THE MATHEMATICS OF LAND VALUES

with an analysis of the effects of speculation on the selling price of land

by Mike Trigg

CHAPTER I

INTRODUCTION

Mr. Smith, who owns no land, wants to build a house. He finds a suitable location, an empty lot belonging to Mr. Jones. After several offers and counter-offers, Jones sells the vacant lot to Smith for $20,000. Why this particular amount? Why is Smith willing to pay $20,000 but no more? Why is Jones unwilling to part with the lot for less? In short, what are the economic considerations that determine the selling price of land?

This is a complex question. How are we to approach it scientifically? A physicist trying to unravel the laws of nature does not begin by studying Einstein. He begins with simple "idealized" situations. He ignores the theory of relativity. He pretends that he never heard of quantum mechanics. He leaves a thousand other things out of his calculations. He concentrates on one small part of the total picture and masters that. Only then does he add another factor to his calculations. As his understanding develops he considers increasingly complicated situations. By proceeding from the simple to the complex, the physicist eventually arrives at a description of the world that is amazingly accurate. Moreoever, the latest physical theories are able to explain complicated situations that really do exist. The same approach can be used in studying the question of land values.

Thus, we will begin by ignoring such things as land speculation, monopoly and fluctuating interest rates. The emphasis throughout will be on the mathematical relationships between ground rent, market value and so on. After developing the appropriate formulas for simple "idealized" situations we will consider the effect of land speculation. The physicist would say, "We will then put another parameter into the equations." Our discussion begins with some basic definitions.

QUESTIONS FOR DISCUSSION

1. A, B, C, D and E are scientists investigating the laws of falling bodies.

   a) Scientist A, after making a number of experiments, formulates the following "law of falling bodies": An object initially at rest relative to the surface of the earth will be falling at a velocity of 32 feet per second after one second, 64 feet/sec after two seconds, 96 feet/sec after three seconds, and so on. How fast will a body be falling at the end of four seconds? After five seconds? Can you write a formula involving v (velocity) and t (time in seconds) which summarizes this law?

   b) Scientist B drops an object from a great height. It hits the ground five seconds later, but it's final velocity does not agree with Scientist A's law. B suspects this is due to friction with the air. He adds a "drag coefficient" to the law of falling bodies.

   c) Scientist C drops an object from an airplane high in the atmosphere. The final velocity does not agree with B's formula - even with the drag coefficient thrown in. He adds another term to the law to account for the fact that the drag is less at high altitudes because the air is thinner.

   d) Scientist D drops an object from a rocket ship a quarter of a million miles from earth. Results do not agree with any of the previous laws. He includes a term to account for the lesser pull of gravity at great distances.

   e) Scientist E releases a balloon filled with helium six feet above the surface of the Earth. Instead of falling it rises! He amends the law of falling bodies to read: "All bodies move according to D's law of falling bodies except for helium baloons."

What did A ignore when he formulated his law? What did B ignore? What did C ignore? Do you think A, B and C are good scientists? What do you think of E's law involving helium balloons? Does it seem like a useful generalization? Do you think science will ever discover a perfect law of falling bodies - valid in all circumstances? Do you think we will ever find a law "close enough for all practical purposes"?

2. Astronomers cannot make experiments on the stars and galaxies. Yet, astronomers claim to know a lot about the universe. How do you explain this?

3. It is difficult to make large-scale experiments in economics. Yet we can formulate theories and make observations. Compare this with the situation in astronomy.

4. Scientists frequently state the "laws of nature" in the form of mathematical formulas. Do you think such a formula, no matter how complex, should ultimately agree with common sense?

5. Comment on the following statement: Modern economists rely heavily on mathematics; therefore, they must know what they are talking about.

6. Mark Twain wrote a witty fantasy called, "Eve's Autobiography." In it, Adam and Eve are making "scientific" experiments to determine "how the milk gets into the cow." They finally conclude that the cow must absorb milk from the air. They already knew, somehow, that air contained water vapor - which they symbolized as H_2O. After the cow experiment they amended this formula to read: H_2O, M. Twain was poking fun at science. Do you think his satire is entirely fair? What failing of human behavior is Twain trying to point out to us?

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7. Let $x = 4$ and $y = 3$. Evaluate the following expressions.

a) $x + y$

b) $x^2 + y^2$

c) $\frac{1}{x + y}$

d) 5% of x

e) $xy + 1$

f) 4% of x^3

g) $y + y^2 + y^3 + y^4 + y^5$

h) $\frac{1}{(x-y)^2} + \frac{1}{(x-y)^3}$

8. Factor the following expressions:

a) $3x + 3y$

b) $5A + 7A$

c) $MR + PR + TR + MQ + PQ + TQ$

d) $u^2 + bu$

9) Solve the following equations for M

a) $5 = M + 30$

b) $x = M + Y$

c) $Z = 5M$

d) $10Z = 5M + 15$

e) $3M + TM = M - 1$

f) $J = (A + B)M - \frac{M}{J}$

10) The law of simple interest states that I = PRT where,

P = amount of principle

R = the annual interest rate

T = time in years

a) What is the interest on $10 invested for one year at 6% (use R = .06 in the formula)?

b) What is the interest after two years? Five years? One hundred years?

