Chapter 11

Some Mathematics of Administering Intergenerational Global Justice

People of all time and all places have equal rights to natural opportunities. If the world is in a steady state with no memory of earlier times, the administration of this principle is a simple matter. The world simply collects the rent from all exclusive use of natural opportunities and uses the proceeds to pay a uniform dividend to all persons in all time and all places. If there is a fund that has been accumulated from rent on opportunities that no longer exist, the dividend is augmented by the per capita interest on this fund.

If the world is not in a steady state and people have uniform preferences, matters are a little more complicated. No generation has the right to claim a greater income from nature than it leaves for future generations. Each generation has an obligation to forecast the future and pay itself a dividend of less than rent plus the interest on previously accumulated rent, if this is necessary to ensure that every future generation will have access to natural opportunities as valuable as the ones to which they had access. The per capita value derived from nature may justly rise over time, but it may not justly fall.

In determining whether future generations will be adequately compensated for the decline in per capita natural opportunities, the present generation may justly include a forecast of the value that future generations will receive from innovations that are not available to the present generation. That is, future generations can be compensated for a decline in per capita rent either by creating a fund of capital for them or by providing innovations for them.

The use of innovations to compensate for diminutions in per capita rent creates additional complications when people have different preferences, since an innovation that compensates some people may not compensate others. In deciding how much of a dividend they can justly pay themselves, if the current generation is counting on future generations to be compensated by improvements in technology they have an obligation to ensure that people with all preferences are compensated, setting aside additional money for the compensation of those who are less fully compensated by the technology.

Yet additional complications are introduced by the possibility of an unjust past. The claims of individuals in the present generation on wealth accumulated in the past are valid only if injustices committed in the accumulation of that wealth have been righted. Thus any just generation must appropriate all of the wealth that

was acquired in the past by unjust private appropriations of natural opportunities and use it to provide at least partial compensation for those past injustices. If (as seems likely) there is not enough money to compensate for all past injustices, a reasonable way to select the ones that shall be compensated is to compensate for all injustices since time t, where t is the most distant time in the past such that compensation for all injustices since then can be provided from the appropriated resources. If there is something left after compensation has been provided for all past injustices, then that money should used to increase future dividends for all persons in all times by the same amount.

These ideas are elaborated mathematically in the analysis that follows. Define:

t, τ	Time
i	Individuals
N	Population
R	Flow of rent from natural opportunities
r	Rate of return on investment
A	Accumulated stock of capital that can be used for compensation
V	Utility as an indirect function of income and prices
D	The part of rent used for citizens' dividends
d	The dividend for each citizen; $d = D/N$
C	Compensation
I_{it}	The private asset income of citizen i at time t
Y_{it}	The total non-wage income of citizen i at time t, equal to $I_{it} + d_t + C_{it}$
P	The vector of prices, relative to the numeraire of the annual rent of one "standard remote acre of agricultural land"

The flow of rent comes from enduring assets such as location, the frequency spectrum and geosynchronous orbits; renewable assets such as ocean fish and topsoil; exhaustible assets such as hydrocarbon deposits; and payments for such externality-generating activities as pollution and adding to the population. All components of rent are to be managed with optimal pricing (price = marginal cost). Because of the chosen numeraire, the citizens' dividend is measured in the number of acres of standard remote agricultural land that one can rent for a year with the dividend.

At time t, there may be an accumulated stock of compensation capital, A_t, purchased with past rent that was not used for contemporaneous citizens' dividends. The first just generation, at $t = j$, should start the fund of compensation capital by appropriating all of the wealth of $t = j$ that had been accumulated by past unjust appropriations of natural opportunities. When not used for compensation or augmented by undistributed rent, A grows at the interest rate, r.

Citizen i has an indirect utility function $V_i(Y_i, P)$.

Generation t decides how much of the flow of rent should be used for a citizens' dividend and how much should be used for an addition to the stock of compensation capital. For this decision, they forecast future prices, future population, and the joint distribution of wages and preferences for future citizens. They forecast out to time S, when the economy reaches a steady state, with population N_S, rent R_S, compensation capital A_S, and interest rate r_S.

