Labour Allocation in a Cooperative Enterprise

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I. INTRODUCTION

Two methods of income distribution have been particularly associated with socialist thinking, " to each according to his needs " and " to each according to his work ". In the literature on socialism it is the latter system that has been mostly studied. Marx felt the former system to be appropriate only at the " higher phase of the communist society," and emphasized the principle of distribution " proportional to the amount of labour they contribute ", in the " first phase " of the communist society.[2] While discussions in the Marxist literature have concentrated mainly on distribution according to work, in particular on the utilisation of the wage system, actual methods of payments in socialist economies have often departed from this rule, the most notable example of this being in Chinese agriculture.

On the other plane of discussion, in the theoretical literature on resource allocation with decentralized planning, the emphasis has been on reaching Pareto-optimality, and that, with the usual assumptions, has been found to fit in well with a wage system.[3] There have of course been discussions on correcting the distribution according to work towards the goal of distribution according to needs through a set of taxes and subsidies, but the basic method of payment that has been considered has always been some variant or other of the wage system.

The actual organization of enterprises in communist countries tend to depart from a pure wage system in at least two different ways: (i) in the use of some variant of profit-sharing over and above a wage system,[4] and (ii) in having a part of the income distributed on some criteria other than that of work, e.g. some interpretation of " needs ".[5] There is not yet a distinct body of literature on the *theory* of non-wage allocation of labour; nevertheless, in the context of policy debates, the following questions have repeatedly cropped up.

(1) What are the difficulties in having a system of distribution purely according to needs? While in the U.S.S.R. and in Eastern Europe there has not been any large-scale attempt to have payments primarily according to needs, the Chinese leaders have tried to break through the problem of incentives involved in this, and it has even been claimed that " ideally, the party leaders would like non-material incentives to become the main motive force impelling the masses on to greater output ".[6]

(2) What difficulties are there in having a system of distribution purely according to work, even profits being shared on that basis? Based on Yugoslav experience Ward [21]

1 I am indebted to Peter Diamond for his helpful comments. This is a revised version of Working Paper No. 67 of the Committee on Econometrics and Mathematical Economics of the Institute of Business and Economic Research at the University of California at Berkeley, April 1965.

2 Marx [14], pp. 29-31; see also Sweezy [20], pp. 10-11.

3 See Lange and Taylor [10], Lerner [11] and Koopmans [8]. See also Robinson [16], Chapter II.

4 See Ward [21]; the discussion is based on the experience of Yugoslavia. The " enterprise funds " in the U.S.S.R. also imply some profit-sharing (see Bergson [2], p. 109). See also Wiles [22], Chapters 1 and 2.

5 Attempted mainly in China, particularly in agriculture; see Li [13], Hoffman [6], Nove [15]. The rejection of the wage system in China may have a lot to do with the well-known problems of utilization of surplus (or near-surplus) labour arising from the rigidity of the wage rate. The efficiency problems arising from a positive wage rate in an economy with surplus labour are discussed in Sen [18].

6 Hoffman [6], p. 110.

has discussed some problems of allocation that arise in this context when the cooperatives are allowed to vary the number of members, and they use it to maximize returns *per person.* The problem does not disappear even when the number of members cannot be varied by the cooperative for this purpose, for there is also the question of the amount of work done. Ward takes the amount of work done per person as given; we shall however treat this as a variable, and study the problem of incentives in that context.

(3) If a mixed system is attempted, what are the " proper " shares of the two methods of distribution? In some respects, all systems actually used in these countries are mixed ones; even in the U.S.S.R. the " social insurance " is basically a method of payments according to needs, and it is " usually considered to add something of the order of the magnitude of one-third to money earnings ".[1] In the Chinese attempt at communization of agriculture, heavy emphasis was put on the so-called " supply portion " of income, distributed on some criteria of needs, and in some cases the proportion of this reached " as high as 80-90 per cent with the slim remainder being distributed as money wages ". But by 1960 payments according to work gained predominance again, when the " ideal " ratio of supplies to wages was put at 30 per cent.[2]

(4) To what extent are these problems dependent on the attitude of the members of the cooperative to each other? It is natural to expect that the results of cooperative efforts will depend crucially on how much concern people have for each other.