11) A certain geometric sequence of numbers is defined as follows:

$a_1 = 2$, $a_2 = 2^2$, $a_3 = 2^3$, and so on. Calculate a_4 and a_5.

12) The T-sequence is defined by the rule: $t_n = n^2 + 1$. Find the first 7 terms of this sequence. Also, calculate t_{100}.

13) The Fibonacci Sequence is defined thus: $F_1 = 1$, $F_2 = 1$, otherwise $F_{n+2} = F_{n+1} + F_n$. In other words, each Fibonacci Number is the sum of the two preceeding Fibonacci Numbers. Find the first eight values of F_n.

CHAPTER II

CAPITALIZATION RATE

We will assume, at the outset, that there is a well-defined thing called the Capitalization Rate. The capitalization rate is defined to be the annual rate, on the average, at which a reasonable and prudent investment of capital will earn money on the open market. It roughly corresponds to what is commonly called the prevailing interest rate. We assume Henry George's definition of capital: wealth used to produce more wealth. Thus, investments in land are specifically excluded by this definition. (The return to land investment is rent.) The capitalization rate is determined by the general state of the economy - it is not something set by law or necessarily determined by such things as prime interest rates. For example, if putting money in a savings and loan institution at 6% for one year is as good an investment as any, then we could conclude that the capitalization rate (at this time) is about 6%.

Throughout most of this discussion we will assume that the capitalization rate stays constant from year to year. It is quite probable, for example, that a general public collection of ground rent would tend to make the capitalization rate go up; but, we will ignore this possibility for now.

> A note on percentages: when we say, "take 5% of x" we really mean "multiply x by 5/100." In general, a percentage must always be divided by 100 before we can use it in a formula. Query: What is 7% of 10 dollars? Answer: $\frac{7}{100} \times 10.00 = \$0.70$. If we let C denote the capitalization rate then C is equal to the percentage number over 100. For example, if the capitalization rate is 4% then C = .04.

We will now use the capitalization rate to derive the law of compound interest. A specific example may be helpful. Assume a capilization rate of 5% (C = .05). If you invest a thousand dollars, how much will you have at the end of one year? Answer: $1000 + .05 \times 1000$ or \$1050. How much will you have after two years? Since you have \$1050 at the end of the first year, at the end of the second year you will have 1050 + .05 × 1050 or \$1102.50.

Let us extend this result to get a general formula. Let M_0 denote the original amount of the investment. Let M_1 be the value after one year; M_2 be the value after two years, and so on.

after one year: $M_1 = M_0 + CM_0 = (1+C)M_0$

after two years: $M_2 = M_1 + CM_1 = (1+C)M_1$

$= (1+C)(1+C)M_0 = (1+C)^2 M_0$

after three years $M_3 = M_2 + CM_2 = (1+C)^3 M_0$.

In general, we have the formula for calculating the value of an investment after n years (where n stands for any number).

$$M_n = (1+C)^n M_0$$

This is the well-known law of compound interest. For instance, we can now calculate how much our one thousand dollar investment would be worth at the end of 10 years (where C = .05).

$$M_{10} = (1+.05)^{10}\ 1000 = (1.05)^{10}\ 1000 = 1.62889 \times 1000 = \$1,628.89$$

Thus, after 10 years the original investment has earned \$628.89.

We will see that the capitalization rate is always one of the factors that determine the selling price of land. Why should this be so? It is because the potential buyer of a piece of land must say to himself, "If, instead of paying this amount of money for the land, I were instead to invest it otherwise, would I be able to make the same amount of money?" That is to say, investors have two different ways of making money - they can invest in land (and collect rent), or they can invest their capital (and collect economic interest). Since each party in a transaction will try to maximize his return, there is necessarily a relation between the selling price of land and the Capitalization Rate.

QUESTIONS FOR DISCUSSION

1. It could be argued that no one really knows what the capitalization rate is; so, it is incorrect to base a formula on it. Consider the following imaginary dialogue.

Joe: I have some capital I would like to loan to you for a year. Will you agree that the current capitalization rate is about 10% and pay me for the use of my capital accordingly?

Moe: Certainly not! I don't even believe there is such a thing as a capitalization rate. However, your capital would be useful. I'll give you $50 to use it because I am a nice guy.

Joe: That is ridiculously low. I can easily find somebody who will give me more than that.

Moe: That is probably true. I'll give you $200 but that's my last offer. If you insist on talking about this imaginary capitalization rate, I will point out that this corresponds to a rate of 4%.