For the steady state, the present generation initially calculates a dividend financed only by rental payments in the steady state, with no provision for a dividend in the steady state from any accumulated capital. Thus the initial dividend for everyone in the steady state is R_S/N_S. Every person receives the same dividend even though people have different preferences and wages.

Now consider year $S-1$, the year before the steady state is reached. Define $P_{t\tau}$ as the prices that would have prevailed at time t if the technology of that time had been that of prior time τ. Define $C_{it\tau}^*$ as the smaller of

1. the decline in d from time τ to time t and
2. the compensation $C_{it\tau}$ for person i at time t such that $V_i(I_{it} + d_t + C_{it}, P_{tt}) = V_i(I_{it} + d_\tau, P_{t\tau})$,

but not less than 0. Thus $C_{iS,S-1}^*$ is the smaller of the amount by which the per capita dividend falls from time $S-1$ to time S and the compensation for person i at time S such that person i is as well off with that compensation as he would have been with the dividend of time $S-1$ and the prices that would have prevailed with the technology of time $S-1$, but not less than 0. $C_{iS,S-1}^*$ is the compensation that justice requires for person i for the decline in per capita dividends from time $S-1$ to time S. Taking account of the fact that money set aside at time $S-1$ will earn a return of r_{S-1} in year $S-1$ and a return of r_S in the steady state, the amount of money that must be set aside at time $S-1$ to pay such a dividend throughout the steady state is

$$\frac{\sum_{i=1}^{N} C_{iS,S-1}^*}{(1+r_{S-1})r_S}$$

A dividend at time $S-1$ is just only if it permits this much money to be set aside for future compensation after paying the dividend. The dividend that should be paid in year $S-1$ is the largest dividend that permits such future compensation. The permitted dividend is positive as long as rent is positive, because no compensation is required if there is no dividend. Now that year $S-1$ is taken into account, people receive different amounts of compensation in the steady state, since people with greater non-wage income and/or greater wages are likely to benefit more from improvements in technology and therefore require less compensation when dividends fall.

Next, consider year $S-2$. The amount of money that must be set aside for compensation in year $S-1$ in view of the dividend paid in year $S-2$ is

$$\frac{\sum_{i=1}^{N} C_{iS-1,S-2}^{*}}{(1+r_{S-2})}.$$

The additional money that must be set aside for compensation in the steady state in view of the dividend paid in year $S-2$ is

$$\frac{\sum_{i=1}^{N} \max\left(0, C_{iS,S-2}^{*} - C_{iS,S-1}^{*}\right)}{(1+r_{S-2})(1+r_{S-1})r_S}.$$

In words, enough money must be set aside to make each person as well off in the steady as he would be with the dividend of time $S-2$ and the prices that would have prevailed with the technology of time $S-2$, taking account of the fact that compensation for the decline in dividends from $S-1$ to S has already been provided.

Next, consider year $S-3$. The amount of money that must be set aside for compensation in year $S-2$ in view of the dividend paid in year $S-3$ is

$$\frac{\sum_{i=1}^{N} C_{iS-2,S-3}^{*}}{(1+r_{S-3})}.$$

The additional money that must be set aside for compensation in year $S-1$ in view of the dividend paid in year $S-3$ is

$$\frac{\sum_{i=1}^{N} \max\left(0, C_{iS-1,S-3}^{*} - C_{iS-1,S-2}^{*}\right)}{(1+r_{S-3})(1+r_{S-2})}.$$

The additional money that must be set aside for compensation in the steady state in view of the dividend paid in year $S-3$ is

$$\frac{\sum_{i=1}^{N} \max\left[0, C_{iS,S-3}^{*} - \max\left(C_{iS,S-2}^{*}, C_{iS,S-1}^{*}\right)\right]}{(1+r_{S-3})(1+r_{S-2})(1+r_{S-1})r_S}.$$

In words, enough money must be set aside to make each person as well off in the steady as he would be with the dividend of time $S-3$ and the prices that would have prevailed with the technology of time $S-3$, taking account of the fact that the compensation already provided for him in the steady state is the greater of the amount of money needed to compensate him for the decline in dividends from time

$S-2$ to time S and the amount of money needed to compensate him for the decline in dividends from time $S-1$ to time S.