In this note we shall go into this collection of questions in terms of a highly simplified model. A cooperative is considered, consisting of N families, identical in every respect.[3] The cooperative uses its own homogeneous labour and its own land, and hires outside factors from a set of perfectly competitive markets. We shall confine our attention to the efficiency problems that arise *within* a cooperative enterprise, and not be concerned with allocational problems *among* them. Problems of taxation are not considered. Nor do we go into saving decisions in this paper, which we assume are left to individual families, and we relate utility to family income and not to family consumption. If the basic framework is found relevant, then the analysis can be easily extended in a variety of directions.

II. INDIVIDUAL AND SOCIAL WELFARE

There are N identical families that make up the cooperative. Member families of the cooperative like more and more income (y^i) and dislike more and more work (l^i), and each family has a given and identical utility function, U. Marginal utility from income is positive and diminishing, and marginal utility from work is negative and diminishing (i.e., marginal disutility from work is positive and increasing), and they are independent of each other.

$$U^i = U(y^i, l^i), \text{ with } U_y>0,\ U_l<0,\ U_{yy}<0,\ U_{ll}<0,\ U_{yl} = U_{ly} = 0. \qquad \text{...(1)}$$

However, the families are not *necessarily* indifferent to the happiness of other families (though they might also be that), and their notion of " social welfare " takes into account the utility of other families.[4] Individual j attaches a weight a_{ij} to a unit of the utility of individual i in aggregating the social welfare,

$$W^j = \sum_{i=1}^{N} a_{ij} . U^i. \qquad \text{...(2)}$$

The utility of his own family can serve as the unit of account ($a_{jj} = 1$), and it is assumed that he attaches a weight somewhere between 0 and 1 to a unit of the utility of other

[1] Dobb [3], p. 448.
[2] Hoffman [6], pp. 104-105.
[3] Since the families share the same needs and have the same productive abilities, we shall find that distribution according to needs as well as that according to work both tend to produce an equal distribution of income, but the *level* of income varies, and so does welfare.
[4] For the underlying concepts, see Harsanyi [4].

families ($0 \leqq a_{ij} \leqq 1$). This means that while he may like other people to be happy, he does not attach greater weight to the happiness of other families than he does to his own.[1] Equation (2) can now be rewritten as:

$$W^j = U^j + \sum_{\substack{i=1 \\ i \neq j}}^{N} a_{ij} \, . \, U^i, \text{ with } 0 \leqq a_{ij} \leqq 1. \qquad \ldots(2.1)$$

While (2) or (2.1) represents the welfare of the cooperative as viewed by individual j, we define the " social welfare " (W) to be simply an aggregate of individual utilities. We assume that this is the Management's notion also, which is assumed to be non-discriminating.

$$W = \sum_{i=1}^{N} U^i. \qquad \ldots(3)$$

The set of a_{ij} for any individual j defines, quite precisely, his attitude to the welfare of other families, and we shall find it convenient to extract from it an aggregate measure of his " sympathy " for other families, which we shall call, in keeping with our subject matter, his " social consciousness " (S^j).

$$S^j = \frac{1}{N} \sum_{i=1}^{N} a_{ij}. \qquad \ldots(4)$$

In view of (2.1), S^j resides somewhere in the closed range $\left[\frac{1}{N}, 1\right]$ The more he values other families' happiness vis-a-vis his own, the closer is the value of S^j to 1.

While S^j measures the sympathy that family j has for the other families, we can define a magnitude T^i that will measure the sympathy that family i *receives* from the other families. This we call the " social goodwill " of family i.

$$T^i = \frac{1}{N} \sum_{j=1}^{N} a_{ij}. \qquad \ldots(5)$$

Once again, in view of (2.1), T^i resides somewhere in the closed range $\left[\frac{1}{N}, 1\right]$, and the more " goodwill " that this family has, the closer will be the value of T^i to 1.