Joe: So, you do admit that such a thing exists!

Moe: I didn't say that. If such a thing did exist, however, your rate is much too high. Would you take $250?

Joe: I admit it. My original offer was too high. Frankly, I have checked and haven't done better. You've got a deal. We seem to have agreed that the capitalization rate is about 5%.

Moe: I admit nothing. However, I am a businessman and I am satisfied with our deal.

Can you see why there must be a capitalization rate? Could capitalists operate without some idea of what it is?

2. What do you think happens to the value of C during boom times? During a depression? Following the opening up of a new frontier? Following a period of intense land speculation? In 1852, where do you think the capitalization rate was higher - New York or California? In which direction did capital tend to flow in that year?

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Assume a value of C = .04 in the following two problems.

3. Mrs. White buys a lot from Mr. Black for $10,000. She sells it one year later for $11,000. Who was the more astute business person?

4. Mrs. White then tries to buy Mr. Green's lot. Green wants $11,000 for it. White knows she will be able to sell this lot in one year for $11,500. Should she buy?

5. Use the law of compound interest to fill in the blanks in the following table. M is the amount of the original investment. The number of years elapsed is listed in the column headed "n."

M	Cap. Rate	n	Final Value	Hints
100.	5%	1		$(1.05)^1 = 1.05$
100.	5%	2		$(1.05)^2 = 1.1025$
100.	1%	2		$(1.01)^2 = 1.0201$
50.	10%	3		$(1.1)^3 = 1.331$
1.	2%	100		$(1.02)^{100} = 7.2446....$
10.	60%		$40.96	$(1.6)^n = 4.096$
100.		2	$108.16	$(1+C)^2 = 1.0816$
	30%	3	$439.40	$(1.3)^3 = 2.197$
100.	100%	5		$1 + 1 = 2$

CHAPTER III

PRIVILEGE

Before deriving a formula for land value, we will digress and discuss a more general question: If we know the capitalization rate, can we calculate the selling price of privileges in general? The answer is manifestly "yes." To demonstrate this we will use several specific examples and calculate their value. To keep things simple we will assume a Capitalization Rate of 5% (C = .05) throughout this section.

Example 1: Brown is a potter who sells his wares through local gift shops. Jones is a street artist with a permit to sell his work on certain busy downtown streets. These permits are no longer available from the city, but it is legal to sell your permit to another artist. Jones is willing to sell his permit and Brown wants to buy it. With a permit, Brown estimates that he could make $100 a year more than he is making now - with the same amount of work. What is the most that Brown should be willing to pay Jones for the permit.

Basically, what Brown must do is to say to himself, "How much money would I have to invest on the open market to make $100?" This is the same as saying, "One hundred dollars is 5% of what?" In symbols:

$$.05x = \$100$$

$$x = \frac{100}{.05} = \$2{,}000$$

This is the most that Brown should be willing to pay; and Jones, if he is a good businessman, will be unwilling to sell for much less.

Two thousand dollars may seem like a lot for a mere permit - but consider Brown's alternatives. If he simply invested the $2,000 his income will be $100. This is the same as the extra income he would realize by using his $2,000 to buy the permit. The two ways of investing are equally attractive.

Example 2: Wallace is an independent licensed cab driver. Richards wants to buy his cab license. He calculates that by driving a cab he will be able to make $500 a year more than he is making now. How much will Richards be willing to pay Wallace for the privilege of owning a cab license. Going through the same kind of reasoning as above, we have: $.05x = 500$ or $x = \$10,000$. This is approximately what Wallace will demand and about what Richards will pay (Cab Licenses in San Francisco, as of 1976, went for about $18,000!).

Example 3: Mr. Jenks is retired. He has some savings and would like to invest in some "income producing" property. He has found a piece of unimproved land which would be ideal for farming. Moreover, a local farmer has said that he would be glad to rent the land from Jenks for $2,250 a year. After paying his property tax, Jenks would still make $1,750. About how much will Mr. Jenks pay for the privilege of owning this land? Answer: $35,000. (Land doesn't come cheap!)

Do you see how the capitalization rate enters into the selling price of <u>any</u> privilege? The investor simply calculates how much he will be ahead after all expenses and then divides by C. Brown's expenses were his pottery materials. Richards had to figure in the operating expenses of his cab. Mr. Jenks had to pay property tax. What do you think raising the property tax would do to the selling price of land?

We can summarize this way of estimating the purchase price of a privilege by the following simple formula, where P denotes the purchase price:

$$P = \frac{\text{annual yield}}{\text{capitalization rate}}$$

Do not make the mistake of assuming that both buyer and seller want to transact business at this value of P. What really happens is this: The buyer is willing to pay anything up to P, but no more (since his effective rate of return would then fall below the current capitalization rate). The seller will sell for anything down to P, but no less (because if he reinvests the receipt from the sale, he wants the annual return to be no less than the annual return to the privilege he now holds.)

