the subject that a good mathematical theorem dealing with economic hypotheses was very unlikely to be good economics: and I went more and more on the rules—(1) Use mathematics as a shorthand language, rather than as an engine of inquiry. (2) Keep to them till you have done. (3) Translate into English. (4) Then illustrate by examples that are important in real life. (5) Burn the mathematics. (6) If you can't succeed in 4, burn 3. This last I did often."

[41] Maloney, *Marshall* (cit. n. 13), pp. 233–234; Marshall to Bowley; and Pigou, as quoted in Vincent Bladen, *From Adam Smith to Maynard Keynes* (Toronto: Univ. Toronto Press, 1974), p. 361. Jacob Viner maintained that Marshall's distrust of mathematical economics "was probably a lurking puritan suspicion of the morality of any highly pleasurable activity" and that he must have delighted in mathematical manipulations: Viner, "Marshall's Economics in Relation to the Man and His Times," *American Economic Review,* 1941, *31*:223–235, on p. 231.



MATHEMATICAL APPENDIX

Note I. (p. 79). The law of diminution of marginal utility may be expressed thus:—If u be the total utility of an amount x of a commodity to a given person at a given time, then marginal utility is measured by $\frac{du}{dx}\delta x$; while $\frac{du}{dx}$ measures the *marginal degree* of utility. Jevons and some other writers use "Final utility" to indicate what Jevons elsewhere calls Final degree of utility. There is room for doubt as to which mode of expression is the more convenient: no question of principle is involved in the decision. Subject to the qualifications mentioned in the text $\frac{d^2u}{dx^2}$ is always negative.

Note II. (p. 81). If m is the amount of money or general purchasing power at a person's disposal at any time, and μ represents its total utility to him, then $\frac{d\mu}{dm}$ represents the marginal degree of utility of money to him.

If p is the price which he is just willing to pay for an amount x of the commodity which gives him a total pleasure u, then

$$\frac{d\mu}{dm}\Delta p = \Delta u; \quad \text{and} \quad \frac{d\mu}{dm}\frac{dp}{dx} = \frac{du}{dx}.$$

If p' is the price which he is just willing to pay for an amount x' of another commodity, which affords him a total pleasure u', then

$$\frac{d\mu}{dm}\cdot\frac{dp'}{dx'} = \frac{du'}{dx'};$$

and therefore

$$\frac{dp}{dx}:\frac{dp'}{dx'} = \frac{du}{dx}:\frac{du'}{dx'}.$$

(Compare Jevons' chapter on the *Theory of Exchange*, p. 151.)

Every increase in his means diminishes the marginal degree of utility of money to him; that is, $\frac{d^2\mu}{dm^2}$ is always negative.

Therefore, the marginal utility to him of an amount x of a commodity remaining unchanged, an increase in his means increases $\frac{du}{dx} \div \frac{d\mu}{dm}$; i.e. it increases $\frac{dp}{dx}$, that is, the rate at which he is willing to pay for further supplies of it. We may regard $\frac{dp}{dx}$ as a function of m, u, and x; and then we have $\frac{d^2p}{dm\,dx}$ always positive. Of course $\frac{d^2p}{du\,dx}$ is always positive.

Note III. (pp. 86–7). Let P, P' be consecutive points on the demand curve; let PRM be drawn perpendicular to Ox, and let PP' cut Ox and Oy in T and t respectively; so that $P'R$ is that increment in the amount demanded which corresponds to a diminution PR in the price per unit of the commodity.

Then the elasticity of demand at P is measured by

$$\frac{P'R}{OM} \div \frac{PR}{PM}, \text{ i.e. by } \frac{P'R}{PR} \times \frac{PM}{OM};$$

$$\text{i.e. by } \frac{TM}{PM} \times \frac{PM}{OM};$$

$$\text{i.e. by } \frac{TM}{OM} \times \text{ or by } \frac{PT}{Pt}.$$

MATHEMATICAL APPENDIX 691

When the distance between P and P' is diminished indefinitely, PP' becomes the tangent; and thus the proposition stated on pp. 86–7 is proved.

It is obvious *à priori* that the measure of elasticity cannot be altered by altering relatively to one another the scales on which distances parallel to Ox and Oy are measured. But a geometrical proof of this result can be got easily by the method of projections: while analytically it is clear that $\frac{dx}{x} \div \frac{-dy}{y}$, which is the analytical expression for the measure of elasticity, does not change its value if the curve $y = f(x)$ be drawn to new scales, so that its equation becomes $qy = f(px)$; where p and q are constants.

If the elasticity of demand be equal to unity for all prices of the commodity, any fall in price will cause a proportionate increase in the amount bought, and therefore will make no change in the total outlay which purchasers make for the commodity. Such a demand may therefore be called a *constant outlay demand*. The curve which represents it, a *constant outlay curve*, as it may be called, is a rectangular hyperbola with Ox and Oy as asymptotes; and a series of such curves are represented by the dotted curves in the following figure.

There is some advantage in accustoming the eye to the shape of these curves; so that when looking at a demand curve one can tell at once whether it is inclined to the vertical at any point at a greater or less angle than the part of a constant outlay curve, which would pass through that point. Greater accuracy may be obtained by tracing constant outlay curves on thin paper, and then laying the paper over the demand curve. By this means it may, for instance, be seen at once that the demand curve in the figure represents at each of the points A, B, C, and D an elasticity about equal to one; between A and B, and again between C and D, it represents an elasticity greater than one; while between B and C it represents an elasticity less than one. It will be found that practice of this kind makes it easy to detect the nature of the assumptions with regard to the character

of the demand for a commodity, which are implicitly made in drawing a demand curve of any particular shape; and is a safeguard against the unconscious introduction of improbable assumptions.

The general equation to demand curves representing at every point an elasticity equal to n is $\frac{dx}{x} + n\frac{dy}{y} = 0$, i.e. $xy^n = C$.

It is worth noting that in such a curve $\frac{dx}{dy} = -\frac{C}{y^{n+1}}$; that is, the proportion

Figure 2. *Two pages from the Mathematical Appendix to Marshall,* Principles of Economics, *with mathematical formulas and "inscrutable curves."*

later years to warn against the excessive use of mathematics, it appears to have been of little or no avail. His own analyses of such concepts as consumer surplus and the price elasticity of demand demonstrate all too clearly his allegiance to the view that mathematics, as he once declared, is "the chief means of scientific progress."[42]

What can we conclude from this? Evidently, Marshall did much to promote mathematical economics in the period 1870–1900. Whether or not he acknowledged a debt to Jevons, or even had contracted one at the substantive level, Marshall's lectures and writings reinforced the radical measures set out by Jevons. This is noted in several firsthand accounts, such as Foxwell's 1888 address, "The Economics Movement in England," or Irving Fisher's appraisal of the field in 1891, a fitting quotation on which to end: "The mathematical method really began with Jevons in 1871. . . . In England, Prof. Edgeworth, noted for his enthusiasm on mathematical economics, has recently been elected to the chair of pol. econ. at Oxford, while Prof. Marshall is carrying forward the same movement at Cambridge. . . . From all apparent evidence the mathematical method has come to stay."[43]

[42] Foxwell to Jevons, 12 Nov. 1879, in Jevons, *Correspondence* (cit. n. 15); and Marshall, *Principles* (cit. n. 19), p. 700.

[43] Irving Fisher, *Mathematical Investigations in the Theory of Value and Price* (New Haven, Conn.: Yale Univ. Press, 1925; rpt. New York: Augustus M. Kelly, 1961), pp. 109–110.