We now introduce two crucial assumptions, one of which will be used immediately, and the other later. The assumption of *symmetric sympathy* is that all families have the same measure of " social consciousness ".

$$S^j = S, \text{ for all } j. \qquad \ldots(4.1)$$

The assumption of *symmetric goodwill* is that all families have the same measure of " social goodwill ".

$$T^i = T, \text{ for all } i. \qquad \ldots(5.1)$$

Note that neither assumption requires symmetry in the exact *distribution* of the values of a_{ij}. Both the notions of symmetry are *aggregate* ones.

III. CENTRALIZED ALLOCATION

The cooperative owns a given amount of land (A) and hires m outside factors (F^k, with $k = 1, 2, \ldots, m$) at constant prices (P^k, with $k = 1, 2, \ldots, m$). Labour (L) is provided by members of the cooperative. The production function Q is given by (6).

$$Q = Q(L, A, F^1, F^2, \ldots, F^m). \qquad \ldots(6)$$

Q is assumed to be a " well-behaved " production function, with the usual nice properties, including twice differentiability throughout, and positive but diminishing marginal product of each factor.

[1] Of course he can attach a greater weight to a unit of another family's *income* than to his own, if there is significant inequality.

Given (1), it is easy to check that welfare maximization requires equal division of total income (V) and of total work (L). Hence, *under centralized allocation*, we should have:

$$y^i = \frac{V}{N} \equiv y, \quad \text{for all } i \qquad ...(7)$$

$$l^i = \frac{L}{N} \equiv l, \quad \text{for all } i. \qquad ...(8)$$

Given (7) and (8), the social welfare function (3) simplifies into:

$$W = N \,.\, U(y, l). \qquad ...(3.1)$$

The income generated in the cooperative (V) is given by the difference between the value of the output (Q) and the purchase costs of the m inputs. We take the factor prices (P^k) to be expressed in units of the output.

$$V = Q - \sum_{k=1}^{m} F^k . P^k. \qquad ...(9)$$

Given (1), (3.1), (6), (7), (8) and (9), maximization of social welfare (W) under centralized allocation requires: [1]

$$Q_k = P^k, \quad \text{for } k = 1, 2, ..., m \qquad ...(10)$$

$$Q_L = -\frac{U_l(y, l)}{U_y(y, l)} \equiv R. \qquad ...(11)$$

Here R is defined as the individual marginal rate of indifferent substitution between income and non-labour (leisure). The rate is the same for all individuals. The interpretation of rules (10) and (11) is obvious.

Note that rules (7), (8), (10) and (11), which describe the allocation decisions, yield not only Paretian optimality (the usual competitive result), but also maximization of total social welfare. This completing of the incomplete Paretian ordering has been possible by assuming a social welfare function with additive cardinal utility.[2]

IV. VOLUNTARY ALLOCATION OF LABOUR

One feature of the allocational rules of the last section is that the individual members of the cooperative are not allowed to determine how much work they would like to put in; it is all decided for them by the Management. We now relax that assumption, and examine the voluntary allocational results given the system of rewards. It is assumed that a proportion α of income is distributed according to " needs ", and the rest $(1-\alpha)$ according to " work ". The value of α lies in the closed interval between 0 and 1. Needs are equal, and thus α proportion of the income is equally distributed; the rest is distributed in a way such that family i gets $\left(\frac{l^i}{L}\right)$ proportion of it. We have then:

$$y^i = V\left(\frac{\alpha}{N} + (1-\alpha)\left(\frac{l^i}{L}\right)\right). \qquad ...(7.1)$$

Individual j maximizes W^j for variations of his own labour l^j, given the amount of labour performed by others,[3] and given the use of other factors of production. He is thus

[1] These are the first order conditions. The second order conditions are given by the usual restrictions on the " bordered " determinants (see Hicks [5], p. 320). Q_k refers to the partial derivative of Q with respect to F^k, and Q_L that with respect to L.

[2] We discuss this question further in section VIII.