QUESTIONS FOR DISCUSSION

1. Based on the examples of this chapter, who is the bestower of most privileges? Do you think that privilege is a good idea? Is not privilege a necessary consequence of the fact that not everyone can do the same thing - drive a cab for instance?

2. Can you think of any other privileges habitually conferred on individuals by the government? Are these privileges usually worth money?

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Assume a capitalization rate of 5% in the following exercises. In each case, assume that the privilege can be held in perpetuity. Assume also that the owner is allowed to resell his privilege on the open market anytime he wishes to do so.

3. Max Young is a bartender and business manager of the Happy Hour Tavern. He has an opportunity to buy the liquor license from the present owner. Max calculates that if he owned the license he could make $1,000 more a year than he is making now, and still be doing the same work. How much will Max pay for this privilege?

4. In a certain eastern kingdom the royal tax collector has the traditional privilege of levying a "head tax" on every subject in the kingdom. This tax must be paid under penalty of death. Typically the total revenue each year is about one million dollars - about 10% of which the royal tax collector quietly pockets. The king has decided to raise money for the royal treasury by selling this position to the highest bidder. What is the most the king can get for selling this privilege?

5. If you can secure exclusive patent rights to a certain invention, you can expect to net $30,000 a year in royalties (forever!). What is the most you should pay a firm of patent lawyers to nail down your claim?

As a matter of fact, patent rights usually expire after a specified number of years. What effect does such a restriction have on the value of a patent?

6. Use the following three equations to find one equation which expresses M in terms of G, R, and C. That is, N and T must not appear in the final equation.

$$T = RM \qquad N = G - T \qquad M = N/C$$

The following example involves a privilege that only lasts one year. The only return expected is an increased selling price.

7. Mr. J. Snively is the corrupt zoning commissioner of Sun City. There is a piece of land in Sun City owned by the Titanic Land Development Company with a present market value of $10,000. This land is currently zoned as "industrial." Titanic has asked for a zoning variance so that they can develop a housing tract. Such a variance would make the land worth about $115,000 one year later. How big a bribe should Snively demand from Titanic to push through the zoning variance?

CHAPTER IV

LAND VALUES WITHOUT SPECULATION

We are now in a position to find a formula for the market value of land. If we disregard speculation we will see that the market value depends on just three things: The gross annual rent, the property tax rate and the capitalization rate.

Throughout this section we are considering the "property tax" only as it affects unimproved land. In today's world property taxes usually have two components - a tax on land and a tax on improvements. We will ignore the effect of the latter for now. In addition, this section will assume that the property tax is calculated as a percentage of the market value of land. Although this corresponds to present day practice, the reader should not infer that this is the only method by which the public could collect economic rent. In fact, we shall come to a surprising conclusion. Under such a system the government can never get all the rent - no matter how high the property tax rate!

Our formula for calculating the Market Value of land will be based on the following simple definitions:

Gross Annual Ground Rent (G). The amount of money which a landholder can get from a tenant in one year for the use of a piece of land as such (i.e., the economic rent). The ground rent may be actual (the land is engaged in production), or it may be potential (the landholder is keeping the land out of production for the purpose of speculation).

Annual Tax (T). The amount of money which the landholder must give to the government for the privilege of "owning" his land. This tax is paid even if the land is not engaged in production.

Net Annual Ground Rent (N). That portion of the ground rent which is left in the hands of the landholder after the tax has been paid. In symbols:

$$\boxed{N = G - T}$$

Tax Rate (R). The rate used by the government to determine the Annual Tax on a piece of land with a given Market Value. (The tax rate is usually expressed as a percentage and should therefore be divided by 100 to get the value of R in the formula below.) In symbols, where M is the Market Value:

$$\boxed{T = R \times M}$$

Capitalization Rate (C). The rate (on the average) at which a reasonable and prudent application of capital will earn money on the open market. The Capitalization Rate has already been discussed in Chapter II.

Market Value of Land (M). In Chapter III we showed how the selling price of any privilege can be calculated if we know the capitalization rate. In the case of land, it is simply the Net Annual Rent divided by the capitalization rate. That is:

$$\boxed{M = N/C}$$

In other words, the higher the Net Annual Rent, the higher the Market Value (obviously); and the higher the Capitalization Rate, the lower the Market Value (for when C gets very large, no one will wish to invest in land).

To gain some further insight into this formula we will consider two cases. Assume that you have an amount of money M and wish to invest it.

1) If you invest your money as capital for one year you will make C X M dollars.

2) If, on the other hand, you buy a piece of land for M dollars, collect one year's rent (N) and then sell it at the end of the year for the same price, you will make N dollars. But, the formula above shows that N = C X M dollars. Thus, you will make the same amount of money with either kind of investment. This relationship necessarily holds if both the buyer and the seller of land are rational men. The landowner would be foolish to charge less than N/C for his land, and the buyer will be unwilling to pay more.