Proceeding to year t, the amount of money that must be set aside for compensation in year $t+1$ in view of the dividend paid in year t is

$$\frac{\sum_{i=1}^{N} C_{it+1,t}^{*}}{(1+r_t)}.$$

For $\tau = t+2$ to $S-1$, the additional money that must be set aside for compensation in year τ in view of the dividend paid in year t is

$$\frac{\sum_{i=1}^{N} \max\left[0, C_{i\tau t}^{*} - \max_{u=t+1}^{\tau-1}\left(C_{i\tau u}^{*}\right)\right]}{\prod_{v=t}^{\tau-1}(1+r_v)}.$$

In words, for each future year τ, from two years into the future until the year before the steady state, enough money must be set aside to make each person as well off in year τ as he would be with the dividend of time t and the prices that would have prevailed with the technology of time t, taking account of the fact that the compensation already provided for him in year τ is the greatest amount of money needed to compensate him for the decline in dividends for any year from $t+1$ to year $\tau-1$, and of the fact that money set aside for compensation will earn interest in each year at that year's rate of return on capital. For the steady state, the compensation must last indefinitely, so it is

$$\frac{\sum_{i=1}^{N} \max\left[0, C_{iSt}^{*} - \max_{u=t+1}^{S-1}\left(C_{iSu}^{*}\right)\right]}{\left[\prod_{v=t}^{S-1}(1+r_v)\right] r_S}.$$

The planned pattern of dividends and compensation for year t and beyond provides that in any year τ after t, the combination of dividend and compensation that any person receives in year τ makes that person as well off in year τ as he or she would have been in year τ with the combination of dividend and compensation of any year u between t and τ and the prices that would have prevailed in year τ with the technology of year u.

The plan above is made under the assumption that $A_t = 0$, that is, that there is no accumulated compensation capital in year t. If $A_t > 0$ the plan must be modified to dispose of A_t. The first claim on A_t is from those whose present and future claims have been diminished by past injustice.

There are two types of past injustice that might be corrected. First, there are persons whose wealth is lower today because they or their ancestors were denied equal access to natural opportunities. Second, there are persons who would have been better off now or in the future with past rent per capita, even if they had only the technology of the past, because they attach little value to technological improvements.

If the amount of money A_t is not great enough to redress all such injustices, then some method of assigning priority must be employed. I suggest that injustices be given priority according to how recent they are. Thus those planning the disposition of A_t would first examine year $t - 1$ and ask how much money would be needed to compensate any persons who did not receive full shares of the dividend that could have been justly distributed from the rent in year $t - 1$. Then they would ask how much additional money for years t and beyond would be needed to compensate those who place little value on technology, for the reduction in dividends and planned compensation from year $t - 1$ to the steady state. If there was enough money to compensate for all such injustices of both types, funds from A_t would be allocated for these purposes, and year $t - 2$ would be addressed in a similar manner. If there was not enough money to redress all of the injustices of year $t - 1$, then all of the money in A_t would be used to compensate the same fraction of all injustices of the year. Injustices would continue to be redressed as far back as possible. If all such past injustices had been redressed and there was still money available, all future dividends would be increase by the same amount.

A valuable property that such a plan possesses is time-consistency. That is, if a plan is made at time t and money is collected and dispersed at time t according to the plan, and then a new plan is made at time $t + 1$ according to the same rules, and the things that were predicted in time t happened as predicted and no new injustices were discovered, then the plan that is made at time $t + 1$ looks exactly like the plan made at time t. If people in year t make predictions that are not fulfilled, then a plan made at time $t + 1$ will differ from the plan made at time t, but the revised plan of time $t + 1$ will still be the just allocation of the available resources.