[3] We are abstracting from behaviour based on " game " considerations. When the number of individuals (N) is small, this may be an important limitation.

guided by (1), (2.1), (6), (7.1), and (9), and his optimal allocation can be seen to require the following condition:

$$-U_l^j = \sum_{i=1}^{N} a_{ij} \cdot U_y^i \left[Q_L \left(\frac{\alpha}{N} + (1-\alpha)\left(\frac{l^i}{L}\right)\right) - \left(\frac{V}{L}\right)\left(\frac{l^i}{L}\right)(1-\alpha) \right] + U_y^j \cdot \left(\frac{V}{L}\right)(1-\alpha). \qquad \ldots(12)$$

Given (1) and (4.1), we shall find at equilibrium, the same amount of labour offered by each family. As a result, the income of each family will also tend to be equal, as is seen from (7.1). Thus conditions (7) and (8) will also hold at equilibrium. There is no inconsistency between this and the assumption of labour allocation underlying (12). Each family decides on how much labour to apply given the amount of labour of other families; but since their calculations are identical they end up by offering the same amount.

Since (7) and (8) hold, and since everyone has the same utility function U, the marginal utilities of income and the marginal disutilities of work that enter in (12) are respectively identical for all families. It is easy to check that under these circumstances (12) simplifies into the following:

$$R = R^j = Q_L \left[S + (1-S)(1-\alpha)\left(\frac{\beta}{\eta}\right)\right], \qquad \ldots(13)$$

where $\beta = \frac{V}{Q}$, the ratio of income to total output, $\eta = \frac{Q_L \cdot L}{Q}$, elasticity of output with respect to labour. S and R have been defined before as the measure of " social consciousness " (see (4)) and the relevant individual marginal rate of indifferent substitution between income and leisure (see (11)). The voluntary allocational result (13) has to be compared and contrasted with the optimal rule (11).

V. PURE SYSTEMS AND OPTIMALITY

If the system that is followed is one purely according to needs, then we have $\alpha = 1$. Under those circumstances, result (13) becomes:

$$R = Q_L \cdot S. \qquad \ldots(13.1)$$

This corresponds to the optimum rule (11) only when $S = 1$, i.e. only when sympathy for other families is perfect, with each family attaching equal weight to the happiness of every family in the community.[1] Barring this special case, the allocation of labour will not be carried to the point required for optimality, but stopped before that, since $S < 1$, and therefore $R < Q_L$.

If, on the other hand, the system of distribution is one purely according to work, we have $\alpha = 0$. Then, result (13) becomes:

$$R = Q_L \left[S + (1-S)\left(\frac{\beta}{\eta}\right)\right]. \qquad \ldots(13.2)$$

This can coincide with the optimality requirement (11) if *either* there is complete sympathy for all, *i.e.* $S = 1$, *or* if $\beta = \eta$. The latter condition can be analysed a little further. It equates the relative share of the income of the cooperative in gross output to the elasticity of output with respect to labour. If we assume that the production function is homogeneous of the first degree,[2] and also that the cooperative does not own any factor (like land) other than labour, then this condition of β being equal to η will indeed be fulfilled.

[1] Cf. Marx's position that payments according to needs will not be the right system in the " first phase " of socialism when the society is " in every respect tainted economically, morally, intellectually with the hereditary diseases of the old society from whose womb it is emerging " ([14], p. 29), and that it could work well only when " all the springs of cooperative wealth are gushing more freely together with the all-round development of the individual " ([14], p. 31).

[2] More generally, in an equilibrium with no " abnormal " profits, as under perfect competition, this result will hold. Constant returns to scale is not needed throughout; only at the point of equilibrium.

The proof is obvious from Euler's Theorem and the fulfillment of the allocational rules for other factors given by (10).

If, however, sympathy or " social consciousness " is not perfect, i.e. $S<1$, and the cooperative owns some factors other than labour (in our assumption land), it is easy to show that with a production function homogeneous of the first degree, we have $\beta>\eta$. And it follows from (13.2) that $R>Q_L$, i.e. labour will be applied *beyond* the point required for optimization. Thus barring the special cases of complete sympathy for all, or of a cooperative devoid of non-labour resources, a pure system of allocation according to work will tend to make the amount of labour offered to be greater than the optimum.