We can now combine all our formulas to get:

$$M = M\frac{R + C}{R + C} = \frac{M \times R + M \times C}{R + C} = \frac{T + N}{R + C} = \frac{G}{R + C}$$

In words:

$$\text{Market Value} = \frac{\text{Gross Annual Ground Rent}}{\text{Tax Rate + Capitalization Rate}}$$

We will refer to this formula as the "static formula for land values."

In the table below the effects of various tax rates on a given piece of land are compared. Assume that the Capitalization Rate is 5% in every case.

Gross Rent (G)	Tax Rate (R)	Market Value (M)	Tax (T)	Net Rent (N)
$1,050.	0% = 0.	$21,000.	$ 0.	$1,050.
1,050.	2% = 0.02	15,000.	300.	750.
1,050.	5% = 0.05	10,500.	525.	525.
1,050.	10% = 0.10	7,000.	700.	350.
1,050.	20% = 0.2	4,200.	840.	210.
1,050.	100% = 1.0	1,000.	1,000.	50.
1,050.	200% = 2.0	512.	1,024.	26.

Summary of Formulas

a) $N = G - T$

b) $T = R \times M$

c) $M = \frac{G}{R + C}$

Notice how rapidly the Net Rent (and hence the Market Value) goes down as the Tax Rate goes up. For example, when the Tax Rate is merely equal to the Capitalization Rate (R = C), the government collects half of the Economic Rent.

Notice that this method of calculating property taxes leads to the paradoxical conclusion that the government can never collect all of the economic rent. One might expect, for instance, that a tax rate of 100% (R = 1.0) would get it all, but the table shows that this is not so. There is nothing wrong with the mathematics; in fact, this same conclusion is dictated by common sense. For, if we confiscated all of the rent the Market Value would be zero. There must be a tax base if we are to calculate the tax at all. A trillion percent of zero is still zero!

Nevertheless, this method of collecting taxes will get most of the ground rent. At this point one might adopt one of two positions:

1) It is all right to leave some rent in the hands of the "landowner." This small fee is fair compensation for the labor he expends in managing his land, After all, he does have to collect rent, pay taxes, keep books and so on.

2) Don't use this method to collect the tax. Simply figure out what the economic rent is and confiscate it all. This has the necessary consequence that land will have a market value of zero.

In the next section we will "add in another parameter" namely land speculation. As Henry George says in Book IV of "Progress and Poverty": "We have discovered the statics of the problem - the dynamics we have yet to seek."

QUESTIONS FOR DISCUSSION

1. What, according to Henry George, is the "mother of all privileges"? Does this privilege, in fact, often have a high selling price?

2. Do you think the high selling price of land in cities is a deterrent to private enterprise? One frequently hears statements like, "Business is leaving the city because of high property taxes." Comment.

3. Do you think it would be all right to leave some economic rent in the hands of landholders? If not, why not? What other ways of implementing the Georgist remedy can you think of?

4. Based on the "static formula," I find a piece of land in San Francisco worth $50,000. When I go to a realtor I find that the asking price is $200,000. Explain.

5. Property in California is, by law, assessed at 25% of real market value. The so-called "property tax rate" in San Francisco is something like 19% of assessed valuation. What is the "real tax rate" in San Francisco, assuming assessments are accurate?

6. Assessors habitually underassess unimproved land. What effect does this have on the "real tax rate"?

7. Typically, property is not reassessed every year. Thus, there is a considerable "lag time" between rising market value and reassessment. What effect does this have on the "real tax rate"?

8. A and B have identical residential lots. Because B has a house on his lot, his tax bill is much higher than A's. Not only is his house taxed, but his <u>lot</u> is assumed to be worth more. Are A and B both paying taxes on <u>land</u> at the same rate?

9. If the California Legislature succeeds in "reducing oppressive property taxes" to aid the "beleaguered homeowner," where do you think the lost revenue will come from?

Assume that rents are not rising. Fill in the blanks in the following table.

G	R	M	T	N	C
Gross Rent	Tax Rate	Market Value	Tax	Net Rent	Cap. Rate
$ 100.	5%	$	$	$	5%
750.	2%			450.	
1,000.	1%		$200		
1,000.	0%	25,000.			
1,000.		10,000.			5%
1,000.		4,000.		200.	
3,000.		5,000.	$2,500.		
3,600.			3,000.		20%
3,300.			3,000.	300.	
4,040.				40	10%
	800%	500.			4%
	0%	80,000.		4,000.	
	5%	40,000	2,000.		
	95%		3,800.		5%
	95%		7,600.	400.	
	0%			8,000.	10%
		2,000.	0		5%
		100.	0	10.	
		1,000.		10.	1%
			1,950.	50.	5%

CHAPTER V

ONE YEAR OF LAND SPECULATION

We have found a static formula for land values. We are now ready to introduce another parameter - land speculation. As usual, we will proceed from the simple to the complex. Thus, we start out by assuming that speculation goes on for only one year! This will, in fact, be the case if we assume a piece of land for which the following is true:

The Gross Rent in year 1 is \$500
The Gross Rent in year 2 is \$750
The Gross Rent is \$750 forever after.

To make our example concrete we will also assume a Tax Rate of 2.5% and a Capitalization Rate of 5%. We will let M_o denote the original market value, M_1 denote the market value at the end of year one and so on. Also, G_n will denote the gross rent in year n. Summing up what we know so far:

$$G_1 = 500 \qquad R = .025$$

$$G_2 = 750 \qquad C = .05$$

Our task is to calculate the original market value. We start by calculating M_1, the market value at the end of one year. We will then be able to work backwards to find the original market value M_o.

Suppose you are the potential buyer at the end of year one. Your thinking will go something like this: "How much should I pay for this land? I know that the gross rent next year will be \$750. I also know that it will never go up again in the forseeable future. Therefore, I can sell this land next year, but I will not be able to sell it for any more than I am paying for it now. Thus, any profit I make must be based solely on the net rent I can make in this coming year. It has become a static situation. I will use the static formula:

$$M = \frac{G}{C + R}$$

Thus, we have our first result.

$$M_1 = \frac{G_2}{C + R} = \frac{750}{.05 + .025} = \$10,000$$

Now we consider what the selling price will be at the <u>beginning</u> of year one. In other words what is the original Market Value M_o? Now the deliberations of the buyer will be of this sort: I can no longer use the static formula because rents are expected to rise. Since rent will be higher next year, the selling price will be higher next year. Thus, as an investor in land, I can expect to make a profit from two sources: (1) I will collect rent in the coming year and (2) I can sell the land at a higher price next year.

From the first source my income will be $G_1 - T_1 = 500 - .025M_o$
From the second source my income will be $M_1 - M_o = 10,000 - M_o$
Thus, my net income for the year will be $\$10,500 - 1.025 \times M_o$

If I invested this same amount of money M_o at the current capitalization rate, my income would be $.05 \times M_o$. These two ways of making money must be equally lucrative. Thus,

$$.05\ M_o = \$10,500 - 1.025M_o$$

Now we can solve this equation to get M_o = $9,767.44(rounded to the nearest cent). Let us check this result. If I invest $9,767.44 for one year at the current capitalization rate, my income will be $488.37 (Check this) If, on the other hand, I buy the land, collect rent, pay taxes and sell the land at a profit next year I will have:

original cost of land:	$ 9,767.44	expenses
taxes for the year:	244.18	
gross rent for year:	500.00	income
selling price next year:	10,000.00	
net income:	$ 488.37	

The two kinds of investment will yield the same income. This must be the case. For if investing capital on the open market was <u>less</u> attractive than buying the land, we would conclude that the landholder was selling his land too cheaply.

And, if investing capital was <u>more</u> attractive, no one would buy the land.

Did you notice that the market value <u>this</u> year was based on knowing what the market value will be <u>next</u> year? We can use this idea to develop a formula for calculating the market value at the end of <u>any</u> given year, say, year number k. This formula will even allow for the possibility of speculation. We assume, for a moment, that we already have the market value for the following year, that is to say, we know the value of M_{k+1}. Once we know this we can compute M_k.

Again we assume that we buy a piece of land, collect rent, pay taxes and sell it for a profit one year later. Here is the bookkeeping:

original cost of land:	M_k	expenses
taxes for the year:	RM_k	
gross rent for the year:	G_{k+1}	income
selling price next year:	M_{k+1}	
net income:	$G_{k+1} + M_{k+1} - RM_k - M_k$	

This net income must equal the income one would get from simply investing M_k dollars at the current capitalization rate. Thus we have the relation:

$$CM_k = G_{k+1} + M_{k+1} - RM_k - M_k$$

Solving this equation for M_k (this year's market value) we get

$$M_k = \frac{G_{k+1} + M_{k+1}}{C + R + 1}$$

At this point we will introduce a constant Q which we define to be the reciprocal of (C + R + 1). This will save a lot of writing and make all our formulas much simpler. Thus

$$M_k = QG_{k+1} + QM_{k+1}$$

$$\text{where } Q = \frac{1}{C + R + 1}$$

The formula relates this year's market value to next year's market value. In the next chapter we will use this simple relation to develop a general formula for land values based on speculation any number of years into the future.

* * * * * * * * *

QUESTIONS FOR DISCUSSION

1. Do land speculators always make money? If the rent goes up next year, but not as much as it was expected to do, will the buyer lose money or break even?