One comment on the distribution system according to work may be worth making here. We have assumed that the part that is distributed according to work is given out directly in terms of the proportion of labour contributed by each family. If instead a part is distributed as straightforward wages, and if the surplus after paying for wages and for the part of the income that is distributed according to needs (still α proportion of total income of the members) is again split up in proportion to the amount of labour contributed, the result will be the same as in our distributional equation (7.1), and all consequences will hold.

$$y^i = \frac{V.\alpha}{N} + l^i.w + [V(1-\alpha) - L.w]\left(\frac{l^i}{L}\right)$$
$$= V\left[\frac{\alpha}{N} + (1-\alpha)\left(\frac{l^i}{L}\right)\right].$$

In particular, when nothing is distributed according to needs ($\alpha = 0$), the same tendency to over-contribute labour will follow, except in the special case of complete sympathy ($S = 1$), or no owned non-labour resource ($\beta = \eta$). This is a basic problem of syndicalism, in fact of any economic system with profit-sharing by workers according to work.

Of the two pure systems, the result that there is *too little* work done in a system of distribution according to *needs* is easier to see intuitively. The result of *too much* work done in a system of distribution according to *work* can be explained in the following heuristic terms. When an individual contributes an additional unit of labour, he gets two compensations for his troubles: first, the income of the cooperative goes up, and he gets a share of the marginal product, though not the whole of it; second, he gets an enlarged share of the *total* income because his share in the total labour contributed is larger. The former, on its own, is insufficient to make him offer the optimum amount of labour, since he gets only a part of his marginal product, but the latter over-compensates for it, as long as the average income per unit of labour is greater than the marginal product of labour. Hence the over-allocation of work. When, however, the cooperative possesses no factor other than labour, and the production function has constant returns to scale, the average income per unit of labour just equals the marginal product, and the two effects exactly balance out. Similarly, when he has complete sympathy for all other families, he does not mind other people getting a share of his marginal product, nor does he see any gain in getting a larger income at the cost of others; and once again the allocation is right. When, however, the cooperative possesses non-labour factors and the sympathy is not quite complete, there is a tendency towards over-contribution of work, arising from a desire to get a higher share of the existing income exploiting the system of distribution according to work.

VI. THE OPTIMAL RULE

In order to achieve welfare maximization, result (13) has to coincide with the requirement (11). This coincidence holds if and only if:

$$S+(1-S)(1-\alpha)\left(\frac{\beta}{\eta}\right) = 1. \qquad \ldots(14)$$

This is always satisfied when:

$$S = 1. \qquad \text{...(14a)}$$

If (14*a*) does not hold, and " social consciousness " is not complete, we require then:

$$(1-\alpha) = \frac{\eta}{\beta}. \qquad \text{...(14b)}$$

This means that the proportion of income to be distributed according to work should equal the ratio of the elasticity of output with respect to labour to the share of cooperative income in total output.

It should be noted that rule (14*b*) has a close affinity to the competitive rule. If the production function is homogeneous of the first degree, and if the allocational rule (9) for hired factors (F^k) is followed, then the share of the cooperative's income in total output, i.e., the value of β, will simply equal the sum of the elasticities of output with respect to all factors of production supplied by the cooperative. Also, under those circumstances, labour's share of output (Q) will be given by η and labour's share in the cooperative income (V) will be given by $\left(\frac{\eta}{\beta}\right)$. And it is this portion that is to be distributed according to work if rule (14*b*) is followed. And the proportion of the cooperative income that is to be distributed according to needs should equal the rest, i.e. what would have been the competitive share of non-labour productive factors (land, in our example) owned by the cooperative.

However, the similarity with the competitive case, while striking, should not be over-stressed. First of all, the correspondence is not complete when we take a production function with diminishing returns to scale. Then it is not clear what the competitive distribution would have been. It is, however, clear from observing (14b) that the right proportion to be distributed according to work is what would have been the share of labour *if* it got its competitive share while the owned non-labour resources obtained the surplus that remained after paying all hired factors and labour their respective shares equal to their elasticities. The result quoted earlier that the right proportion to be distributed according to needs is what would have been the competitive share of non-labour factors of production (land) owned by the cooperative, does not any longer hold.