2. Did you notice that this year's tax must necessarily be based on last year's market value? That is, $T_1 = RM_o$ and $T_2 = RM_1$. Why not base the tax on next year's market value?

3. Land speculators frequently do not even bother to collect the rent. Yet, they still make money. Explain.

4. In this chapter we assumed that the market value stabilized at the beginning of the second year. Yet, the expected rent in year 2 was higher than in year one. Why wouldn't M_3 be even larger than M_2?

5. If rents rise for 10 years, when will the market value stabilize? At the beginning of year 10 or the end of that year?

6. Recompute the example used in the text, but with R = .05 and C = .025. Compare the resulting value of M_o with the value found before. That is, compute M_o with G_1 = \$500, G_2 = \$750 and G = \$750 forever after.

7. Fill in the following table, where $Q = 1/(C + R + 1)$.

C	R	Q	Q^2
.05	.05		
.04	.04		
.01	.01		
.07	.03		
.05	.2		

8. Is Q always less than 1? Is Q^2 always less than Q? What can you say about the size of numbers like Q^7? How about Q^{50}?

9. Assume that market values stabilize at the end of year one. This means you can use the static formula $M_1 = \frac{G_2}{R + C}$. Then use the relation $M_k = QG_{k+1} + QM_{k+1}$ to find the value of M_o in each of the following cases.

R	C	Q	G_1	G_2	M_1	M_o
.05	.05		\$1,155	\$1,650		
.2	.05		1,155	1,650		
1.05	.05		1,155	1,650		
0	.05		1,155	1,650		

CHAPTER VI

LAND VALUES WITH SPECULATION

We are now ready to treat land speculation in its full generality. We will project our calculations n years into the future. However, we must make one last assumption before letting n sail off into infinite posterity. We will assume, for the moment, that land values rise for n years and then finally stabilize. Later, we will relax even this restriction and show how our formulas can correctly calculate present land values even if speculation goes on forever!

Our assumption means that we can immediately calculate the market value at the end of year n. We simply use the static formula. That is

$$M_n = \frac{G_{n+1}}{R + C} \tag{1}$$

Now we will work backwards, year by year, until we finally have M_o, the original market value. All we need is the formula developed in the previous chapter. Thus, if we know the market value in any future year, say year k + 1, we can immediately get the market value in year k. The formula is repeated here for convenience.

$$\begin{aligned} M_k &= QG_{k+1} + QM_{k+1} \\ \text{where} \quad Q &= 1/(C + R + 1) \end{aligned} \tag{2}$$

Now, we know the market value in year n. Thus, we know the market value in year n - 1 based on equation (2).

$$M_{n-1} = QG_n + QM_n \tag{3}$$

One might object at this point and say, "Wait a minute - you are basing your calculations on knowing future rents." The answer to that is: this is the only thing that land speculation can be based on. Ultimately, all market values are based on expected rents. We must assume that the land speculator has a good estimate of G_1, G_2, and so on. Speculators are businessmen - not fortune tellers.

We now know M_n and M_{n-1}. We use formula (2) again to find M_{n-2}.

$$\begin{aligned} M_{n-2} &= QG_{n-1} + QM_{n-1} \\ &= QG_{n-1} + Q(QG_n + QM_n) \\ &= QG_{n-1} + Q^2G_n + Q^2M_n \end{aligned} \tag{4}$$

Applying (2) repeatedly we get (check those for yourself):

$$M_{n-3} = QG_{n-2} + Q^2G_{n-1} + Q^3G_n + Q^3M_n \tag{5}$$

$$M_{n-4} = QG_{n-3} + Q^2G_{n-2} + Q^3G_{n-1} + Q^4G_n + Q^4M_n \tag{6}$$

⋮

and so on

All such formulas can be summarized in one equation. We let r stand for any number between 1 and n.

$$M_{n-r} = QG_{n-r+1} + Q^2G_{n-r+2} + \ldots + Q^rG_n + Q^rM_n \tag{7}$$

Don't let this abstract looking formula scare you. Try plugging values of r equal to 1, 2, 3 and 4 into the equation. You will automatically get equations (3), (4), (5) and (6), respectively.

Now we let $r = n$ in equation (7) to get our final result. M_{n-n} is simply M_o. That is, M_o is just the market value n years before year number n. Thus,

$$M_o = QG_1 + Q^2G_2 + Q^3G_3 + \ldots + Q^nG_n + Q^nM_n \tag{8}$$

The market value in year n still appears in our formula. We can get rid of it by substituting in the known value of M_n from equation (1).

$$M_o = QG_1 + Q^2G_2 + Q^3G_3 + \dots + Q^nG_n + Q^n\left(\frac{G_{n+1}}{C + R}\right) \tag{9}$$

$$Q = 1/(C + R + 1)$$

Finally we have it! Equation (9) exactly expresses M_o in terms of the capitalization rate, the tax rate, and expected future rents. To figure out the current market value, we simply choose a value of n as large as we please and apply this formula. We will refer to formula (9) as the "dynamic formula for land values".