Second, while rule (14*b*) corresponds to the competitive solution, the method of distribution need not involve any wage system at all, and might therefore be free from sociological constraints that apply to boundary values of the wage rate as such. For example, the well-known allocational problems raised by the existence of a minimum level of the wage rate in preventing proper utilisation of surplus or near-surplus labour in the underdeveloped countries,[1] might not necessarily apply to this case where no explicit use of a wage rate need be made. That is, if the constraint applies not the minimum marginal return to labour (irrespective of the form of it) but to the minimum *wage rate* as such,[2] then rule (14*b*) need not be interfered with by such a constraint while the competitive solution might be.

Third, rule (14*b*) is not strictly necessary for optimality and condition (14*a*) is quite sufficient. That is if people do have complete social consciousness ($S = 1$), any choice of α will do just as well.[3] This introduction of external concern makes our model different from that of the usual competitive models. However, for any value of $S<1$, rule (14*b*) is necessary for optimality in the voluntary system.

1 See Sen [18], Chapters II, V, and Appendix A.

2 This will be the case when there is a conventional minimum level of the wage rate which does not come in when the method of rewards is altogether different from the wage system.

3 This might explain why the Chinese attempt at having a system of payment not closely related to work in agriculture in the " Great Leap Forward " period was accompanied by attacks on the " family-centered psychology " (Nove [15], p. 22; Hoffman [6], p. 100).

VII. THE ROLE OF EXTERNAL CONCERN

It is interesting to note that the optimal rule (14*b*) is completely independent of the extent of external concern that people have for each other. The coincidence of (13) with the optimality requirement (11) when rule (14*b*) is followed, is independent of the value of *S*.

One reason why this appears surprising is the practice, in " new welfare economics ", of ruling out *all* types of external effects while deriving propositions about the optimality of competitive equilibrium.[1] But it can be checked that some types of external concern do not matter at all as far as the optimum allocation is concerned. At the competitive equilibrium an individual is paid at a rate just equal to the productivity of his last unit of labour, so that the real income enjoyed by the others is not affected by this allocational decision, and how much weight he wants to attach to other people's happiness does not make any difference to his choice. Similarly, since he stops at the point where his marginal net gain from the last unit of labour is nil, other people also do not care whether he applies this unit of labour or not, even though they may attach value to his utility.

It is to be noted that the external effects allowed here are of a kind different from the usual utilitarian presentation. In the utility functions of each family we have introduced only the labour and income of that family, but we have assumed that members try to maximize not this utility function of their family but a weighted sum of the utilities of all families, representing their notions of welfare. There is, however, a well-established tradition in economics of taking as people's " utility " whatever it is that they try to maximize, so that family utility in this model might be taken to be equivalent not to our set of (U^i) but to the set of (W^j). The consequences of taking social welfare to be the sum of (W^j) rather than of (U^i) may be pursued.

$$\overline{W} = \sum_{j=1}^{N} W^j = \sum_{j=1}^{N} \sum_{i=1}^{N} a_{ij} \,.\, U^i = N \,.\, \sum_{i=1}^{N} T^i \,.\, U^i. \qquad \ldots(3.2)$$

At this stage, the assumption of symmetric goodwill (5.1) is helpful. With that assumption, we have:

$$\overline{W} = N.T.W. \qquad \ldots(15)$$

Since N and T are given, maximization of W is equivalent to that of $\overline{W}$.[2] Aggregate social welfare is still maximized by following rule (14*b*), even though the identification of the set of individual utilities as the set (W^j) rather than the set (U^i), makes it a straightforward case of " direct (i.e. non-market) interdependence," where " the individual person's satisfaction . . . depends not only on the quantities of product he consumes and services he renders but also on the satisfaction of other persons." [3] The optimality of the rule seems to be completely independent of the exact size of such external concern.

One extreme case of *symmetric goodwill* is the case when people are completely egocentric (family-centric) and other people's (families') satisfactions simply do not enter into the individual utility functions, which is the favourite *neo-classical* assumption, which corresponds here to $T = \frac{1}{N}$. The other extreme case is that of full " social consciousness," which Marx expected in, and only in, the " higher phase " of socialism,[4] and which corresponds here to the case of $T = 1$. The value of T can lie *anywhere* in

1 " The new welfare economists, despite their name, actually said little that was new. They accepted the usual simplifying assumptions of Pareto and Barone: to wit, the independence of different people's satisfactions and the absence of external economies and diseconomies." (Scitovsky, in " The State of Welfare Economics ", in [17], p. 79.)

2 Even the condition of " symmetric goodwill " can be relaxed when the object is to achieve only Pareto optimality through the competitive mechanism, as can be seen from the argument outlined in the previous paragraph in the text.

3 Scitovsky, " Two Concepts of External Economies," in [17], pp. 70-71.

4 Marx [14], pp. 29-31.

the closed interval $\left[\frac{1}{N}, 1\right]$, and irrespective of where it lies, rule (14*b*) yields the maximization of the aggregate welfare of the cooperative. However, in the extreme case of $T = 1$, we also have $S = 1$, i.e. condition (14*a*) holds, so that the policy implied by rule (14*b*), while still optimal, is redundant, and any proportion of total income (even all) can be distributed directly according to needs.

VIII. CONCLUDING REMARKS

We have examined the problem of labour allocation in a cooperative system both in terms of centralized decisions as well as in terms of voluntary allocation. The optimal rules of allocation in the former system are straight-forward (rules 7, 8, 10 and 11), and it is the latter system that raises interesting problems. The conflicting principles of distribution according to " work " and according to " needs " were specifically examined in terms of maximizing aggregate social welfare. The following are the main conclusions.

(1) Distribution purely according to " needs " tends to result in an under-allocation of labour in the cooperative enterprise, and that purely according to " work " tends to produce an over-allocation of it.

(2) Optimization requires a mixed system of distribution according to work and needs. More specifically, the proportion of income to be distributed according to work should equal the ratio of the elasticity of output with respect of labour *to* the relative share of cooperative income in the value of total output. The correspondence between this rule for a cooperative enterprise with the result of competitive equilibrium is striking, but some differences are also noted.

(3) An exception to conclusions (1) and (2) is provided by the case when there is complete " social consciousness ", i.e. in the case in which every individual attaches the same weight to his own happiness as he does to that of everyone else. In this case, a system of distribution according to work, or one according to needs, or *any* mixture of the two, produces the optimum allocation of labour.

(4) Barring the special case discussed in (3), the optimum proportion to be distributed according to needs, or according to work, is completely independent of the amount of sympathy that the members of the cooperative have for each other (i.e. is independent of their " social consciousness ").

(5) A corollary of conclusion (4) is that the optimal distribution rule, which closely corresponds to the competitive result, is not influenced by the existence or not of " external effects " in the shape of concern for each other. An incidental observation is that to show the optimality of competitive equilibrium, all " external effects " do not have to be ruled out, as is the practice in the now classic presentation of " New Welfare Economics ".[1] Existence of concern for each other's happiness is seen to be harmless for the optimality of competitive allocation, provided there is symmetry (strictly defined as " symmetric sympathy " and " symmetric goodwill ") in the pattern of such external effects. Zero external effects amount to no more than a *special case* of that symmetry.

So much about the conclusions. Now about the assumptions underlying the analysis. Some assumptions are easily removable, e.g. that the cooperative does not own any resource other than land. Even if the cooperative owns some other resource (e.g. capital goods) along with (or without) land, it makes no difference to the results. Some other assumptions are serious but not especially odd in this branch of economics. We have assumed well-behaved utility and production functions; automatic fulfilment of the second order conditions of welfare maximization and of equilibrium; no uncertainty; perfectly competitive markets; homogeneity of labour; and other assumptions commonly employed in this field.