One might suppose that the dynamic formula will do us no good. After all, we can speculate a thousand years into the future and we would still be leaving out an infinite number of terms. Where does it all end? How many years ahead do we have to look to get an accurate value? In the next section we will show that this objection is easily overcome. In fact, we will show even more: The dynamic formula tells the whole story.

QUESTIONS FOR DISCUSSION

1. In our derivation of the dynamic formula we started with a known value of the land in year n and worked backwards to year zero. It would seem less confusing to start with a known value in year zero and work forward. Why can't this approach be taken?

2. Do you think our analysis of land values is complete yet? What factors might we still be ignoring?

3. It was stated in the text that market prices are ultimately based on expected future rents. Next year's rent may be known with certainty - the following year's rent is less certain. What can you say about the expected gross rent 10 years from now? How about 50 years from now?

4. Do you think the theory of probability might be needed to make a thorough analysis of land values? Why?

5. Considering exercise 8 in Chapter V, what can you say about the size of the "higher terms" in the dynamic formula?

* * * * * * * * * * * * *

6. Using only equation (2) of this chapter, fill in the blanks in the following chart. You will use the equation three times. Round out to the nearest cent when necessary.

Assume R is 4% and C is 6%.

M_3	G_3	M_2	G_2	M_1	G_1	M_o
\$464,000	\$46,000		\$44,000		\$35,000	

The following optional problems require a little more mathematical sophistication.

7. The Geometric Series: Let $S_n = a + ar + ar^2 + \dots + ar^n$. A sum such as this is called a geometric series. (It is understood that n is a positive integer.) For any particular n, the number S_n is said to be "the sum of the first n terms of the geometric series." It is easy to find a simple formula for S_n.

$$S_n = a + ar + ar^2 \dots + ar^n$$
$$rS_n = ar + ar^2 + ar^3 + \dots ar^{n+1}$$

Subtracting the top series from the bottom one we get

$$rS_n - S_n = ar^{n+1} - a$$

For all the terms in the middle cancel out. This equation is easily solved for S_n.

$$\boxed{S_n = \frac{a(1 - r^{n+1})}{1 - r}}$$

For example, if $a = 2$, $r = \frac{1}{2}$ and $n = 3$ we have:

$$S_3 = 2 + 2(\tfrac{1}{2}) + 2(\tfrac{1}{2})^2 + 2(\tfrac{1}{2})^3$$

Do the addition to find S_3. Then use the formula above to calculate S_3. You should get the same answer.

8. Use the geometric series formula to find a simple expression for H, where

$$H = Q + Q^2 + Q^3 + \dots + Q^n$$

Hint: Let $a = Q$ and $r = Q$.

9. Recalling that $Q = 1/(C + R + 1)$, show that $1 - Q = Q(C + R)$. Use this result to simplify the formula for H which you found in problem 8.

10. The static formula must be consistent with the dynamic formula. Thus, if all future rents are equal, the dynamic formula must reduce to the static formula. Prove it. Hint: Let $G = G_1 = G_2 = \ldots = G_n$ in the dynamic formula and use the results of the previous two problems to prove the identity:

$$M_o = \frac{G}{C + R} + Q^{n+1}$$

If we let n be very large, Q^{n+1} will be so close to zero that it can be disregarded (see problem 8, Chapter V). This shows that M_o is then given by the static formula.

AUG 23 1982

HENRY
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The Earth is the Birthright of all People

Mr. Oscar B. Johannsen
Dir., Schalkenbach Foundation
5 East 44th St.
New York, NY 10017

August 18, 1982

OJ response 9/20 - will contact you as soon as one read (no action taken)

Dear Mr. Johannsen,

Enclosed is a copy of a manuscript done by Mr. Mike Trigg here in San Francisco. Mike has spent many hours on his own studying the mathematical implications of both our present land tenure system and the effects of a land value tax. Perhaps you have seen this work, for I am not sure when it was actually written, but would you consider publishing something like this?

We find in our classes economists who are used to the traditional approach to economics which includes graphs and mathematical equations. They are sometimes impatient with theory, and would enjoy information such as that in this manuscript.

Here in San Francisco, we are now attempting to become accredited with a local university that specializes in economic degree programs. Perhaps a publication like Mike's would bring added credibility to the academia.

Of course, this is all punctuated by the fact that I have taken many courses from Mike, and know he is very willing to think through the mathematics of LVT. He is quite an asset to the Georgist ranks. I hope you too think his work will be worth publishing and making available to our members.

I look forward to your comments. Thank you.

Cordially,
Laraine Stiles
Laraine Stiles
Assistant Director

3410 19th Street   San Francisco   California 94110   Tel: (415) 864-1014