We have also abstracted from some of the more perplexing problems in the practical

[1] Perhaps the best presentations are to be found in Lange [9], and Koopmans [8].

running of a cooperative. In particular, we have avoided the complexities of resource allocation when different members have differently shaped utility functions, or have different degrees of " social consciousness " (or of " social goodwill "), if we take the social welfare function given by (3.2) as opposed to (3). These are relatively restrictive assumption. However, both *symmetric sympathy* and *symmetric goodwill* are aggregate constraints and do not impose any detailed pattern of actual sympathies (a_{ij}). The values of (a_{ij}) can vary in many manner within the two sets of linear constraints.[1]

The nature of the social welfare function used, given by (3), (3.1) and (3.2), is also open to question. There are, first of all, the general difficulties of the impossibility of a social ordering based on individual orderings satisfying the set of conditions postulated by Arrow [1]. This problem we avoided by deliberately violating Arrow's condition of " the independence of irrelevant alternatives ".[2] The use of *cardinal* individual welfare to arrive at a social ordering always violates this condition.[3] The acceptability of the particular social ordering used depends on our assessment of the relevance of the condition of the " independence of irrelevant alternatives ". Secondly, even within this general framework, the use of the Marshallian method of simply *aggregating* unweighted individual welfare indices may be found objectionable. However, its high intuitive appeal (from Bentham onwards) is an argument for the retention of this simple formula. Thirdly, non-utilitarian considerations are excluded from the social welfare function, which is a limiting assumption.

Similar problems arise with the individual welfare functions also. The general homogeneous, linear form (equation 2) of individual welfare based on the set of individual utilities is open to challenge. While Harsanyi [4] has shown that this is the only acceptable form for an individual's judgments about social welfare, if a number of highly appealing postulates have to be fulfilled,[4] an individual may not act, even in a cooperative, to maximize what he recognizes to be the social welfare of that cooperative, as opposed to his own welfare.

However, even if we do not restrict ourselves to the linear form used in equation (2), our analysis need not require substantial change. Consider W^j in the more general form below:

$$W^j = W^j(U^1 . U^2, \ldots, U^N), \text{ with } 0 \leqq \frac{\partial W^j}{\partial U^i} \leqq 1, \qquad \ldots(2.2)$$

$$\frac{\partial^2 W^j}{\partial U^{i2}} \leqq 0, \text{ and } \frac{\partial W^j}{\partial U^j} = 1.$$

Equation (12) will still give the first order conditions of individual equilibrium if we interpret a_{ij} as the partial derivative of W^j with respect of U^i, i.e. $a_{ij} = \frac{\partial W^j}{\partial U^i}$. The assumptions of

1 There are $2(N-1)$ linear constraints given by symmetric sympathy and symmetric goodwill.

$$\sum_{i=1}^{N} a_{i1} = \sum_{i=1}^{N} a_{i2} = \ldots = \sum_{i=1}^{N} a_{iN} \qquad \ldots(4.2)$$

$$\sum_{j=1}^{N} a_{1j} = \sum_{j=1}^{N} a_{2j} = \ldots = \sum_{j=1}^{N} a_{Nj}. \qquad \ldots(5.2)$$

The set of (a_{ij}), in number N^2, has to satisfy these $2(N-1)$ linear equations.

2 An alternative way out of the " impossibility " problem is to assume that the individuals have a certain pattern of " similarity ", e.g. " single peaked preferences ", or more generally " value restricted preferences " (see Arrow [1], Inada [7], and Sen [18]).

3 On this see Arrow [1], Chapter III.

4 See Theorem V in Harsanyi [4]. Note that when Harsanyi speaks of " social " preferences, he " always mean[s] preferences based on a given individual's value judgment concerning ' social welfare ' " ([4], p. 310). Incidentally, to justify that equation (2) is of the right form for W^j using Harsanyi's proof, we may assume that (*a*) W^j for each j satisfies Marschak's Postulates I, II, III′ and IV; (*b*) U^i for each i satisfies the same postulates, and (*c*) if each U^i is the same in two situations, W^j must have the same value in both the situations.

symmetry will need to be redefined to take account of the variability of a_{ij}. *Symmetric sympathy* can be defined as equal " social consciousness ", i.e. equations (4.1), *whenever* the utilities of all individuals are equal. Similarly *symmetric goodwill* will now require the fulfilment of (5.1) in a situation of equal utility. It is easy to check that our main results stand even with this more general definition of the individual welfare function (2.2) instead of the homogeneous, linear form (2) used in the foregoing analysis.

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